

Determining the Maximum Lateral Displacement Due to Severe Earthquakes without Using Nonlinear Analysis

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Abstract—For Seismic design, it is important to estimate, maximum lateral displacement (inelastic displacement) of the structures due to severe earthquakes for several reasons. Seismic design provisions estimate the maximum roof and storey drifts occurring in major earthquakes by amplifying the drifts of the structures obtained by elastic analysis subjected to seismic design load, with a coefficient named “displacement amplification factor” which is greater than one. Here, this coefficient depends on various parameters, such as ductility and overstrength factors. The present research aims to evaluate the value of the displacement amplification factor in seismic design codes and then tries to propose a value to estimate the maximum lateral structural displacement from severe earthquakes, without using non-linear analysis. In seismic codes, since the displacement amplification is related to “force reduction factor” hence; this aspect has been accepted in the current study. Meanwhile, two methodologies are applied to evaluate the value of displacement amplification factor and its relation with the force reduction factor. In the first methodology, which is applied for all structures, the ratio of displacement amplification and force reduction factors is determined directly. Whereas, in the second methodology that is applicable just for R/C moment resisting frame, the ratio is obtained by calculating both factors, separately. The acquired results of these methodologies are alike and estimate the ratio of two factors from 1 to 1.2. The results indicate that the ratio of the displacement amplification factor and the force reduction factor differs to those proposed by seismic provisions such as NEHRP, IBC and Iranian seismic code (standard no. 2800).

Keywords—Displacement amplification factor, Ductility factor, Force reduction factor, Maximum lateral displacement.

I. INTRODUCTION

ESTIMATING the maximum lateral displacement of the structures in the wake of massive earthquakes is considered to be widely important for seismic design. These include: estimating minimum separation joint width to avoid pounding, estimating maximum storey drifts to avoid destruction of non-structural elements and performance of p-delta analysis.

Due to economic reasons, the present seismic codes often allow structures to undergo inelastic deformations in the event of strong ground motions. Consequently, the demand lateral strength is lower than the required strength to maintain the structure in the elastic range.

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The demand lateral strength is obtained by dividing the required fully elastic strength to the force reduction factors (R). As such, the displacement (or drifts), calculated by analyzing structures under the lateral design force is not the real displacement, rather it is less than the maximum structural displacement during strong motions.

Seismic design provisions, estimate the maximum roof displacement and storey drifts by augmenting the displacement and drifts obtained from elastic analysis by displacement amplification factor (C_d):

$$\Delta_{\max} = \Delta_e \times C_d \quad (1)$$

Where Δ_{\max} is the maximum inelastic displacement (roof or storey drifts), Δ_e is the displacement calculated by elastic analysis and C_d is the displacement amplification factor. Since, C_d depends on force reduction factor (R), therefore, it is important to evaluate the ratio of both displacement amplification factor (DAF) and force reduction factor (FRF). Here, two approaches are selected to evaluate C_d/R ratio. The first approach generally determines the ratio of C_d/R for all structures, but the second approach, which is just acceptable for R/C moment resisting frames, obtains the values of C_d and R separately and then calculates their ratios.

II. FIRST APPROACH: DIRECT EVALUATION OF THE RATIO

Fig. 1 shows the actual response envelope and idealized elasto-plastic response curves and the followed by three quantities [1]:

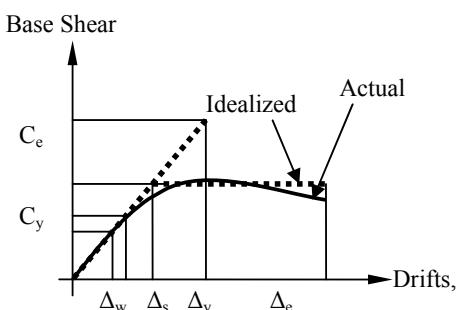


Fig. 1 General Structural Response

$$\mu = \Delta_{\max} / \Delta_y \quad (2)$$

$$R_\mu = C_e / C_y \quad (3)$$

$$R_s = C_y / C_s \quad (4)$$

Where μ = system ductility factor, R_μ = ductility reduction factor, R_s = structural overstrength factor, Δ_y = system yield displacement, C_e = fully elastic base shear ratio, C_y = yield strength level and C_s = first significant yield level base share ratio. It is shown [1] that the force reduction and displacement amplification factors, for working stress design case, can be expressed as:

