International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:10, 2010

Refined Buckling Analysis of Rectangular Plates Under Uniaxial and Biaxial Compression

V. Piscopo

***Abstract*—**In the traditional buckling analysis of rectangular plates the classical thin plate theory is generally applied, so neglecting the plating shear deformation. It seems quite clear that this method is not totally appropriate for the analysis of thick plates, so that in the following the two variable refined plate theory proposed by Shimpi (2006), that permits to take into account the transverse shear effects, is applied for the buckling analysis of simply supported isotropic rectangular plates, compressed in one and two orthogonal directions.

The relevant results are compared with the classical ones and, for rectangular plates under uniaxial compression, a new direct expression, similar to the classical Bryan's formula, is proposed for the Euler buckling stress.

As the buckling analysis is a widely diffused topic for a variety of structures, such as ship ones, some applications for plates uniformly compressed in one and two orthogonal directions are presented and the relevant theoretical results are compared with those ones obtained by a FEM analysis, carried out by *ANSYS*, to show the feasibility of the presented method.

***Keywords*—**Buckling analysis, Thick plates, Biaxial stresses

I. Introduction

THE buckling problem of rectangular plates, under the action of uniaxial and biaxial stresses, is generally solved applying the classical thin plate theory, in which the transverse shear deformation is neglected. Obviously, when the thickness is not negligible, as regards the plate dimensions, the classical theory becomes not totally appropriate and a new formulation, that permits to take into account the shear effects, is required. In the past, to overcome this lack, different shear deformable theories were presented by several authors, such as Reissner [7], Mindlin [8], Levinson [9], Reddy [10], Shimpi [2].

In the following the Shimpi theory, based on two variable coupled governing equations for the bending and shear displacement fields, is applied for the buckling analysis of simply supported rectangular plates under the action of uniaxial and biaxial stresses. The theory accounts for the cubic variation of the in-plane displacements through the plate thickness and the transverse shear strains, which vanish on the top and bottom faces of the plate. The main advantage of the theory is that the governing equations have been derived using the Hamilton's principle, so that they are certainly consistent with the assumed displacement field.

The obtained results are compared with the classical project formulas for buckling analysis and particularly with the Bryan expression for simply supported rectangular plates under uniaxial compression. This last formula is easily obtained by developing into appropriate double sine trigonometric series the deflection surface of the buckled plate. So, denoting by *m* the number of half-waves in the direction of compression, by t the plating thickness, E and ν the Young and Poisson modulus, a and b the longer and shorter sides of the panel and by $\alpha = a/b$ the plating aspect ratio, the Bryan's formula for the Euler buckling stress can be so expressed:

$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2\left(\frac{\alpha}{m}+\frac{m}{\alpha}\right)^2 \tag{1}$$

The magnitude of the Euler load depends on the panel aspect ratio α and also on *m*, which gives the number of half-waves into which the plate buckles. The eq. (1) is generally developed as follows:

$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 K_1 \tag{2}$$

having denoted by K_1 the buckling factor, defined as:

$$K_1 = \begin{cases} 4.00 & if \quad \alpha > 1 \\ \left(\alpha + \dfrac{1}{\alpha}\right)^2 & if \quad \alpha \le 1 \end{cases} \tag{3}$$

The classical approach for rectangular plates under biaxial compression is quite similar and some project curves are generally used to evaluate the Euler stresses at which buckling occurs.

In the following the buckling analysis of thick rectangular plates is carried out applying the Shimpi theory and so taking into account the shear deformations. A method, different from the classical energy one and substantially based on the nontrivial solution of the two variable coupled governing equations for the bending and shear displacement fields, is adopted to evaluate the critical buckling load. A new project formula, that differs from the Bryan's one for a corrective factor function of the ratio *t/b*, is also proposed for plates under uniaxial compression.

Finally, some applications are presented for platings uniformly compressed along one and two orthogonal directions, varying the thickness and the plating aspect ratio. The theoretical Euler buckling stresses, corrected taking into

V. Piscopo is, since December 2009, Ph.D. in Aerospace, Naval and Quality Engineering at the University of Naples "Federico II", Department of Naval Architecture and Marine Engineering, Via Claudio 21 - 80125 Napoli Italy (e-mail: vincenzo.piscopo@ unina.it).

account the shear effects, are also compared with those ones obtained by a FEM analysis carried out by *ANSYS*.

