

# Fluid Flow and Heat Transfer Structures of Oscillating Pipe Flows

Yan Su, Jane H. Davidson and F. A. Kulacki

**Abstract**—The RANS method with Saffman's turbulence model was employed to solve the time-dependent turbulent Navier-Stokes and energy equations for oscillating pipe flows. The method of partial sums of the Fourier series is used to analyze the harmonic velocity and temperature results. The complete structures of the oscillating pipe flows and the averaged Nusselt numbers on the tube wall are provided by numerical simulation over wide ranges of  $Re_A$  and  $Re_R$ . Present numerical code is validated by comparing the laminar flow results to analytic solutions and turbulence flow results to published experimental data at lower and higher Reynolds numbers respectively. The effects of  $Re_A$  and  $Re_R$  on the velocity, temperature and Nusselt number distributions have been discussed. The enhancement of the heat transfer due to oscillating flows has also been presented. By the way of analyzing the overall Nusselt number over wide ranges of the Reynolds number  $Re$  and Keulegan-Carpenter number  $KC$ , the optimal ratio of the tube diameter over the oscillation amplitude is obtained based on the existence of a nearly constant optimal  $KC$  number. The potential application of the present results in sea water cooling has also been discussed.

**Keywords**—Keulegan-Carpenter number, Nusselt number, Oscillating pipe flows, Reynolds number

## I. INTRODUCTION

OCEAN wave energy [1] and sea water cooling [2] have been potential sources as renewable energy with high energy densities [3]. The fluid mechanics and heat transfer of oscillating pipe flows are very important for ocean energy applications [4].

The early studies of oscillating flows were highly concentrated on flow structures [5-9]. The flow was observed to be laminar at low  $Re_A$  and become turbulent at high  $Re_A$  by Ohmi *et al.* [10]. They demonstrated that for  $Re_R > 8$ , the critical Reynolds number  $(Re_A)_{cr}$  for the onset of transition is independent of  $Re_R$ : the value of  $(Re_A)_{cr} = 4.00 \times 10^4$  corresponds to the onset of disturbed laminar flow superimposed with small perturbations, while  $(Re_A)_{cr} = 1.51 \times 10^5$  for the onset of intermittently locally-bursting turbulent flow. Akhavan *et al.* [11] also investigated experimentally the transition of oscillating flows in circular pipes. Hino *et al.* [7] observed that the value of  $(Re_A)_{cr}$  increased when  $Re_R < 5.12$ , because the viscous Stokes layers at the pipe wall become relatively thicker when  $Re_R$  decreases

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and the interaction of the Stokes layers from the wall restricts the viscous diffusion for boundary layer growth. Further decrease in  $Re_R$  leads to the limit case of a quasi-steady laminar Poiseuille flow. Conversely, the increase of  $Re_R$  to a very large value will lead to the other limit case where the Stokes layer becomes much thinner than the pipe radius. To investigate the flow transition as well as its associated wall shear stress, Blondeaux [12] investigated numerically the oscillating flows in a semi-infinite fluid domain over a flat plate by implementing the Reynolds Averaged Navier-Stokes (RANS) method with Saffman's turbulence model [13, 14]. However, he reported only amplitude but no phase angle information on the oscillatory shear stress. The Lam-Bremhorst form of the low-Reynolds number  $k-\epsilon$  turbulence model was chosen for oscillating-flow modeling by [15], while there are still some deficiencies due to the shortcomings of the low-Reynolds number computational model. Also the higher order harmonics are not decomposed from the first order one. Hsu *et al.* [16] demonstrated that Saffman's turbulence model is applicable for unsteady oscillating flows and they also provided a complete account for the oscillatory shear stress on the flat plate. Hsu *et al.* [17] revealed the flow structure of oscillating channel flows and obtained  $(Re_A)_{cr} = 2.00 \times 10^4$ .

Recently, more studies are concentrated on heat transfer of oscillating pipe flows due to the increasing importance of the application in the ocean energy. Experimental studies show that the oscillating flows can enhance heat transfer. Experimental results of Chai *et al.* [18] show that the heat exchange capability of the oscillating heat pipe heat exchanger is about 3 times higher than that of a common tube heat exchanger. However their measurement is under the laminar flow region and they did not show the relationship of the enhancement of the heat transfer with governing parameters such as the Reynolds numbers. Wang and Lu [19] applied large eddy simulation (LES) technique to simulate heat transfer between the two constant temperature endplates of oscillating channel flow at  $Re = 350$ . They find out that the heat transfer takes place in a much thinner region near the wall at  $Pr = 100$  than at  $Pr = 1$ . Due to the limitation of the computational speed of the LES method, they did not give out the full structures for heat transfer and fluid flow from laminar to transient and turbulent range. Also the assumption of the constant temperature difference between the two endplates does not fit for the present mode for sea water cooling heat exchangers, because the characteristic temperature difference is not on the pipe wall itself, but between inlet/outlet oscillating sea water and the pipe wall.

In the present study, the RANS method with Saffman's

turbulence model was employed to study overall structures of the axis direction dominated flow and two dimensional heat transfer of oscillating flows in circular pipes. The experimental studies of [20] and [21] show that the entrance length of oscillating pipe flow can be approximated by  $L_{entrance}/R = 8.76 \times 10^{-3} Re_R$ . So the entrance length is a small value comparing to the transportation length of the water pipe before the water coming into the heat transfer part. Thus the flow can be assumed to be fully developed one dimensional dominated oscillating flow. Experimental study of a pulse combustor tail pipe in [22] showed that the mean temperature was as high as 800 K, while the surface temperature oscillated only about 0.56K. So the two dimensional heat transfer model with constant wall temperature  $\Theta_w$  and inlet/outlet temperature  $\Theta_0$  boundary conditions is applied in the present study. Also the present model can speed up the simulation and make it possible for us to provide a complete picture of the oscillating flow structures and the overall heat transfer enhancement over a wide range of  $Re_A$  and  $Re_R$ . Results of oscillating velocity and temperature fields are decomposed by the method of partial sums of the Fourier series. The overall heat transfer enhancement comparing to a based line case of pure conduction will be presented. By the way of analyzing the overall Nusselt Number over wide ranges of the Reynolds number  $Re$  and Keulegan-Carpenter number  $KC$ , their effects on heat transfer and the potential application of the results in sea water cooling will also be discussed.