$$R = R_\mu \times R_s \times Y \quad (5)$$

$$C_d = \mu \times R_s \times Y \quad (6)$$

$$Y = C_s / C_w \quad (7)$$

Where Y denote allowable stress factor applied for working stress design and, C_w is corresponding design force level. The force reduction and displacement amplification factors, for ultimate strength design case, could be expressed as:

$$R = R_\mu \times R_s \quad (8)$$

$$C_d = \mu \times R_s \quad (9)$$

Where the value of Y is equal to one. ($C_w = C_s$). Considering the equations (5) to (9), the ratio of C_d and R is thus:

$$\frac{C_d}{R} = \frac{\mu \times R_s}{R_\mu \times R_s} (\text{or } \frac{\mu \times R_s \times Y}{R_\mu \times R_s \times Y}) = \frac{\mu}{R_\mu} \quad (10)$$

Equation (10) shows that the ratio of C_d/R for ultimate strength design and working stress design are the same hence; it would be better to evaluate the ratio of μ/R_μ instead of C_d/R . It is concluded that the ratio of C_d/R depends on the ductility factor and the parameters affecting R_μ , such as system ductility factor, fundamental period of structures, material load-displacement models, damping ratio, site effects and the characteristics of earthquake (PGA, duration and frequency contents) [2].

A. Determination of μ/R_μ ratio

With regard to equation (10), it is thus feasible to evaluate the ratio of μ/R_μ instead of C_d/R . Several formulas have been suggested as force reduction factor (R_μ) by Newmark and Hall [2], Riddell [3], Krawinkler [4] and Miranda [5]. The

R_μ factor, proposed by Newmark-Hall, depends on ductility factor (μ) and structural fundamental period (T):

$$R_\mu = \sqrt{2\mu - 1} \quad T \leq 0.5 \quad (11)$$

$$R_\mu = \mu \quad T > 0.5$$

At the same time, ductility factor and structural fundamental period also affect on the formula proposed by Riddell:

$$R_\mu = 1 + \frac{R^* - 1}{T^*} \quad T \leq T^* \quad (12)$$

$$R_\mu = \mu \quad T > T^*$$

Where R^* and T^* are determined from the table proposed by Riddell in terms of system ductility factors.

Krawinkler's factor depends on fundamental period of system (T), ductility factor (μ) and strain hardening ratio (α). It is assumed the value of strain hardening ratio is equal to zero in this paper:

$$R = [C(\mu - 1) + 1]^{1/C} \quad (13)$$

$$C = \frac{T^a}{T^a + 1} + \frac{b}{T}$$

According to Krawinkler's Table, when $\alpha=0$, the values of a , and b are equal to one and 0.42 respectively.

The force reduction factor as suggested by Miranda depends on ductility factor (μ), structural fundamental period (T), predominant period of the ground motion (T_g) and site characteristics. The current paper thus applies the formula for rock sites as:

$$R_\mu = \frac{\mu - 1}{\Phi} + 1 \geq 1 \quad (14)$$

$$\Phi = 1 + \frac{1}{10T - \mu T} - \frac{1}{2T} \exp\left[-\frac{3}{2} \left(\ln T - \frac{3}{5}\right)^2\right]$$

Fig. 2 to 11 show the ratio of μ to R_μ (C_d/R) (or DAF/FRF) which have been calculated in terms of μ and T. Fig. 2 indicates relationship between the ratio of C_d/R and system fundamental period as determined by aforementioned formulas. Based on Fig. 2, several conclusions can be drawn. The minimum value of C_d/R is about 0.85, extracted by Miranda equation. The maximum value for C_d/R is 1.35, which is related to the equations of Miranda and Krawinkler. It is found that the ratio of C_d/R is high when the value of the period is low and becomes equal to one when the period is high.

Figs. 3-6 show the ratio of C_d/R , which was computed using equations and has already been explained above for $\mu=3, 4, 6$ and 8 respectively. The minimum value for all cases is approximately 0.8 . The value increases with increasing ductility factor. The ratio of C_d/R is higher than one when the fundamental period is less than 0.7 sec.