II. Theoretical development

Let us refer to the coordinate system of Fig.1 with z axis having the origin on the plate middle plane. The basic assumptions of the two variable refined Shimpi theory are:

1. the displacements are small, if compared with the plating thickness;
2. the stress σ_z is negligible respect to the in-plane stresses σ_x and σ_y;
3. the displacement $w(x,y)$ normal to the plate middle plane is the sum of two components of bending and shear $w_b(x,y)$ and $w_s(x,y)$ respectively;
4. the in-plane displacements $u(x,y)$ and $v(x,y)$ along the x and y axes include two components of bending and shear and one component $u_0(x,y)$ and $v_0(x,y)$ due to the in-plane normal forces;
5. the bending components $u_b(x,y)$ and $v_b(x,y)$ are similar to those ones of the classical thin plate theory: $u_b(x,y) = -z\frac{\partial w_b}{\partial x}$ and $v_b(x,y) = -z\frac{\partial w_b}{\partial y}$;
6. the shear components $u_s(x,y)$ and $v_s(x,y)$ are related to the vertical shear displacement field w_s.

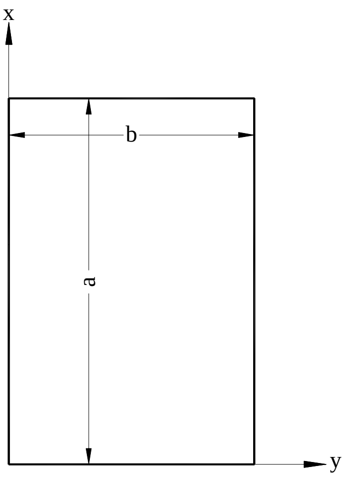

Fig. 1 Plate reference system

Starting from these basic assumptions, the displacement field becomes:

$$\begin{cases} u(x,y) = u_0(x,y) - z\dfrac{\partial w_b}{\partial x} + z\left[\dfrac{1}{4} - \dfrac{5}{3}\left(\dfrac{z}{t}\right)^2\right]\dfrac{\partial w_s}{\partial x} \\ v(x,y) = v_0(x,y) - z\dfrac{\partial w_b}{\partial y} + z\left[\dfrac{1}{4} - \dfrac{5}{3}\left(\dfrac{z}{t}\right)^2\right]\dfrac{\partial w_s}{\partial y} \\ w(x,y) = w_b(x,y) + w_s(x,y) \end{cases} \tag{4}$$

The Hamilton's principle is used to derive the equations of motion appropriate to the assumed displacement field (4), so imposing the following condition:

$$\int_0^T (\partial U + \partial V - \partial T)dt = 0 \tag{5}$$

having denoted by U the strain energy, V the work done by the applied forces and T the kinetic energy. Denoting by $D = \frac{Et^3}{12(1-\nu^2)}$ the plate flexural rigidity and by G the Coulomb modulus, the governing equations can be finally expressed as follows, [2]:

$$\begin{cases} D\nabla^4 w_b = p + N_x\dfrac{\partial^2 w}{\partial x^2} + N_y\dfrac{\partial^2 w}{\partial y^2} + 2N_{xy}\dfrac{\partial^2 w}{\partial x \partial y} \\ \dfrac{D}{84}\nabla^4 w_s = p + \dfrac{5}{6}Gt\left(\dfrac{\partial^2 w_s}{\partial x^2} + \dfrac{\partial^2 w_s}{\partial y^2}\right) + N_x\dfrac{\partial^2 w}{\partial x^2} + N_y\dfrac{\partial^2 w}{\partial y^2} + 2N_{xy}\dfrac{\partial^2 w}{\partial x \partial y} \end{cases} \tag{6}$$

where N_x and N_y are the in-plane forces per unit of length, directed along the x and y axes respectively and N_{xy} is the shear in-plane force per unit of length. Assuming that these forces are constant throughout the plate, $N_{xy} = 0$ and $N_y = \gamma N_x$, with $0 \le \gamma \le 1$, the eq. (6) can be rewritten as follows:

$$\begin{cases} D\nabla^4 w_b = N_x\left(\dfrac{\partial^2 w}{\partial x^2} + \gamma\dfrac{\partial^2 w}{\partial y^2}\right) \\ \dfrac{D}{84}\nabla^4 w_s = \dfrac{5}{6}Gt\left(\dfrac{\partial^2 w_s}{\partial x^2} + \dfrac{\partial^2 w_s}{\partial y^2}\right) + N_x\left(\dfrac{\partial^2 w}{\partial x^2} + \gamma\dfrac{\partial^2 w}{\partial y^2}\right) \end{cases} \tag{7}$$

As the plate is considered as simply supported along all edges, the boundary conditions are satisfied by taking for the deflection surface of the buckled plate the following double sine trigonometric series for the bending and shear components:

$$\begin{cases} w_b(x,y) = \displaystyle\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{m,n}^{(b)} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \\ w_s(x,y) = \displaystyle\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{m,n}^{(s)} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \end{cases} \tag{8}$$