## II. GOVERNING EQUATIONS AND GOVERNING PARAMETERS

Consider an oscillating flow in a pipe with radii  $R$ , the cylindrical coordinate is chosen such that  $x$  is in the flow direction parallel to the centerline of the pipe. The pressure gradient in the  $x$  direction that drives the flow is assumed to be cosinusoidal with a frequency  $f$  as:

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \alpha_p \cos(2\pi ft) \quad (1)$$

where  $\rho$  is the fluid density,  $p$  the pressure, and  $\alpha_p$  is the amplitude of negative pressure gradient which is assumed to be constant. Using  $\alpha_p$  and  $f$ , a displacement length scale  $A$  is now defined as  $A = \alpha_p / (2\pi f)^2$ . Similarly as the oscillating channel flows discussed in [17], there are three length scales for oscillating flows in circular pipes: the displacement amplitude of fluid oscillation  $A$ , the pipe radius  $R$ , and the Stokes layer thickness  $\delta$ . The Stokes layer thickness  $\delta = \sqrt{\nu / 2\pi f}$  measures the viscous diffusion distance in one cycle of oscillation, where,  $\nu$  is the fluid viscosity and  $f$  is the oscillation frequency. The ratios of  $A$  and  $R$  to  $\delta$  then give two important independent parameters defined respectively by  $Re_A = A^2 / \delta^2 = 2\pi f A^2 / \nu$  and  $Re_R = R^2 / \delta^2 = 2\pi f R^2 / \nu$ . The oscillating period number, i.e. the Keulegan-Carpenter number  $KC$  (based on the characteristic length scale  $R$ ) is defined as:  $KC = 2\pi A / R = 2\pi (Re_A / Re_R)^{1/2}$ , and the Reynolds number  $Re$  is defined as  $Re = (2\pi f A) R / \nu = (Re_A Re_R)^{1/2}$ .

Thus, in previous literatures we can see two set of governing parameters ( $Re_A, Re_R$ ) and ( $KC, Re$ ) for oscillating flow studies. The characteristics of the oscillating pipe flows then depend entirely on ( $Re_A, Re_R$ ) or ( $KC, Re$ ). While for heat transfer in oscillating pipe flows will also have Prandtl number  $Pr$  as the third governing parameter. The coordinate systems for the two groups of governing parameters ( $\log(Re_A), \log(Re_R)$ ) and ( $\log(KC), \log(Re)$ ) differ  $\pi/4$ , so results can be presented in either of them.

Using  $R$  as the length scale,  $2\pi f A$  as the velocity scale,  $R / (2\pi f A)$  as the time scale,  $\rho A (2\pi f)^2$  as the scale for negative pressure gradient,  $\Theta_w - \Theta_0$  as the temperature scale, and the scales  $(2\pi f A)^2$  and  $2\pi f A / R$  for the pseudo-energy  $e$  and the pseudo-vorticity  $\omega$  respectively, the non-dimensional governing equations can be obtained based on Saffman's turbulence model [13] as:

$$\frac{\partial u}{\partial t} = \frac{2\pi}{KC} \cos(2\pi / KC) + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{1}{Re} + \gamma \frac{e}{\omega} \right) \frac{\partial u}{\partial r} \right] \quad (2)$$

$$\frac{\partial e}{\partial t} = \alpha_e e \left| \frac{\partial u}{\partial r} \right| - \beta_e e \omega + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{1}{Re} + \sigma_e \gamma \frac{e}{\omega} \right) \frac{\partial e}{\partial r} \right] \quad (3)$$

$$\frac{\partial \omega^2}{\partial t} = \alpha_\omega \omega^2 \left| \frac{\partial u}{\partial r} \right| - \beta_\omega \omega^3 + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{1}{Re} + \sigma_\omega \gamma \frac{e}{\omega} \right) \frac{\partial \omega^2}{\partial r} \right] \quad (4)$$

$$\frac{\partial \Theta}{\partial t} + u \frac{\partial \Theta}{\partial x} = \frac{\partial}{\partial x} \left[ \left( \frac{1}{Re Pr} + \gamma_T \frac{e}{\omega} \right) \frac{\partial \Theta}{\partial x} \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{1}{Re Pr} + \gamma_T \frac{e}{\omega} \right) \frac{\partial \Theta}{\partial r} \right] \quad (5)$$

where, the Keulegan-Carpenter number  $KC = 2\pi A / R$  is also the dimensionless period, and the Reynolds number  $Re = (2\pi f A) R / \nu$ , the  $Pr = 7.0$  is selected to simulate water cooling in the present study.

The proper boundary conditions are:

$$u = 0, e = 0, \omega = \frac{S}{\alpha_e} \left| \frac{\partial u}{\partial r} \right|, \Theta = 1.0 \quad \text{at } r = \pm 1, \quad (6)$$

$$\frac{\partial u}{\partial r} = \frac{\partial e}{\partial r} = \frac{\partial \omega}{\partial r} = \frac{\partial \Theta}{\partial r} = 0 \quad \text{at } r = 0, \quad (7)$$

$$\Theta = 0 \quad \text{at } x = \pm Lx / 2R. \quad (8)$$

In (2)-(5),  $\alpha_e, \alpha_\omega, \beta_e, \beta_\omega, \sigma_e, \sigma_\omega, \gamma$  and  $\gamma_T$  are universal constants. In the present computation, we followed Saffman & Wilcox [14] and Jacobs [23] to use  $\alpha_e = 0.3$ ,  $\alpha_\omega = 0.18$ ,  $\beta_e = 0.09$ ,  $\beta_\omega = 0.15$ ,  $\sigma_e = 0.5$ ,  $\sigma_\omega = 0.5$ ,  $\gamma = 1.0$  and  $\gamma_T = 1/0.89$ . The value of  $S$  in the wall boundary condition (6) depends on the surface roughness and is equal to 100 for a smooth wall [14].

## III. NUMERICAL PROCEDURE AND RESULT VALIDATION

### A. Numerical Procedure

The equation system (2)-(5), subjected to boundary conditions (6)-(8), was solved numerically with the following procedures: (i) Central difference scheme for spatial

derivatives, (ii) Second order Adams-Bashforth scheme for time advancement of the source terms, and (iii) Implicit scheme for the viscous terms.