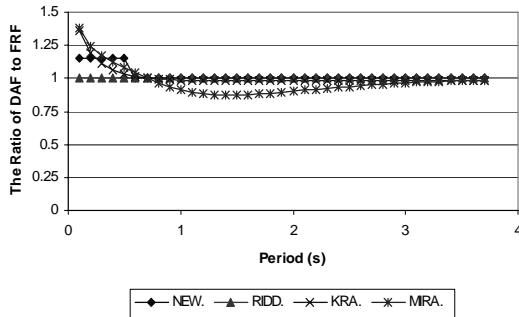


Fig. 2 The ratio C_d/R versus fundamental period for system ductility factor=2

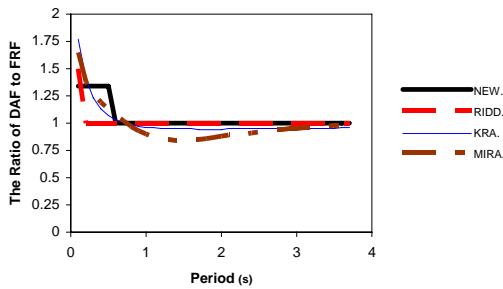


Fig. 3 The ratio C_d/R versus fundamental period for system ductility factor=3

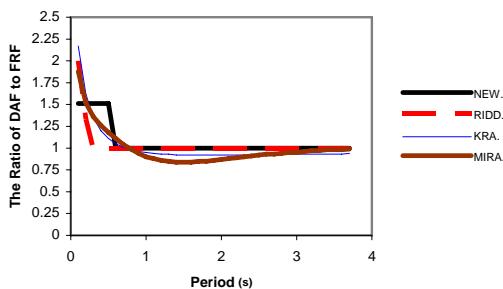


Fig. 4 The ratio C_d/R versus fundamental period for system ductility factor=4

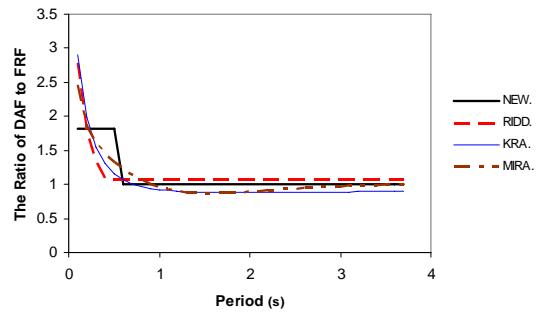


Fig. 5 The ratio C_d/R versus fundamental period for system ductility factor=6

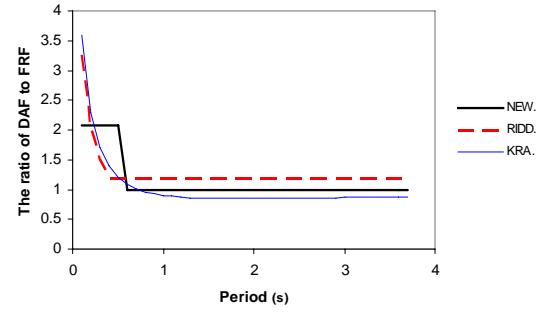


Fig. 6 The ratio C_d/R versus fundamental period for system ductility factor=8

Fig. 7 highlights the variation of the C_d/R in terms of ductility factor for $T=0.1$. As such, the figure shows that the ratio strongly depends on ductility factor. The minimum value is one for all formulas. Figs. 8 to 11 show the relationship between the ratio C_d/R in terms of ductility factor computed as $T=0.3, 0.5, 1$, and 4 sec. The minimum value for the ratio C_d/R in Figs. 8 and 9 is one and equal to 0.9 and 0.85 in Figs. 10 and 11, respectively. Meanwhile, the maximum value of the ratio C_d/R increase with increasing ductility factor and decreases with increasing T .

