International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:6, No:11, 2012

Analysis of S.P.O Techniques for Prediction of Dynamic Behavior of the Plate

Byung-kyoo Jung, and Weui-bong Jeong

***Abstract*—**In most cases, it is considerably difficult to directly measure structural vibration with a lot of sensors because of complex geometry, time and equipment cost. For this reason, this paper deals with the problem of locating sensors on a plate model by four advanced sensor placement optimization (S.P.O) techniques. It also suggests the evaluation index representing the characteristic of orthogonal between each of natural modes. The index value provides the assistance to selecting of proper S.P.O technique and optimal positions for monitoring of dynamic systems without the experiment.

***Keywords*—**Genetic algorithm, Modal assurance criterion, Sensor placement optimization.

I. Introduction

IT is important to know dynamic behavior of structures due to the structural vibration to improve product quality with regard to human comfort, operation and safety in noise and vibration. There are various methods to identify the structural vibration but commonly an experimental method and numerical methods such as finite element model (FEM) are used. In general, the experimental method is much more accurate than numerical analysis methods to define the structural vibration of the machine, however, there are limitations on time and equipment cost. So reducing the number of sensors in the experiment is a common problem encountered in many engineering applications. For this reason, many scholars were studying about the sensor placement optimization (SPO) to get more accurate monitoring results and minimize the number of sensors required.

Krammer (1991) presented the effective independence method (EFI) to select optimal sensor placement by the evaluation of contribution about the independence in target modes [1]. Larson (1994) suggested the eigenvalue vector product (EVP) method preventing the choice of sensors placed on nodal lines of a target mode and to maximize their deflection energy [2]. Imamovic (1998) made the method, called average driving point residue (ADPR), selecting the largest energy position in target modes by using eigenvalues and eigenvectors [3]. EFI-DPR, a hybrid of EFI and ADPR, was developed by Worden (2001) [4]. Algorithms to define sensor placement were also progressed. Especially, the genetic algorithm (GA), as a globally optimal method, was commonly applied to this problem by many researchers [5]-[7].

W. B. Jeong is the professor of Mechanical Engineering, Pusan National University, Republic of Korea (e-mail: wbjeong@pusan.ac.kr).
B. K. Jung is with School of Mechanical Engineering, Pusan National University, Republic of Korea. (e-mail : only417@hanmail.net)

These studies were concerned with methods to get more accurate modal parameters such as natural frequency, natural mode and modal damping in measurement points [8]-[9]. However, these studies did not evaluate whether selected sensor positions are good to monitor of dynamic behavior or not without the experiment. For this reason, this paper suggested the evaluation index for a proper decision of sensor placement optimization to predict dynamic behavior due to the structural vibration.

II. Evaluation Index for S.P.O

A. The Object Structure

In this paper, the simply supported aluminum plate was using as an object structure of sensor placement optimization. The finite element model has 651 nodes, if you choose 10 nodes as sensor position to get structural vibration responses, there are ${}_{651}C_{10}$ cases. So it is impossible to select optimal positions by just user's instinct. Therefore, it needs techniques for sensor placement optimization.

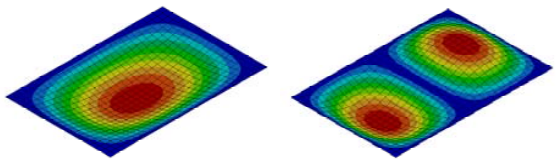

Fig. 1 Natural modes (a) 1st mode (b) 2nd mode

Table I summarizes the first to tenth natural frequencies of this plate and Fig. 1 (a), (b) represents the first and the second natural mode.

TABLE I
NATURAL FREQUENCIES

Mode No.	Frequency	Mode No.	Frequency
1st	*43.6851 Hz*	*6th*	*242.5169 Hz*
2nd	*84.0456 Hz*	*7th*	*247.9148 Hz*
3rd	*135.2536 Hz*	*8th*	*290.3842 Hz*
4th	*151.8781 Hz*	*9th*	*329.9188 Hz*
5th	*175.2408 Hz*	*10th*	*337.8960 Hz*

B. Evaluation Index

Since the structural vibration responses consist of super-position of several modes, if modes composed of values at sensor positions are represented well, we can get reliable structural vibration results which also correspond to goal of sensor placement optimization. Thus, this paper suggested the evaluation index using modal assurance criterion (MAC) [10]. MAC is given by the following formula.