The main reason for adapting Saffman's turbulence model rests on its applicability to flows over the entire range of Reynolds number to provide a first estimate of flow transition. Meanwhile, the simplicity of RANS method, especially the less time-consuming feature, enabled us to compute the flow and heat transfer characteristics over wide ranges of Reynolds numbers to provide a complete picture of the oscillating flow structure and the overall heat transfer enhancement.

In the present simulations, the length of the tube is 8 times of the diameter. A mesh with grid size  $160 \times 200$  is used and the mesh is stretched by exponential function to provide more points near the wall and inlet/outlet of the pipe to resolve the Stokes layer near the tube wall and the entrance heat transfer. The dimensionless time step  $\Delta t$  was chosen as  $\Delta t / KC = 10^{-6}$ . The convergence error is less than  $10^{-6}$  for the velocity field and less than  $10^{-5}$  for the temperature field.

#### B. Validation by Comparing to Analytical Results of Laminar Oscillating Pipe Flows

When the Reynolds number  $Re_A$  is sufficiently low, the flow is laminar, i.e.,  $\langle u'w' \rangle = 0$  or  $\gamma(e/\omega) = 0$ . In term of the complex expression,  $u = [\hat{u} \exp(i2\pi / KC) + c.c.] / 2$  where  $\hat{u}$  is the complex amplitude,  $i = \sqrt{-1}$ , and  $c.c.$  denotes the complex conjugate, the solution to (2) with boundary condition (6) and (7) is given by

$$\hat{u} = -i \left[ 1 - \frac{J_0(r\sqrt{-i Re_R})}{J_0(\sqrt{-i Re_R})} \right] \quad (9)$$

where,  $J_0$  denotes the Bessel function of the first kind and of zero order. The amplitude and the phase angle of  $u$  are then obtained by taking the absolute value and the argument to  $\hat{u}$ , respectively.

The friction coefficient defined by  $C_F = 2\tau_w / \rho(2\pi fA)^2 = [\hat{C}_F \exp(i2\pi / KC) + c.c.] / 2$  with  $\tau_w$  being the wall shear stress, can be obtained by taking the derivative to (9) with respect to  $r$  and evaluating the resultant equation at the wall to give the following expression:

$$\hat{C}_F = -\frac{2}{Re} i \left[ \frac{J_0'(r\sqrt{-i Re_R})}{J_0(\sqrt{-i Re_R})} \right]_{r=1} \quad (10)$$

Also, the amplitude and the phase angle of  $C_F$  are the absolute value and the argument of  $\hat{C}_F$ , respectively.

Two limit cases of high and low  $Re_R$  are of great interest. When  $Re_R \rightarrow \infty$ , Eq. (9) reduces to

$$\hat{u} = -i \left\{ 1 - \frac{1}{r} \exp \left[ -(1+i) \sqrt{\frac{Re_R}{2}} (1-r) \right] \right\} \quad (11)$$

Equation (11) indicates that the oscillating velocity is composed of two Stokes layers near the walls (whose scale is of order  $\delta$ ) and the centerline velocity has an amplitude one and  $90^\circ$  phase-lag to the negative pressure gradient. By the same token, Eq. (10) reduces to

$$\hat{C}_F = (1-i) \frac{\sqrt{2 Re_R}}{Re} = (1-i) \sqrt{\frac{2}{Re_A}} \quad (12)$$

which indicates that the amplitude of wall shear stress depends solely on  $Re_A$  and has the phase angle  $45^\circ$  leading the centerline velocity. On the other hand, when  $Re_R \rightarrow 0$ , Eq. (9) becomes the parabolic profile of a quasi-steady flow given by

$$\hat{u} = (1-r^2) Re_R / 4 \quad (13)$$

which shows that the amplitude decreases with decreasing  $Re_R$  and the phase angle becomes in-phase with the negative pressure gradient. The wall shear stress of Eq. (10) now reduces to

$$\hat{C}_F = Re_R / Re = (Re_R / Re_A)^{1/2} \quad (14)$$

which shows that the amplitude of shear stress decreases with decreasing  $Re_R$  in  $1/2$  power if  $Re_A$  is fixed (i.e., when the amplitude of negative pressure gradient is fixed) and the phase angle is in-phase with the velocity (or negative pressure gradient).

To validate the present numerical code, computational results of the amplitudes and phase angles of centerline velocity  $u_c$  at  $r=0$  and wall shear stress  $C_F$  obtained by present code are compared with the analytical results from (9) and (10). As shown in Fig. 1, five cases for various  $Re_R$  ( $< 0.8 \times 10^4$ ) are in excellent agreement with those predicted from a laminar-flow analytic solution calculated from (9) and (10). From Fig. 1, we can see that when  $Re_R$  is larger than  $10^{2.5}$  and less than  $0.8 \times 10^4$ ,  $|\hat{u}_c|$  equals to 1.0 and  $\theta_{u_c}$  maintains constant at  $-90^\circ$ . Similarly,  $|\hat{C}_F| Re_A^{1/2}$  is constant equal to 2.0 and  $\theta_{C_F}$  remains constantly at  $-45^\circ$ . These amplitudes and phase angles equal to those predicted by (11) and (12) for the limit case of  $Re_R \rightarrow \infty$ . For laminar oscillating flows in two flat plate channels [17], as  $Re_R$  decreases, the amplitudes  $|\hat{u}_c|$  and  $|\hat{C}_F| Re_A^{1/2}$  (shown in dashed lines in Fig. 1) overshoot to the maximum values greater than 1.0 and 2.0 respectively. For the present laminar oscillating flows in circular pipes, the amplitudes  $|\hat{u}_c|$  overshoot to the maximum values greater than 1.0, however there is not overshooting for  $|\hat{C}_F| Re_A^{1/2}$ . When  $Re_R$  becomes very low, the amplitudes of both  $u_c$  and  $C_F$  decrease and their phase angles approach zero. In fact,  $|\hat{u}_c|$  varies linearly with  $Re_R$  and  $|\hat{C}_F| Re_A^{1/2}$  varies with  $(Re_R)^{1/2}$ , which are agree with (13) and (14). The agreement of the numerical results

and the analytical solution shown in Fig. 1, show that the present code works very well at low  $Re_R$  number range.