Substituting eq. (8) into (7), the unknown amplitudes $w_{m,n}^{(b)}$ and $w_{m,n}^{(s)}$ are solution of the following homogeneous equation system:

$$\begin{cases} A_{11}w_{m,n}^{(b)} + A_{12}w_{m,n}^{(s)} = 0 \\ A_{21}w_{m,n}^{(b)} + A_{22}w_{m,n}^{(s)} = 0 \end{cases} \tag{9.1}$$

with:

$$\begin{cases} A_{11} = D\pi^2\left(\dfrac{m^2}{a^2}+\dfrac{n^2}{b^2}\right)^2 - N_x\left(\dfrac{m^2}{a^2}+\gamma\dfrac{n^2}{b^2}\right) \\ A_{12} = -N_x\left(\dfrac{m^2}{a^2}+\gamma\dfrac{n^2}{b^2}\right) \\ A_{21} = A_{12} \\ A_{22} = \dfrac{D\pi^2}{84}\left(\dfrac{m^2}{a^2}+\dfrac{n^2}{b^2}\right)^2 - N_x\left(\dfrac{m^2}{a^2}+\gamma\dfrac{n^2}{b^2}\right) + \dfrac{5}{6}Gt\left(\dfrac{m^2}{a^2}+\dfrac{n^2}{b^2}\right) \end{cases} \tag{9.2}$$

The equation for calculating the Euler stress value σ_E can be now derived by equating to zero the determinant of the above system of equations, so obtaining:

$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \frac{\left(\dfrac{m^2}{\alpha^2}+n^2\right)^2}{\dfrac{m^2}{\alpha^2}+\gamma n^2} F\left(\frac{t}{b},m,n,\nu,\alpha\right) \tag{10.1}$$

with:

$$F\left(\frac{t}{b},m,n,\nu,\alpha\right) = \frac{1+\left(\dfrac{t}{b}\right)^2 \dfrac{\pi^2}{420(1-\nu)}\left(\dfrac{m^2}{\alpha^2}+n^2\right)}{1+85\left(\dfrac{t}{b}\right)^2 \dfrac{\pi^2}{420(1-\nu)}\left(\dfrac{m^2}{\alpha^2}+n^2\right)} \tag{10.2}$$

For plates under uniaxial compression the Euler buckling stress can be immediately derived considering that the plate buckles in such a way that there can be several half-waves in the direction of compression, but only one half-wave in the perpendicular direction. Thus, a new expression for the Euler stress, that differs from the Bryan's formula for the shear correction factor $F(t/b,m,n,\nu,\alpha)$, is obtained:

$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2\left(\frac{\alpha}{m}+\frac{m}{\alpha}\right)^2 F\left(\frac{t}{b},m,\nu,\alpha\right) \tag{11.1}$$

with:

$$F\left(\frac{t}{b},m,\nu,\alpha\right) = \frac{1+\left(\dfrac{t}{b}\right)^2 \dfrac{\pi^2}{420(1-\nu)}\left(\dfrac{m^2}{\alpha^2}+1\right)}{1+85\left(\dfrac{t}{b}\right)^2 \dfrac{\pi^2}{420(1-\nu)}\left(\dfrac{m^2}{\alpha^2}+1\right)} \tag{11.2}$$

Obviously, the classical Bryan's formula for thin plates can be derived putting in eq. (11.1) $F(0,m,\nu,\alpha)=1$.

Introducing, now, the buckling coefficient k_1, as defined in eq. (2), in fig. 2 several curves, as function of the ratio *t/b*, are shown: the thick curve refers to the classical theory for thin plates.