$$MAC_{ij} = \frac{\left| \left\{ \varphi_i^{ref} \right\}^* \left\{ \varphi_j^{cal} \right\} \right|^2}{\left(\left\{ \varphi_i^{ref} \right\}^* \left\{ \varphi_i^{ref} \right\} \right) \left(\left\{ \varphi_j^{cal} \right\}^* \left\{ \varphi_j^{cal} \right\} \right)} \tag{1}$$

Here, $\left\{ \varphi_i^{ref} \right\}$ means *i-th* mode vector as reference, $\left\{ \varphi_j^{cal} \right\}$ means *j-th* mode vector got from FEM analysis. MAC represents the orthogonality between two modes and has a value between 0 and 1; it has a value of 1 at the perfectly matched case, a value of 0 at the completely different case. The evaluation index is induced by using this value and given by the expression.

$$Evaluation\ index = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} MAC_{ij} \right) - n \tag{2}$$

Here, n means the number of target modes. As the value calculated by this formula is small, modes are orthogonal by each other. Therefore, we can simply estimate whether selected points calculated by sensor placement optimization techniques are good for prediction of dynamic behavior of structures or not.

III. Sensor Placement Optimization

This paper used advanced sensor placement optimization techniques which have a constraint equation of the minimum distance between each of sensors. The equation is given by the following formula

$$Constraint : \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} > 50 \tag{3}$$

We extracted 10 of 651 points as optimal sensor placement of the plate model by EFI, EVP, EFI-DPR method and genetic algorithm, and then represented MAC figure as an evaluation index using values of selected sensor positions.

A. EFI Method

The effective independence (EFI) method selecting optimal sensor placement by estimation of contribution about the independence in target modes is given by expression.

$$\begin{aligned} [E]_{m \times m} &= [\Phi]_{m \times n} \left([\Phi]^T [\Phi] \right)_{n \times n}^{-1} [\Phi]_{n \times m}^T \\ EFI &= \{E_d\}_{m \times 1} = diagonal\left([E]_{m \times m} \right) \end{aligned} \tag{4}$$

From the formula, a component of vector $\{E_d\}$ represents contribution of the sensor about target modes, so the component which has the smallest value should be removed. This process was repeated until the number of sensors equaled to 10 to get optimal placement. Fig. 2 shows optimal sensor placement, Fig. 3 shows MAC calculated by mode vectors composed of values at selected points.

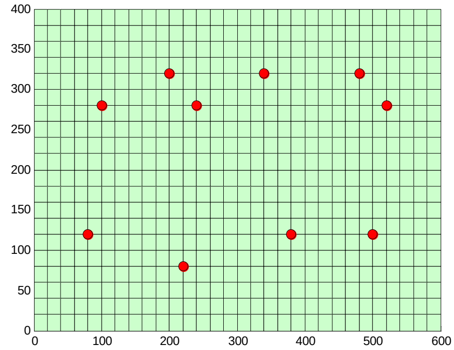

Fig. 2 Optimal sensor placement by EFI method

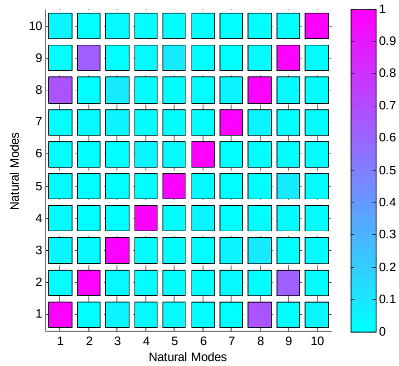

Fig. 3 MAC calculated by EFI method

B. EVP Method

The eigenvalue vector product (EVP) method is an energy based technique calculated using the following expression.

$$EVP_i = |\varphi_{i1}| \times |\varphi_{i2}| \times \cdots \times |\varphi_{in}| = \prod_{j=1}^{n} |\varphi_{ij}| \tag{5}$$

Here, subscript *i* means node number or sensor position and *j* means a number of modes. This technique prevents to select the sensor on nodal lines of a vibration mode and picks the energy maximum position. In short, it is the method to choose positions which have higher EVP values. From this method, optimal sensor positions appear in Fig. 4, MAC is showed in Fig. 5.