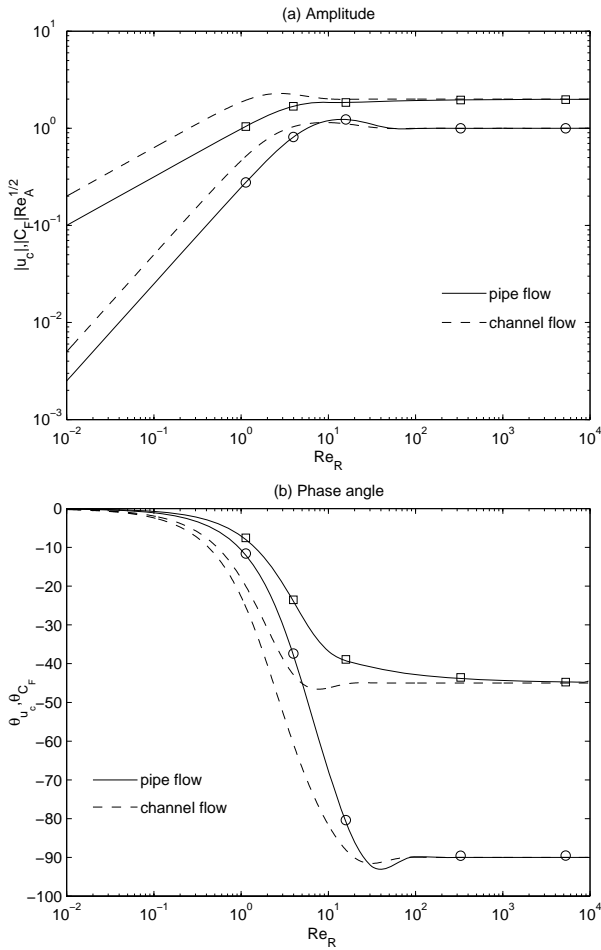


Fig. 1 Variations of centerline velocity  $u_c$  and wall frictional coefficient  $C_f$  with  $Re_R$  for oscillating laminar flow, (a) amplitude and (b) phase angle difference. Solid lines: analytical solution for laminar oscillating pipe flows from Eqs. (9) and (10); Dash lines: analytical solutions for laminar oscillating channel flows Hsu *et al* [17];  $\square$ -  $u_c$  and  $\circ$ -  $C_f$ : computational results from present model

### C. Validation by Comparing to Experimental Data of Turbulent Oscillating Pipe Flows

In order to validate the present code at high Reynolds number range, a comparison between our numerical result for one turbulent flow case ( $Re_R = 224.767$  and  $Re_A = 6.2 \times 10^5$ ) and the experimental data of Akhavan (1991) has been done and shown in Fig. 2. The good agreement between the numerical results and experimental data on the transient velocity for the eight phase angles in a period guaranteed that the present code is also valid in high Reynolds number range.

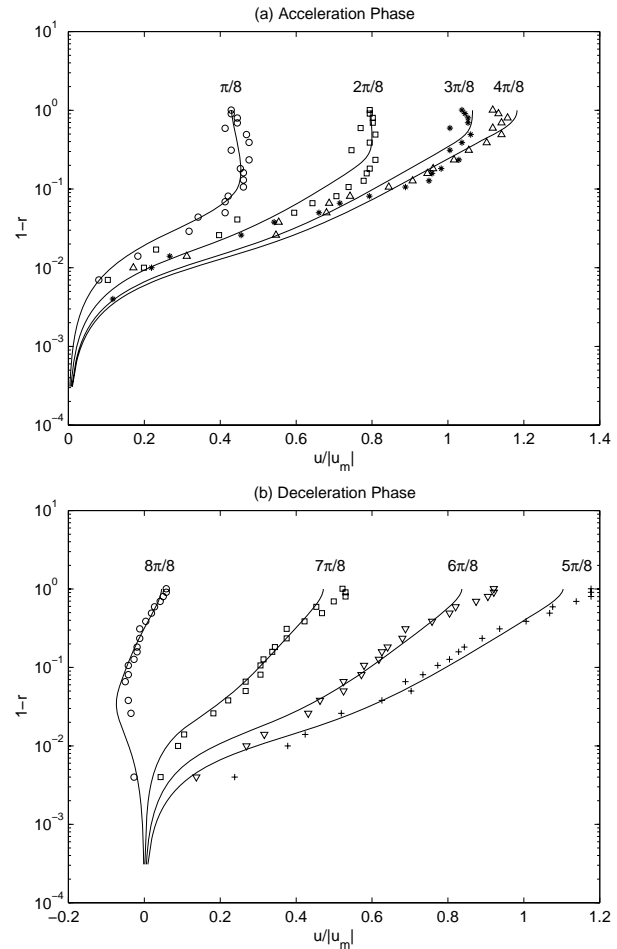


Fig. 2 A comparison between our numerical result and the experimental data of Akhavan [11]

## IV. RESULT AND DISCUSSION

The method of partial sums of the Fourier series is used to decompose the velocity and temperature results. The velocity and temperature can be decomposed based on the dimensionless period of the oscillating pressure (i.e. the  $KC$  number) as:

$$u(r, t) = u_0(r) + \sum_{k=1}^N \hat{u}_k(r) \cos(k2\pi / KC + \theta_k(r)) \quad (15)$$

and

$$\Theta(x, r, t) = \Theta_0(x, r) + \sum_{k=1}^N \hat{\Theta}_k(x, r) \cos(k2\pi / KC + \theta_k(x, r)) \quad (16)$$

where,  $k$  is the order of the harmonic term and  $\theta_k$  is phase angle difference comparing to the pressure phase angle. Zero order harmonic terms ( $u_0, \Theta_0$ ) are the cycle averaged values. The integer  $N$  larger than 3 is enough for decomposition of present numerical results, and higher orders of harmonic terms have maximum values of amplitude less than  $10^{-5}$  and  $10^{-3}$  for velocity and temperature field respectively, which are negligible.