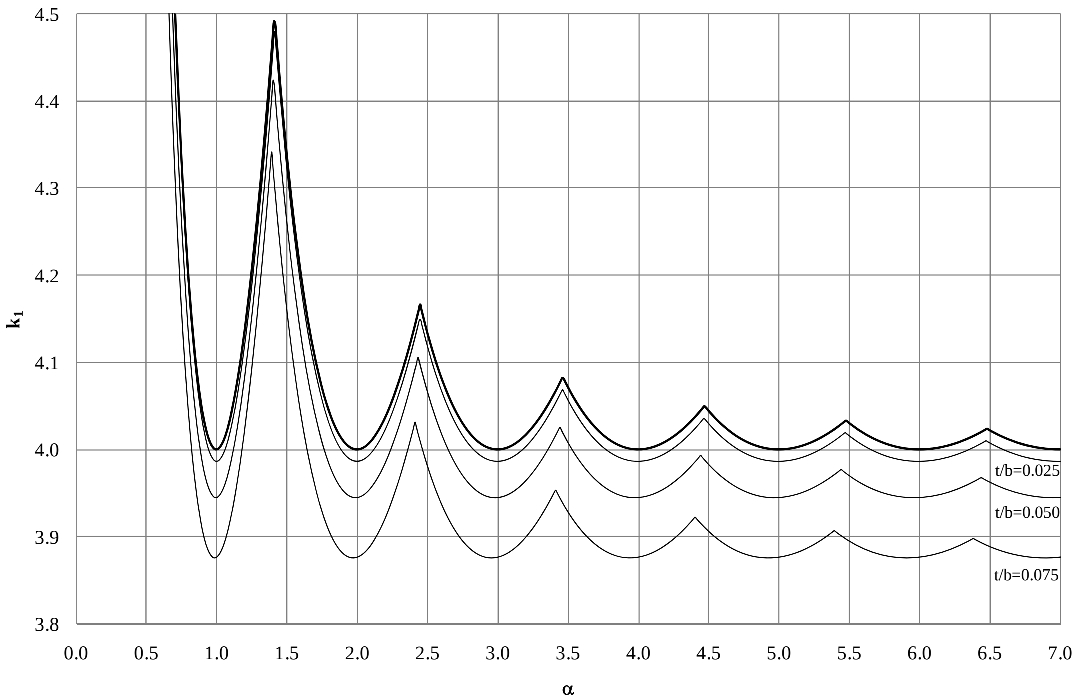

Fig.2 Buckling factor k_1 for thick-plates under uniaxial compression ($\nu = 0.30$; $\gamma = 0.0$)

For plates under biaxial compression with $0 < \gamma < 0.5$ it is possible to verify that the plate always buckles in such a way that *n*=1,while the number of half-waves in *x*-direction varies; instead when $0.5 \leq \gamma \leq 1.0$ the plate buckles in such a way that there is only one half-wave in both directions. In figg.3 and 4,for $\gamma = 0.1$ and $\gamma = 0.4$,the buckling factors are shown for different values of the plating aspect ratio. In figg. 5 and 6 the same factors are shown for $\gamma = 0.7$ and $\gamma = 1.0$ (in this case, for scale reasons, the only curves with *t/b*=0.075 are presented). It is noticed that the thick curves always refer to the classical values for thin plates and the Poisson modulus has been always assumed equal to 0.30.

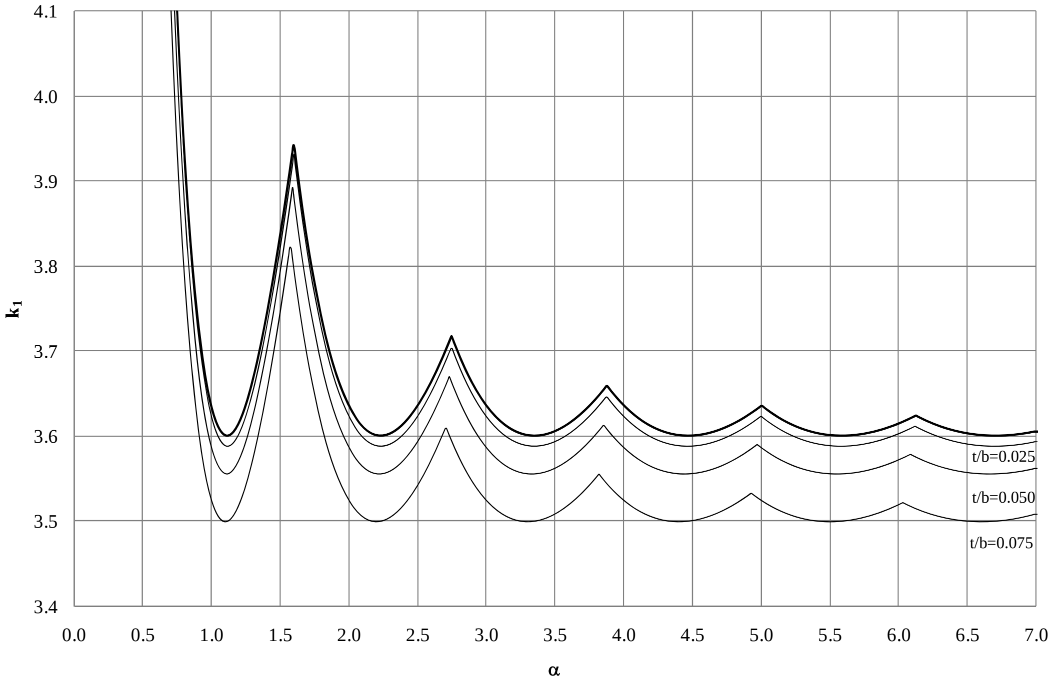

Fig.3 Buckling factor k_1 for thick-plates under biaxial compression ($\nu = 0.30;\ \gamma = 0.1$)