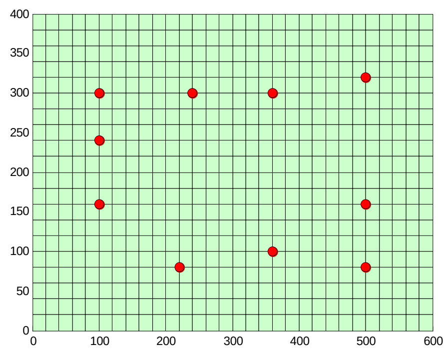

Fig. 4 Optimal sensor placement by EVP method

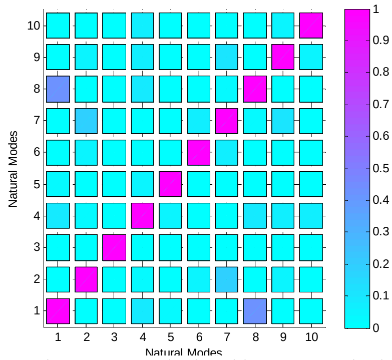

Fig. 5 MAC calculated by EVP method

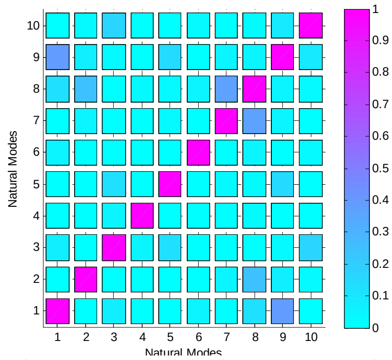

Fig. 7 MAC calculated by EFI-DPR method

C. EFI-DPR Method

EFI-DPR method, a hybrid of effective independence (EFI) method and average driving point residue (ADPR) method, is calculated by the following formula.

$$EFI\text{-}DPR_i = diagonal([E])_i \times \sum_{j=1}^{n} \frac{\varphi_{ij}^2}{\omega_j} \quad (6)$$

Here, subscript *i* means node number or sensor position and *j* means mode number. This technique is also energy based method and selects positions in higher EFI-DPR value order. Fig.6 shows optimal sensor placement, Fig. 7 shows MAC calculated by mode vectors composed of values at selected points.

D. Genetic Algorithm

The genetic algorithm (GA), as a globally optimal method, imitates evolution phenomena of natural and flow of algorithm appears in Fig. 8. In this paper, we defined the fitness function as follow expression.

$$Fitness\ function = n \times \left(\sum_{i=1}^{n} \sum_{j=1}^{n} MAC_{ij} \right)^{-1} \quad (7)$$

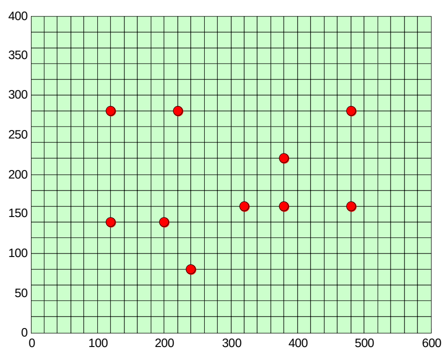

Fig. 6 Optimal sensor placement by EFI-DPR method

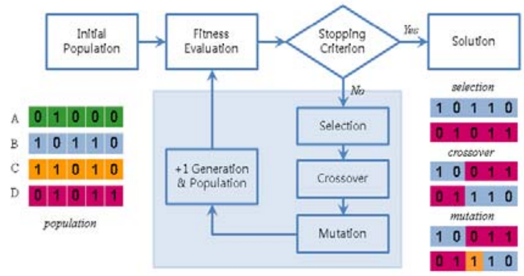

Fig. 8 Flow chart of genetic algorithm

Here, *n* means the number of target modes, *i* and *j* means specific mode. The genetic algorithm with above fitness function is finding the optimal sensor positions to minimize the summation of MAC value. In this GA, the number of population set 100, probability of crossover set 60% and probability of mutation set 1%, it was continuously repeated until all of gene became convergence. Fig. 9 shows the maximum and mean fitness value over the increase of generation, optimal sensor positions appear in Fig. 10, MAC is showed in Fig. 11.