### A. Numerical Results for Fluid Flow

Numerical simulations for five different values of  $Re_R$  ( $= 5255, 328.4, 15.90, 3.974$  and  $1.131$ ) and wide ranges of  $Re_A$  (from  $10^3$  to  $10^{8.5}$ ) have been shown to cover the flow regimes from laminar to turbulent. It is noted that the present numerical simulation results contain in general the second and higher harmonics due to nonlinear nature in the equation system (2-4). Based on the decomposition of velocity as shown in (15), we find out that the fluid flow is dominated by the first order of harmonics.

#### A.1 Velocity Distribution along the $r$ Direction

As our objective is to explore the flow structure for transition rather than non-linearity, only the amplitudes and the phase angles of the first order harmonic of the velocity results are presented in this paper.

To illustrate the flow transition under different conditions of  $Re_R$ , the velocity profiles of three different values of  $Re_A$ , corresponding to laminar, transitional and fully turbulent flows, are plotted in Fig. 3 for the two extreme cases of  $Re_R = 5255$  (solid lines) and  $3.974$  (dashed lines). For the case of  $Re_R = 5255$ , Fig. 3 shows that the velocity profile at  $Re_A = 10^3$  is of a typical laminar oscillating flow with a thin Stokes layer near the wall and a potential core. When the flow becomes transitional at  $Re_A = 10^5$ , the velocity profile shown in Fig. 3 indicates that the turbulent mixing is still confined in the turbulent boundary layer near the wall whose thickness is much thicker than the laminar Stokes layer. The locations of amplitude overshoot and phase-angle undershoot are shifted toward the pipe centerline due to turbulent mixing, even though the potential flow remains in the core region. At  $Re_A = 10^{8.5}$ , the turbulent boundary layer apparently has occupied the whole pipe, the amplitude overshoot disappears. Alternatively, this can be interpreted as the location of overshoot has moved to the pipe centerline. The phase angle  $\theta_u$  then becomes quite uniform across the pipe, but remains to be close to  $-90^\circ$ . On the other hand, for the case of  $Re_R = 3.974$  the velocity profile shown in Fig.3 indicates that the flow is laminar and nearly quasi-steady when  $Re_A = 10^3$ , with an almost parabolic profile in  $|\hat{u}|$  and phase angles  $\theta_u$  ranging from  $-23.5^\circ$  to  $-37.4^\circ$ . When the flow becomes transitional at  $Re_A = 10^6$ , the enhancement of the fluid diffusion by turbulent eddy viscosity apparently has flattened the velocity profile near the pipe core region to result in lower velocity amplitude. Meanwhile, the phase angle  $\theta_u$  ranges from  $-18.1^\circ$  to  $-21.8^\circ$ , which indicates that the eddy viscosity effect renders the flow to approach toward the quasi-steady state. Further increase of the Reynolds number seems only to provide high eddy viscosity to further enhance the turbulent mixing effect toward a fully developed quasi-steady turbulent pipe flow, as shown by  $Re_A = 10^{8.5}$  in Fig. 3 and the phase angle  $\theta_u$  ranges from  $-6.5^\circ$  to  $-8.1^\circ$ . The data also show that second order harmonic term of velocity is negligible comparing to the first order term and its phase angle is not as regular as that of the first hand harmonic term.

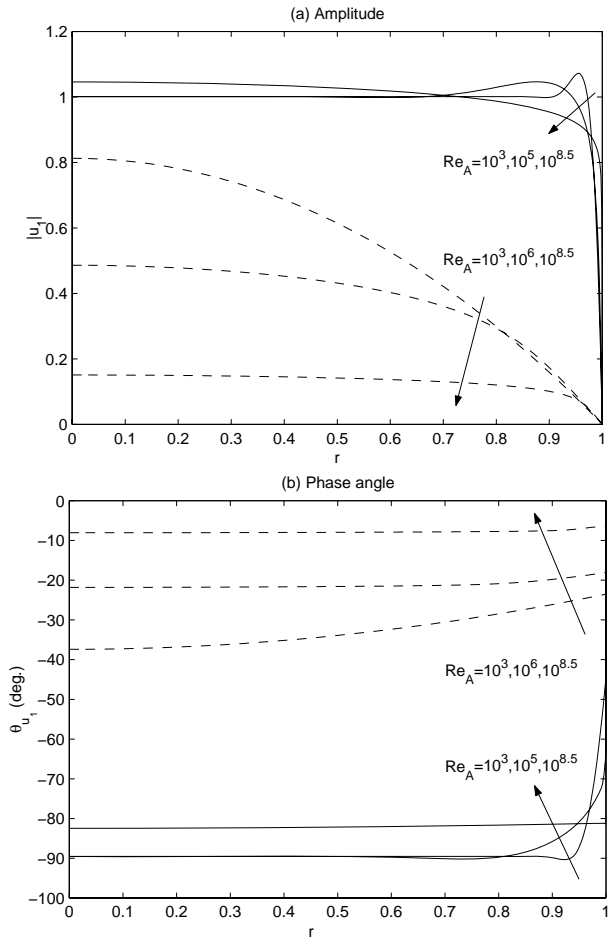


Fig. 3 First term of velocity profiles of laminar, transitional and turbulent oscillating flows for  $Re_R = 5255$  (solid line) and  $Re_R = 3.974$  (dashed line), (a) amplitude of  $u_1$ , (b) phase angle of  $u_1$