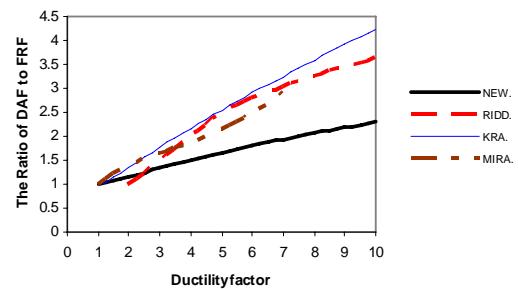


Fig. 7 The ratio C_d/R versus system ductility factor for fundamental period=0.1 sec.

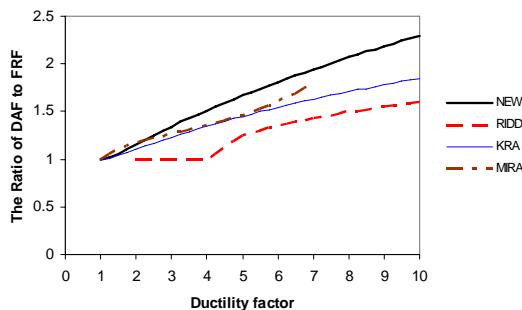


Fig. 8 The ratio C_d/R versus system ductility factor for fundamental period=0.3 sec.

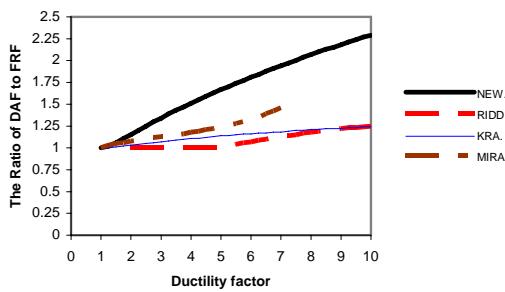


Fig. 9 The ratio C_d/R versus system ductility factor for fundamental period=0.5 sec.

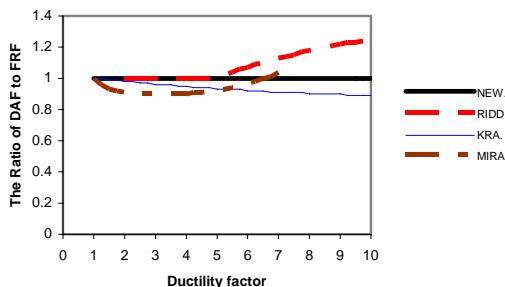


Fig. 10 The ratio C_d/R versus system ductility factor for fundamental period=1 sec.

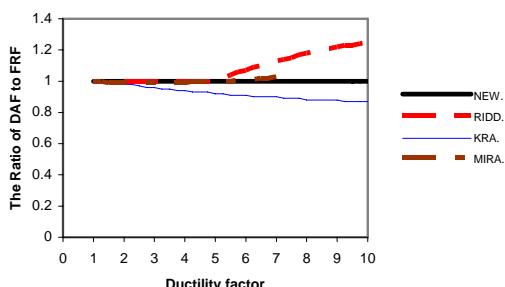


Fig. 11 The ratio C_d/R versus system ductility factor for fundamental period=4 sec.

III. SECOND APPROACH: SEPARATE EVALUATION OF C_d AND R FACTORS

In this approach, values of the displacement amplification factor and the force reduction factor are determined using equations (5) and (6) separately and then their ratios are obtained.

$$R = R_\mu \times R_s \times Y \quad (5 \text{ repeat})$$

$$C_d = \mu \times R_s \times Y \quad (6 \text{ repeat})$$

The present research evaluates the values of C_d and R for R/C moment resisting frames hence; it is necessary here to calculate the values of ductility factor, overstrength factor, force reduction factor and safety factor. The calculations and their values for structures with one to fifteen stories are presented in [6], [7].

A. Determination of the Force Reduction Factors

Step 1: determination of members rotational ductility factors

Rotational ductility factors for beams and columns (μ_l) is expressed as follow:

$$\mu_l = \frac{\theta_u}{\theta_y} \quad (15)$$

Where θ_u is the ultimate rotation for plastic hinges and its values will be selected according to FEMA (Federal Emergency Management Agency) shown in Table I [8].