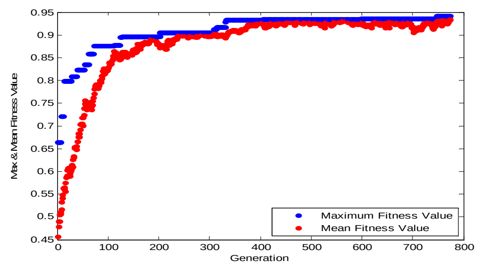

Fig. 9 Fitness value over generation

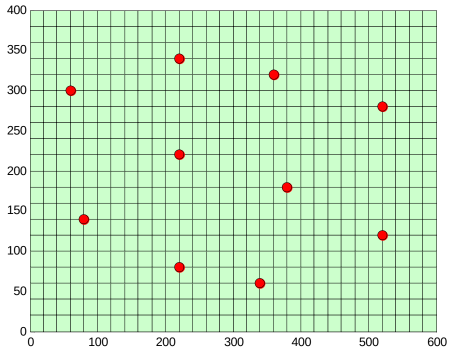

Fig. 10 Optimal sensor placement by genetic algorithm

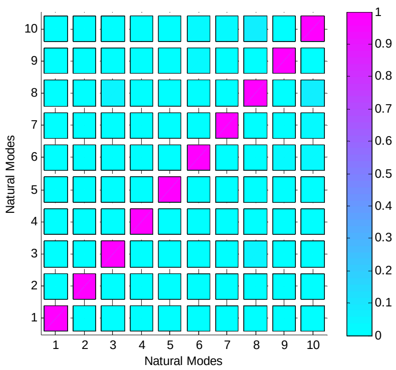

Fig. 11 MAC calculated by genetic algorithm

TABLE II
EVALUATION INDEX VALUE OF S.P.O TECHNIQUES

Method	EFI	EVP	EFI-DPR	Genetic algorithm
Evaluation Index value	*4.2140*	*3.1283*	*4.8732*	*0.9196*

Table II shows the evaluation index values computed by each of S.P.O techniques. As the diagram indicates, MAC calculated by using genetic algorithm shows good results since each of component values in Fig. 11 almost closes to zero. Unexpectedly, EFI method shows better results than EFI-DPR method. It was predicted that selected sensors were slightly concentrated in the center because of the effect of ADPR.

The procedure and these results can provide assistance to selecting of proper method and optimal positions for monitoring of the model on dynamic behavior without the experiment.

IV. Conclusion

This is a comparative study of sensor placement optimization (S.P.O) techniques for monitoring of the structural vibration using the evaluation index which is suggested in this paper. It used 4-techniques, EFI, EVP, EFI-DPR and GA, to find 10 points as optimal positions of the simply supported plate model. Modal assurance criterion was used as the evaluation index for the decision of mode vectors reconstructed by values of optimal sensor positions. From the results in previous chapter, we can predict GA for S.P.O may show better performance for monitoring of the plate model since each of MAC values computed by GA was nearly zero.

This procedure studied in the paper can apply to find suitable S.P.O techniques of other systems to monitor dynamic behavior by using the evaluation index without the experiment. Also, it is possible to increase accuracy for monitoring even though we minimize the number of sensors required.

Acknowledgment

This work was supported by Agency for Defense and Development (ADD).

References

[1] D.C. Krammer, "Sensor placement for on-orbit modal identification and correlation of large space structure," *Control dynam*, 1991.
[2] C.B. Larson, D.C. Zimmerman, E.L. Marek, "A comparison of modal test planning techniques: excitation and sensor placement using the NASA 8-bay truss," *International modal analysis conference*, 1994.
[3] N. Imamovic, "Model validation of large finite element model using test data," *Imperial College, London, UK*, 1998
[4] K. Worden, A.P. Burrows, "Optimal sensor placement for fault detection," *Engineering structures*, 2001.
[5] C. Mares, C. Surace, "An application of genetic algorithms to identify damage in elastic structures," *Journal of sound and vibration*, 1996, pp. 195-215.
[6] Deshan Shan, Zhenhua Wan, "Optimal sensor placement for long-span railway steel truck cable-stayed bridge," *Measuring technology and mechatronics automation conference*, 2011.
[7] Zhang Lianzhen, Huang Qiao, Wang Chaohai, "Optimal sensor placement based on multi-object genetic algorithm," *Engineering mechanics*, 2007, pp. 168-172.
[8] M. Meo, G. Zumpano, "On the optimal sensor placement techniques for a bridge structure," *Engineering structures*, 2005.
[9] Wei Liu, Wei-cheng Gao, "Optimal sensor placement for spatial lattice structure based on genetic algorithm," *Journal of Sound and Vibration*, 2008.
[10] R.J. Allemang, D.L. Brown, "A correlation for modal vector analysis," *International modal analysis conference*, 1982, pp. 110-116.