#### A.2 Full Structures of Velocity and the Wall Shear Stress

To obtain overall flow structures, the results of centerline velocity  $u_c$  and the wall shear stress  $C_F$  for all computed cases of  $Re_R$  and  $Re_A$  are plotted in Figs. 4 and 5, respectively, for (a) amplitude and (b) phase angle. In Figs. 4 and 5, the solid lines represent the analytical laminar flow results of low  $Re_A$  from Eq. (9) and (10). We now first examine the result of  $u_c$  given in Fig. 4. For the two sets at high Reynolds numbers of  $Re_R = 5255$ , the amplitudes  $|\hat{u}_c|$  as shown in Fig. 4a maintain at one and the corresponding phase angles as shown in Fig. 4(b) have the value of  $-90^\circ$ , except at very high  $Re_A$ . This suggests that the turbulent oscillating boundary layer for high  $Re_A$  is still thinner than  $R$  and is unable to produce noticeable effect on the centerline velocity, until  $Re_A$  becomes very high. For the cases of  $Re_R = 328.4, 15.90, 3.974$  and  $1.131$ , the values of  $|\hat{u}_c|$  however drop monotonically with increasing  $Re_A$  in the turbulent regime. This is accompanied by the continuing shift of phase angle from  $-90^\circ$  toward  $0^\circ$  as indicated in Fig. 4(b). Apparently, when  $Re_R$  is sufficiently small, the oscillating turbulent boundary layer

becomes quasi-steady as  $Re_A \rightarrow \infty$ . Assuming the flow is quasi-steady, the expression for  $|\hat{u}_C|$  at very high  $Re_A$  can be devised by following Saffman's derivation [13] to give:

$$|\hat{u}_C| = \frac{1}{\kappa} \left( \frac{Re_R}{Re_A} \right)^{0.25} [0.25 \ln(Re_A) + 0.75 \ln(Re_R) + 1.92] \quad (17)$$

where,  $\kappa$  ( $0.38 < \kappa < 0.47$ ) is the von Kármán constant. The results calculated from (17) for high  $Re_A$  and low  $Re_R$  are shown as the dashed lines in Fig. 4a. They agree very well with the numerical results. It is recalled that in the quasi-steady limit where the shear layer covers the entire pipe, the pressure force is balanced totally by the shear. Therefore, we conclude that at very high  $Re_A$ , the eddy viscosity effect has greatly enhanced the shear force to render the transient inertia force negligible.

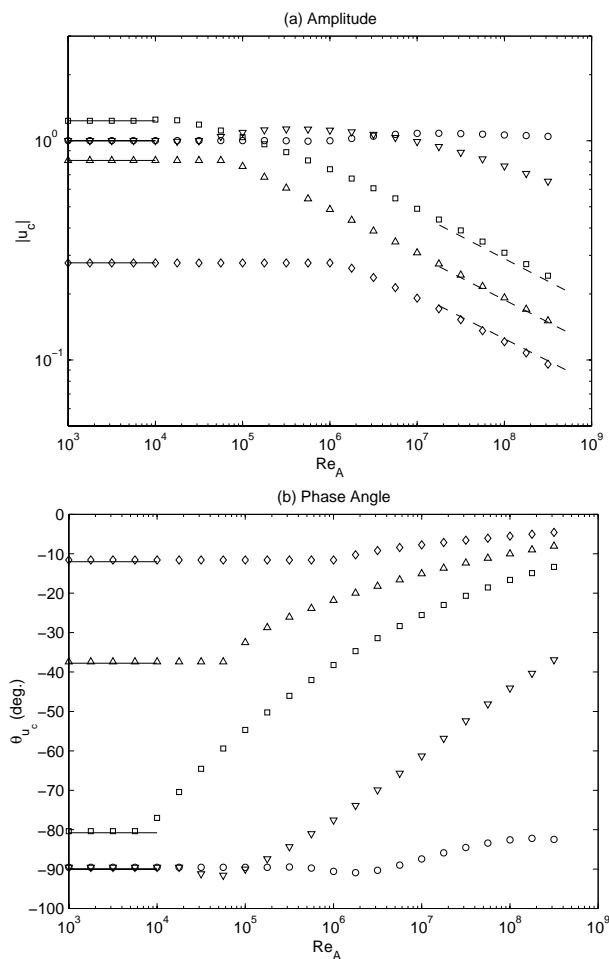


Fig. 4 Variations of (a) amplitude and (b) phase angle of  $u_C$  with  $Re_A$  for five values of  $Re_R$ . Solid lines: laminar solution; Dashed lines: quasi-steady analytical solution using Saffman's model [13]

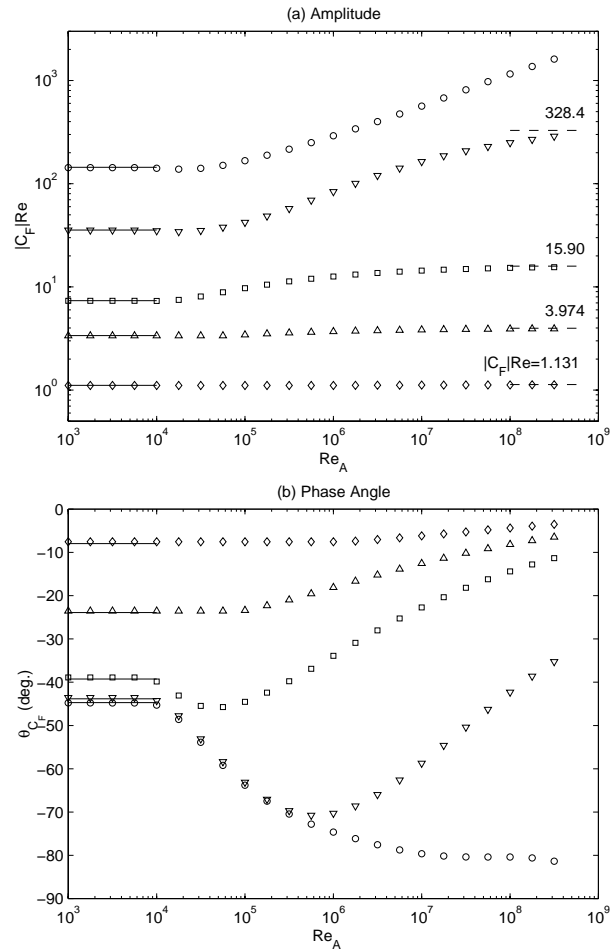


Fig. 5 Variations of (a) amplitude of  $C_F$  (b) phase angle of  $C_F$  with  $Re_A$  for five values of  $Re_R$ . Solid lines: laminar solution; Dashed lines:  $|\hat{C}_F|_{Re = Re_R}$ .