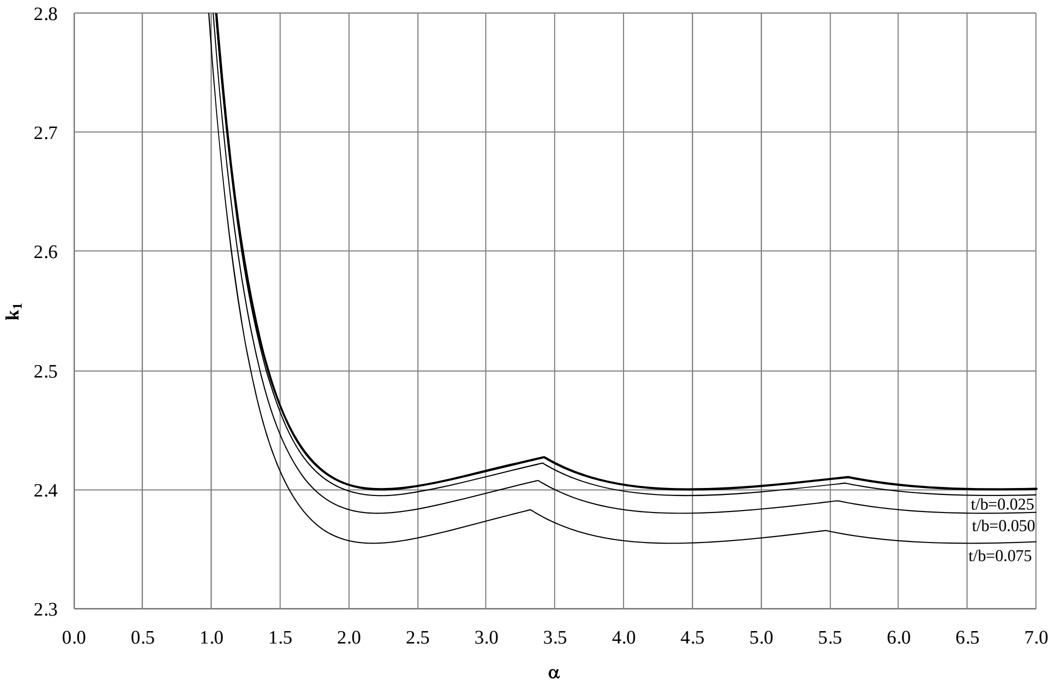

Fig.4 Buckling factor k_1 for thick-plates under biaxial compression ($\nu = 0.30;\ \gamma = 0.4$)

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:10, 2010

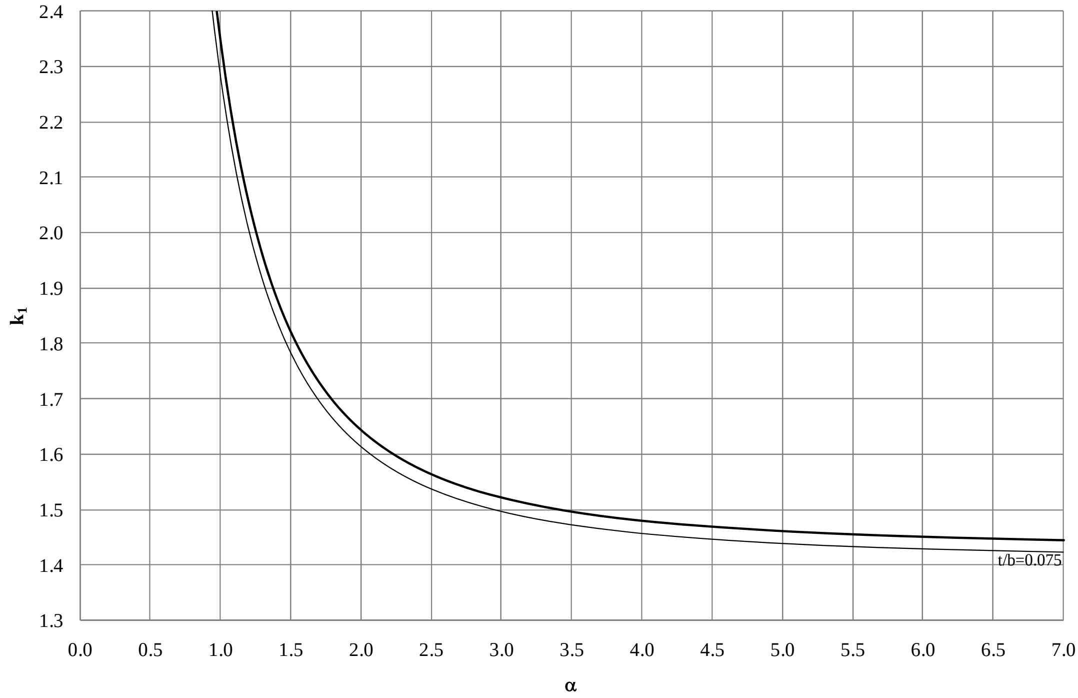

Fig.5 Buckling factor k_1 for thick-plates under biaxial compression ($\nu = 0.30$; $\gamma = 0.7$)