TABLE I

THE VALUES OF θ_u

| Members | θ_u |
|---------|------------|
| Beams | 0.02 |
| Columns | 0.015 |

θ_y is the yield rotation and calculated according to FEMA-273. In Table II the values of μ_l is presented for all frames (beams and columns separately) [8].

TABLE II

THE VALUES OF MEMBERS ROTATION DUCTILITY (μ_l)

| Frames (no. of stories) | Beams | Columns |
|----------------------------|-------|---------|
| 1 | 2.24 | 4.01 |
| 2 | 3.34 | 8.7 |
| 3 | 2.85 | 7.14 |
| 4 | 4.26 | 9.6 |
| 5 | 4.64 | 8.1 |

| | | |
|----|------|------|
| 6 | 4.54 | 9.6 |
| 8 | 4.26 | 15.0 |
| 10 | 4.22 | 11.8 |
| 15 | 3.64 | 15.0 |

Step 2: Calculation of global ductility factors

The relationships between global ductility factor (μ_g) and local ductility factor (μ_l) for R/C moment resisting frames are as follow [2]:

A- Relation between global ductility and beams ductility factor:

$$\mu_g = a_b(\mu_{lb} - 1.4) + 1 \quad (16)$$

$$a_b = 0.21 + \frac{2.4}{N} - \frac{1.13}{N^2} \quad (17)$$

B- Relation between global ductility and columns ductility factor:

$$\mu_g = a_c(\mu_{lc} - 1.4) + 1 \quad (18)$$

$$a_c = 0.085 + \frac{0.57}{N} \quad (19)$$

Where N is the number of stories, μ_{lb} and μ_{lc} are the critical beam ductility and critical column ductility respectively in moment resisting frames. The minimum value of μ_g in equations 16 to 19 is as the global ductility factor of frames (μ_g). Table III shows the values of global ductility factors for all frames.

Step 3: determination of R_μ

Using μ_g determined in step 2 and assuming stress hardening (α) equal to 0.02 (a=1, b=0.37) and using equations 13, the force reduction factors due to ductility (R_μ) are calculated. The values of (R_μ) is presented in Table IV.

TABLE III
GLOBAL DUCTILITY FACTORES

| Frames (no. of stories) | μ | Frames (no. of stories) | μ |
|----------------------------|-------|----------------------------|-------|
| 1 | 2.24 | 6 | 2.47 |
| 2 | 3.19 | 8 | 2.42 |
| 3 | 2.29 | 10 | 2.45 |
| 4 | 2.86 | 15 | 2.82 |
| 5 | 2.33 | | |

TABLE IV
FORCE REDUCTION FACTORES DUE TO DUCTILITY

| Frames (no. of stories) | R_μ | Frames (no. of stories) | R_μ |
|----------------------------|---------|----------------------------|---------|
| 1 | 1.77 | 6 | 2.49 |
| 2 | 2.57 | 8 | 2.48 |
| 3 | 2.13 | 10 | 2.32 |
| 4 | 2.72 | 15 | 1.87 |
| 5 | 2.3 | | |

Step 4: Calculation of overstrength factor

In ref. [2], the overstrength factor (R_s) was determined for all frames. The values of overstrength factors is presented in Table V.

TABLE V
OVERSTRENGTH FACTORES

| Frames (no. of stories) | R_s | Frames (no. of stories) | R_s |
|----------------------------|-------|----------------------------|-------|
| 1 | 1.61 | 6 | 1.35 |
| 2 | 1.45 | 8 | 1.30 |
| 3 | 1.36 | 10 | 1.29 |
| 4 | 1.43 | 15 | 1.26 |
| 5 | 1.40 | | |

Step 5: Determination of safety factor

The value of safety factor depends on amplification load factor and strength reduction factors. For example, in ACI89 code, the value of safety factor equals to 1.4 ($1.1 \times 1.7 \times 0.75 = 1.4$).

Step 6: Evaluation of the force reduction factors

In the previous sections the items affecting on force reduction factors (R_μ , R_s and Y) were determined. Using these factors, the values of R is calculated according to equation 5 and shown in Table VI.

B. Determination of the displacement amplification factors

The values of displacement amplification factors (C_d) is obtained using equation 6. The values of C_d is shown in Table VI. The ratio of the force reduction factors and the displacement amplification factors is presented in Table VI too.