For a better understanding of the flow structure, we shall examine the wall shear stress shown in Fig. 5. Attention is first given to the case of  $Re_R = 328.4$ , i.e, when  $R$  is about eighteen times the Stokes layer thickness  $\delta$ . When  $Re_A$  is sufficiently low, say  $Re_A < 0.8 \times 10^4$  before flow transition, the oscillating flows are laminar. The results of  $C_F$  as computed according to the RANS method with Saffman's turbulence model agree excellently with the analytical predictions from (10), i.e.,  $|\hat{C}_F|_{Re} = 35.54$  and  $\theta_{C_F} = -43.84^\circ$ . As  $Re_A$  increases, the transition from laminar to turbulent occurs approximately at  $(Re_A)_{cr} = 0.8 \times 10^4$  as shown in Fig. 5b where  $\theta_{C_F}$  starts to decrease from  $-43.84^\circ$ . Under the condition of  $Re_R = 328.4$ , the oscillating turbulent flow after transition remains as a boundary layer flow confined near the wall, with a potential flow in the pipe core region. Interestingly, the amplitude  $|\hat{C}_F|_{Re}$  does not change noticeably until  $Re_A = 7.5 \times 10^5$ , and hence is not a good indicator for flow transition. It is found out that the phase angle of the wall shear stress is a

more sensitive gauge than amplitude for the determination of flow transition. Further increase in  $Re_A$  leads to higher value of the eddy viscosity  $\nu_T$  that thickens the thickness  $\delta_T$  of the oscillating turbulent boundary layer. The phase angle  $\theta_{C_F}$  continues to decrease with increasing  $\delta_T$ , until reaches a minimum value of about  $\theta_{C_F} = -71^\circ$  at  $Re_A = 7.5 \times 10^5$  where  $\delta_T$  has become sufficiently thick that the effect due to the mutual interaction of the boundary layers at top and bottom of the channel is appreciable. For  $Re_A > 7.5 \times 10^5$ , the phase angle  $\theta_{C_F}$  increases with increasing  $Re_A$ . In the limit of  $Re_A \rightarrow \infty$ ,  $R$  becomes the governing length scale since  $\delta_T \gg R$  and the oscillating turbulent flow becomes a quasi-steady turbulent flow where  $\theta_{C_F} \rightarrow 0$ . The amplitude of wall shear stress given in Fig. 5a for  $Re_R = 328.4$  shows that  $|\hat{C}_F|Re$  increases with increasing  $Re_A$  and asymptotically reaches a constant when  $Re_A \rightarrow \infty$ . For a fully developed turbulent flow in a channel that is steady in mean and driven by a mean pressure gradient  $\partial p / \partial x$ , a simple momentum balance results in  $\tau_w / R = -\partial p / \partial x$  which in terms of the friction coefficient becomes (14). This implies that Eq. (14), which was originally obtained for quasi-steady oscillating laminar flows, applies equal well to quasi-steady oscillating turbulent flows. The asymptotic value of  $|\hat{C}_F|Re$  is  $Re_R$ . This gives  $|\hat{C}_F|Re = 328.4$  if  $Re_R = 328.4$ , which as shown as dashed line in Fig. 5a agrees very well with the computed result.

With the above comprehension of the flow for  $Re_R = 328.4$ , we now examine the flows at different  $Re_R$ . For higher  $Re_R$  such as the cases of  $Re_R = 5255$ , the oscillating laminar Stokes layer at low  $Re_A$  is much thinner than  $R$ . The transition from laminar to turbulent still occurs near  $(Re_A)_{cr} = 0.8 \times 10^4$ ; however, after the transition it requires much higher  $Re_A$  than that of  $Re_R = 328.4$  for  $\delta_T$  to become comparable with  $R$ . Fig. 5b indicates that  $\theta_{C_F}$  reaches a minimum of  $-71^\circ$  at  $Re_A = 7.5 \times 10^5$  for  $Re_R = 328.4$  and is still decreasing at  $Re_A = 1.5 \times 10^7$  for  $Re_R = 5255$ . Figure 8a also shows that the amplitude results of this study never reach the asymptotic values of  $|\hat{C}_F|Re = 5255$  for  $Re_R = 5255$ . Apparently, for the cases of  $Re_R = 5255$  the computed range of  $Re_A$  in this study covers only the laminar Stokes layer flow and the oscillating turbulent boundary layer flow regimes. On the other hand, for the cases of low  $Re_R$  the thickness of the Stokes layer is already comparable with  $R$  when  $Re_R = 3.974$  and much thicker than  $R$  when  $Re_R = 1.131$ . At low  $Re_A$  the flow is laminar with  $\theta_{C_F} = -23.914^\circ$  for  $Re_R = 3.974$  and  $\theta_{C_F} = -7.968^\circ$  for  $Re_R = 1.131$ . The oscillating laminar flow is already in the quasi-steady flow regime. As  $Re_A$  increases passing the critical value  $(Re_A)_{cr}$ , the oscillating flow moves directly from the quasi-steady laminar flow regime to the quasi-steady turbulent flow regime. Therefore, the phase angle  $\theta_{C_F}$  increases from its respective laminar flow value toward  $0^\circ$ . Fig. 5a shows that for both cases of  $Re_R = 3.974$  and  $1.131$  the amplitudes  $|\hat{C}_F|Re$  increases with increasing  $Re_A$  starting

from  $(Re_A)_{cr}$  and reaches the asymptotic values of  $|\hat{C}_F|Re = 3.974$  and  $1.131$ , respectively, as plotted again as dashed lines. There is a delay in flow transition depending on  $Re_R$ . The lower the  $Re_R$ , the higher will be the  $(Re_A)_{cr}$  and the earlier the oscillating turbulent flow will reach the asymptotic results of quasi-steady state.

### A.3 Flow Regimes

From the results given above, the structure of the oscillating pipe flows is constructed using parameters  $(Re_A, Re_R)$  as shown in Fig. 6. Each point on Fig. 6 represents one computed case. The open circle represents the laminar flow, the star represents the oscillating turbulent boundary layer flow and the triangle represents the quasi-steady turbulent flow. The transition from laminar regime to turbulent regime is marked by the sudden change of  $\theta_{C_F}$  from the constant laminar values.