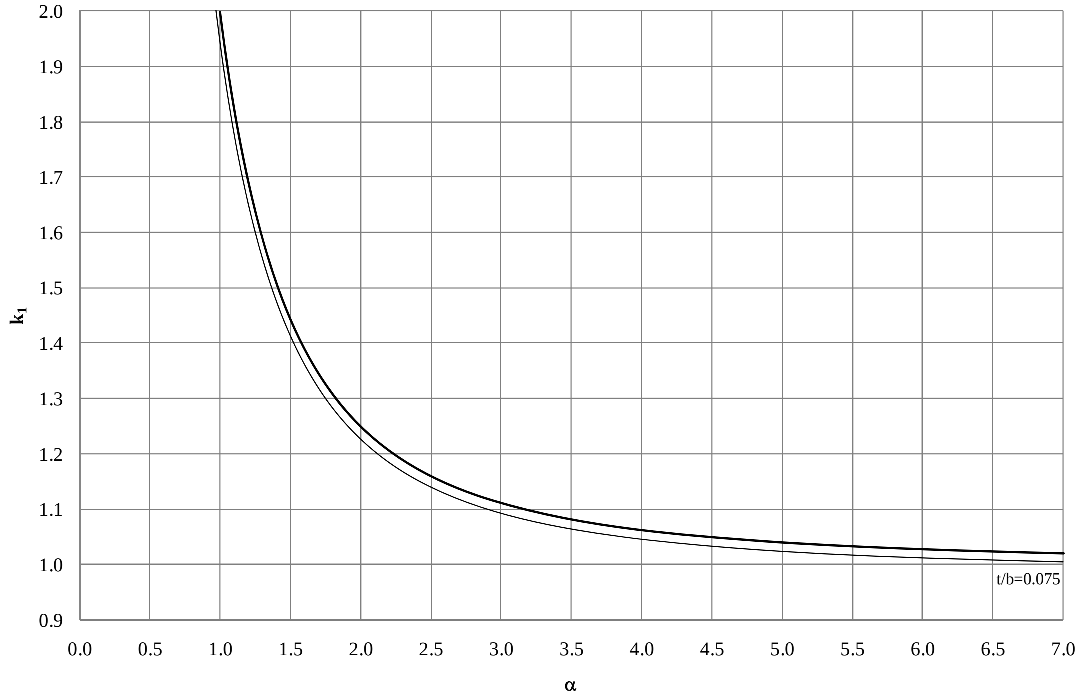

Fig.6 Buckling factor k_1 for thick-plates under biaxial compression ($\nu = 0.30$; $\gamma = 1.0$)

III. NUMERICAL APPLICATIONS

To show the feasibility of the presented formulas, several buckling analyses for steel platings of ship structures, subjected to uniaxial and biaxial compression, are shown. A comparison with the relevant results obtained by a FEM analysis, carried out by *ANSYS*, is also presented to validate the proposed formulas. In *ANSYS* some eigenvalue buckling analyses have been performed and the convergence of the solution has been studied by thickening the mesh (the last one with a mean element length of 0.005 m). The chosen element is the 4-node finite strain SHELL181, suitable for analyzing thin to moderately thick structures and well-suited for linear, large rotation, and/or large strain nonlinear applications. The analyzed panels are:

1. Case 1: a=0.25 m; b=1.00 m; t=10-20-30 mm;
2. Case 2: a=1.00 m; b=1.00 m; t=10-20-30 mm;
3. Case 3: a=4.00 m; b=1.00 m; t=10-20-30 mm.

It was assumed that they are in high strength steel with E=2.06E11 Pa, ν=0.3, R_{eH} =355 N/mm^2 .

A. Plates under uniaxial compression

In tables I.A, II.A and III.A the Euler forces per unit of length $N_E = \sigma_E t$ are shown for different values of the ratio *t/b*. The convergence of the solution obtained by *ANSYS* is also studied: in all cases it is quite quickly achieved and a very good accordance with the proposed buckling formulas is found. In tables I.B, II.B and III.B the relevant critical buckling stresses are shown.