TABLE VI
FORCE REDUCTION FACTORS, DISPLACEMENT AMPLIFICATION FACTORS AND THEIR RATIO

| Frames (no. of stories) | R | C_d | C_d/R |
|----------------------------|------|-------|---------|
| 1 | 3.99 | 5.05 | 1.26 |
| 2 | 5.21 | 6.48 | 1.24 |
| 3 | 4.06 | 4.35 | 1.07 |
| 4 | 5.45 | 5.73 | 1.05 |
| 5 | 4.52 | 4.57 | 1.01 |
| 6 | 4.70 | 4.68 | 0.99 |
| 8 | 4.51 | 4.4 | 0.98 |
| 10 | 4.18 | 4.06 | 0.97 |
| 15 | 3.29 | 3.22 | 0.98 |

Table VI shows the ratio of displacement amplification factor to force reduction factor is equal to one approximately, except for buildings having short period.

IV. SEISMIC PROVISION OF C_d/R

This section deals with C_d/R as recommended by NEHRP, IBC and Iranian seismic code (standard no. 2800). Table VII lists the maximum and minimum values of C_d/R ratio, given by NEHRP [9].

TABLE VII
NEHRP- RECOMMENDED MAXIMUM AND MINIMUM RATIO OF C_d AND R

| Structural systems | Maximum value | Minimum value |
|---|---------------|---------------|
| Bearing wall system | 1 | 0.62 |
| Building frame system | 1 | 0.5 |
| Moment resisting frame system | 0.92 | 0.69 |
| Dual system with a special moment frame ... | 0.85 | 0.5 |
| Dual system with an intermediate moment frame ... | 0.9 | 0.64 |
| Inverted pendulum structures seismic force resisting system | 1 | 1 |

Table IIX indicates the maximum and minimum values for C_d/R as recommended by IBC code [10].

TABLE VIII
IBC- RECOMMENDED MAXIMUM AND MINIMUM RATIO OF C_d AND R

| Structural systems | Maximum value | Minimum value |
|---|---------------|---------------|
| Bearing wall system | 1 | 0.67 |
| Building frame system | 1 | 0.5 |
| Moment resisting frame system | 0.92 | 0.69 |
| Dual system with a special moment frames | 0.93 | 0.5 |
| Dual system with an intermediate moment frames | 0.91 | 0.75 |
| Inverted pendulum structures seismic force resisting system | 2 | 0.5 |

Iranian seismic code uses only one value for C_d/R which is equal to 0.7 [11]. Based on Tables VII and IIX, it is observed that the ratio of C_d/R is never larger than one except for inverted pendulum structures seismic force resisting systems.

V. CONCLUSION

The present study has applied two methodologies to discuss, in detail, about the displacement amplification factor and the force reduction factor. In the first methodology, which is applied for all structures, the ratio of displacement amplification factor and force reduction factor is determined directly. Whereas, in the second methodology that is just applicable for R/C moment resisting frames, two factors are

calculated separately and then their ratio have been obtained. The results from first method indicate that the minimum value for the ratio of displacement amplification and force reduction factors is 0.8. It showed that the minimum value increases with increasing ductility factor and decreasing fundamental periods. The ratio C_d/R could be much higher than 1.0 for ductile frame systems (high ductility) and stiff buildings (low fundamental period). Meanwhile, the ratio C_d/R went up to more than 2.5 for low period systems.

The results acquired from the second methodology show that the C_d/R ratio is about 1.0 except for short period buildings. With reference to the results from these two approaches, it seems that the values for displacement amplification factors to be down estimated, especially for buildings with short period. The C_d factors, recommended by NEHRP, IBC and Iranian seismic code (standard no. 2800) are low and therefore they need to be modified, especially for buildings having high ductility and low periods.

In order to calculate the maximum lateral displacement, the modified values of displacement amplification factor can be suggested as:

$$C_d = 1.2 R \quad T < 0.5 \text{ sec.}$$

$$C_d = R \quad T \geq 0.5 \text{ sec.}$$

Where, T=structural fundamental period and R=force reduction factor.

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