TABLE I.A
CASE 1 – α = 0.25, γ = 0

$\frac{t}{b}$	Mean element length	ANSYS (A)	Thin plate (B)	Thick plate (C)	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	m	kN/m	kN/m	kN/m	%	%
0.01	0.050	3553	3363	3347	0.87	0.39
	0.025	3391				
	0.010	3343				
	0.005	3334				
0.02	0.050	27924	26904	26398	2.80	0.87
	0.025	26623				
	0.010	26233				
	0.005	26171				
0.03	0.050	91643	90800	87046	5.66	1.30
	0.025	87372				
	0.010	86121				
	0.005	85932				

TABLE I.B
CASE 1 – α = 0.25, γ = 0

$\frac{t}{b}$	ANSYS (A)	Thin plate (B)	Thick plate (C)	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	N/mm^2	N/mm^2	N/mm^2	%	%
0.01	260.5	261.3	260.9	0.31	0.14
0.02	330.9	331.6	331.1	0.20	0.06
0.03	344.0	344.6	344.1	0.17	0.04

TABLE II.A
CASE 2 – α = 1.00, γ = 0

$\frac{t}{b}$	Mean element length	ANSYS (A)	Thin plate (B)	Thick plate (C)	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	m	kN/m	kN/m	kN/m	%	%
0.01	0.050	747	745	744	0.68	0.54
	0.025	743				
	0.010	741				
	0.005	740				
0.02	0.050	5930	5958	5945	1.95	1.73
	0.025	5883				
	0.010	5852				
	0.005	5844				
0.03	0.050	19811	20108	20006	3.14	2.62
	0.025	19616				
	0.010	19513				
	0.005	19496				

TABLE II.B
CASE 2 – α = 1.00, γ = 0

$\frac{t}{b}$	ANSYS (A)	Thin plate (B)	Thick plate (C)	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	N/mm^2	N/mm^2	N/mm^2	%	%
0.01	74.0	74.5	74.4	0.68	0.54
0.02	247.2	249.2	249.0	0.83	0.74
0.03	306.5	308.0	307.8	0.48	0.40

TABLE III.A
CASE 3 – α = 4.00, γ = 0

$\frac{t}{b}$	Mean element length	ANSYS (A)	Thin plate (B)	Thick plate (C)	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	m	kN/m	kN/m	kN/m	%	%
0.01	0.050	747	745	744	0.68	0.54
	0.025	744				
	0.010	741				
	0.005	740				
0.02	0.050	5945	5958	5945	1.34	1.12
	0.025	5908				
	0.010	5886				
	0.005	5879				
0.03	0.050	19917	20108	20006	2.12	1.60
	0.025	19767				
	0.010	19694				
	0.005	19690				

TABLE III.B
CASE 3 – α = 4.00, γ = 0

$\frac{t}{b}$	ANSYS (A)	Thin plate (B)	Thick plate (C)	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	N/mm^2	N/mm^2	N/mm^2	%	%
0.01	74.0	74.5	74.4	0.68	0.54
0.02	247.8	249.2	249.0	0.57	0.48
0.03	307.0	308.0	307.8	0.33	0.25

From the obtained results, it seems quite clear that the refined buckling analysis always furnishes, respect to the classical thin plate theory, results closer to those ones obtained by the FEM analysis and the relevant percentage differences of theoretical results grow up, when the ratio *t/b* increases, as it would be predictable.

Anyway, it is fundamental to note that these differences are certainly lower if referred to the critical stress value, the most important parameter in a buckling analysis. It implies that the introduction of the shear correction factor always permits to obtain results closer to the FEM values, even if not particularly different by those ones obtained applying the classical thin plate theory.

B. Plates under biaxial compression

In tables IV, V and VI the Euler forces per unit of length are shown for platings under biaxial compression with $\gamma = 1.0$, for different values of the ratio *t/b*.

TABLE IV
CASE 1 – $\alpha = 0.25$, $\gamma = 1.0$

$\frac{t}{b}$	Mean element length	*ANSYS (A)*	Thin plate *(B)*	Thick plate *(C)*	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	m	kN/m	kN/m	kN/m	%	%
0.01	0.050	3356	3165	3150	0.86	0.38
	0.025	3194				
	0.010	3147				
	0.005	3138				
0.02	0.050	26377	25321	24845	2.80	0.87
	0.025	25080				
	0.010	24693				
	0.005	24631				
0.03	0.050	86563	85459	81926	5.67	1.30
	0.025	82304				
	0.010	81063				
	0.005	80875				

TABLE V
CASE 2 – $\alpha = 1.00$, $\gamma = 1.0$

$\frac{t}{b}$	Mean element length	*ANSYS (A)*	Thin plate *(B)*	Thick plate *(C)*	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	m	kN/m	kN/m	kN/m	%	%
0.01	0.050	373	372	372	0.54	0.54
	0.025	371				
	0.010	370				
	0.005	370				
0.02	0.050	2965	2979	2972	1.95	1.71
	0.025	2942				
	0.010	2926				
	0.005	2922				
0.03	0.050	9906	10054	10003	3.14	2.62
	0.025	9809				
	0.010	9758				
	0.005	9748				

TABLE VI
CASE 3 – $\alpha = 4.00$, $\gamma = 1.0$

$\frac{t}{b}$	Mean element length	*ANSYS (A)*	Thin plate *(B)*	Thick plate *(C)*	$\frac{B-A}{A}\cdot 100$	$\frac{C-A}{A}\cdot 100$
---	m	kN/m	kN/m	kN/m	%	%
0.01	0.050	199	198	198	0.00	0.00
	0.025	198				
	0.010	198				
	0.005	198				
0.02	0.050	1585	1583	1581	0.51	0.38
	0.025	1580				
	0.010	1577				
	0.005	1575				
0.03	0.050	5337	5341	5327	0.70	0.43
	0.025	5317				
	0.010	5309				
	0.005	5304				

The convergence of the solution obtained by *ANSYS* has been studied and in all cases it was quite quickly achieved. From the analysis it is clear that there is a very good accordance with the proposed formulas and the FEM values and also in this case the thick plate theory furnishes values of the Euler buckling loads lower than the classical ones.

IV. CONCLUSIONS

In this paper the refined plate theory proposed by Shimpi (2006) has been applied for the buckling analysis of platings, simply supported along all edges and compressed in one and two orthogonal directions. For plates under uniaxial compression a new direct formula, similar to the Bryan's one, has been derived: this expression permits to take into account the shear effects by means of a shear correction factor, as function of the half waves' number in the loaded direction, the panel aspect ratio, the Poisson modulus and the thickness ratio *t/b*. Some curves, that permit to easily evaluate the buckling coefficient for plates under uniaxial and biaxial compression (assuming in this case γ=1.0), are also presented.

Finally, some numerical applications have been carried out for steel platings of ship structures. The theoretical buckling loads have been compared with the relevant values obtained by some eigenvalue buckling analyses carried out by *ANSYS*. It was found that the refined theory always furnishes, respect to the classical one, results closer to the values obtained by the FEM analysis. The convergence of the solution obtained by *ANSYS* has also been studied by thickening the mesh.

As this method permits to obtain simple closed project formulas, it can be satisfactory applied to the buckling analysis of plates. Obviously, this refined buckling analysis can also be extended to platings under the combined action of uniaxial and edge shear forces, even if in this case the proposed solution method is not available and the classical energy method must be applied.

REFERENCES

[1] S. Timoshenko, J. Gere, *Theory of Elastic Stability*, McGraw-Hill International Book Company, 17th edition, 1985.
[2] R.P. Shimpi and H.G. Patel, "A two variable refined plate theory for orthotropic plate analysis", International Journal of Solid and Structures (43), pp. 6783-6799, 2006.
[3] H. Tai, S. Kim, J. Lee, "Buckling analysis of plates using two variable refined plate theory", *Proceedings of Pacific Structural Steel Conference 2007, Steel Structures in Natural Hazards*, Wairakei, New Zeland, 13-16 March, 2007.
[4] S. Timoshenko, N. Goodier, *Theory of Elasticity*, McGraw-Hill International Book Company, 1951.
[5] O. Hughes, *Ship Structural Design: a Rationally-Based Computer-Aided Optimization Approach*, SNAME Edition, 1988.
[6] RINA Rules, 2010.
[7] E. Reissner, "The effect of transverse shear deformation on the bending of elastic plates", *Journal of Applied Mechanics Vol. 12* (Transactions ASME 67), pp. 69-77, 1945;
[8] R.D. Mindlin, "Influence of rotary inertia and shear on flexural motions of isotropic, elastic plates ", *Journal of Applied Mechanics Vol. 18* (Transactions ASME 73), pp. 31-38, 1951;
[9] M. Levinson, "An accurate simple theory of the statics and dynamics of elastic plates", *Mechanics Research Communications Vol. 7*, pp. 343-350, 1980;
[10] J.N. Reddy, "A refined non linear theory of plates with transverse shear deformation", *International Journal of Solid and Structures Vol. 20*, pp.881-896, 1984.

V. Piscopo. Bachelor Degree in Naval Engineering in 2004, Master Degree in Naval Engineering in 2006, Ph.D. in Aerospace, Naval and Quality Engineering at the University of Naples "Federico II" in 2009