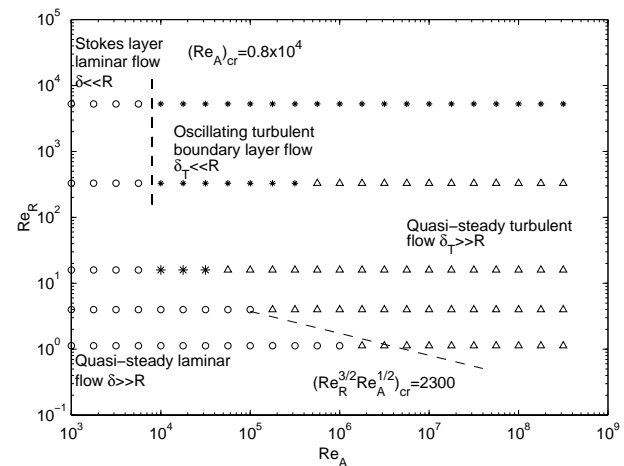


Fig. 6 Flow regimes of oscillating flows in channels in term of coordinates  $(Re_A, Re_R)$ .  $\circ$ : laminar flow;  $\star$ : oscillating turbulent boundary layer flow;  $\triangle$ : quasi-steady turbulent flow; Dashed lines: flow transition lines at the two extremes corresponding to boundary layer flows and quasi-steady viscous flows

At low  $Re_A$ , say  $Re_A < 0.8 \times 10^4$ , the oscillating flows are laminar and two flow regimes showing low and high  $Re_R$  respectively, are identified. The domain of  $Re_R \ll 1$  represents the quasi-steady laminar flow regime where the velocity profile is parabolic and the domain of  $Re_R \gg 1$  represents the Stokes layer laminar flow regime where the velocity profile decays exponentially from the wall. As  $Re_A$  increases, the Stokes layer at high  $Re_R$  becomes unstable. The transition from laminar to turbulent occurs approximately at  $(Re_A)_{cr} = 0.8 \times 10^4$  and is plotted as the dashed line in Fig. 6. For  $Re_A > (Re_A)_{cr}$  and high  $Re_R$ , the flow is in the oscillating turbulent boundary layer flow regime where the thickness  $\delta_T$  of turbulent boundary layer remains thinner than  $R$ , with a potential flow in the channel core. The increase in  $Re_A$  will lead to thicker  $\delta_T$  and, in the limit of  $Re_A \gg (Re_A)_{cr}$  but still of high  $Re_R$ , the thickness  $\delta_T$  becomes much thicker than  $R$  so that the flow is governed by  $R$  and belongs to the quasi-steady turbulent flow regime. The flow transition is delayed to

higher  $(Re_A)_{cr}$  when  $Re_R$  decreases. In the limit of  $Re_R \ll 1$  where flows become quasi-steady, the transition is expected to occur at the same critical condition of a fully developed steady channel flow at  $(\bar{U}2R/\nu)_{cr} = 2300$ , where  $\bar{U}$  is the mean velocity. For full developed pipe flow  $\bar{U} = U_{max} / 2$ . We have  $U_{max} = 2\pi f A (Re_R) / 4$  from (13) and  $(U_{max} R / \nu)_{cr} = (Re_R)_{cr} / 4 = 2300$ ; hence  $(Re_R^{3/2} Re_A^{1/2})_{cr} = 9200$ . This limit case of critical condition is also plotted in Fig. 6. Since the critical value of 9200 was found from experimental observation, which is less sensitive than our classification using phase angle change, the dashed line predicts a slightly higher value of  $(Re_A)_{cr}$ . As  $Re_A$  passes  $(Re_A)_{cr}$ , the oscillating flows move directly from the quasi-steady laminar flow regime into the quasi-steady turbulent flow regime.

**B Numerical Results of Heat Transfer**

Numerical results for the same five values of  $Re_R$  ( $= 5255, 328.4, 15.90, 3.974$  and  $1.131$ ) and  $Re_A$  (from  $10^{1.5}$  to  $10^{7.5}$ ) have been presented to cover the heat transfer regimes from nearly conduction to laminar and turbulent flow convection. Also a pure conduction case is simulated as a base line for the study of heat transfer enhancement.

**B.1 Temperature Distribution**

It is noted that the present numerical simulation results for the temperature field contain higher orders of harmonics than the velocity field, because the fluid field will affect the temperature field as shown in (5). Based on the decomposition of the transient results of the temperature field, as shown in (16), we find out that the temperature field is dominated by the zero order of harmonic. The amplitude of the temperature harmonics will decrease with the increase of the order of harmonics. Phase angles are zero at the inlet/outlet of the pipe, while the phase angles at the center of the pipe ( $x=0$ ) are the maximum value along the x direction. The phase angles will increase with the order of homogeneous, and the center phase angle is larger than  $360^\circ$  at orders higher than the first order. For example, when  $Re_R = 1.131$  and  $Re_A = 100$ , the maximum amplitude of the zero order harmonic temperature is 1, and is about 0.4 for the first order. It is about 0.15 and 0.05 for the second and third harmonics respectively. The maximum phase angles appear in the mid of the tube at  $x=0$ . The maximum first order phase angle is about  $120^\circ$  and it is about  $390^\circ$  and  $660^\circ$  for the second and the third order respectively. Our data also show that the thermal boundary layer thickness decreases with increase of  $Re_R$  or  $Re_A$ . Now that the amount of net heat transfer is based on the cycle averaged temperature gradient on the tube wall, so we will only discuss the Nusselt number based on the zero order of temperature gradient and the length scale  $R$  in the following parts to obtain the effects of parameters on net heat transfer.

**B.2 Axis Direction Distribution of Nusselt Number**

The axis direction distributions of the Nusselt Number on the pipe wall for  $Re_R = 328.4$  and  $Re_A$  from  $10^2$  to  $10^{7.5}$  are plotted as solid lines in Fig. 7. The dash line is the pure conduction result. With  $Re_A$  increased from  $10^2$  to  $10^{7.5}$ , the

center point Nusselt number  $Nu_R(x=0)$  will increase from 0 to 222. There is always a heat transfer leading edge near the inlet/outlet of the pipe due to the assumption of the constant inlet/outlet water temperature. The leading edge Nusselt numbers are much larger than the center ones. From Fig. 7, we can also see that the center point of Nusselt number equals 0 for the cases  $Re_A < 0.8 \times 10^4$ , which is for nearly conduction to laminar flow regions. For the oscillating turbulent boundary layer flow region, when  $Re_A > 0.8 \times 10^4$ , the center point Nusselt numbers will increase with  $Re_A$ .

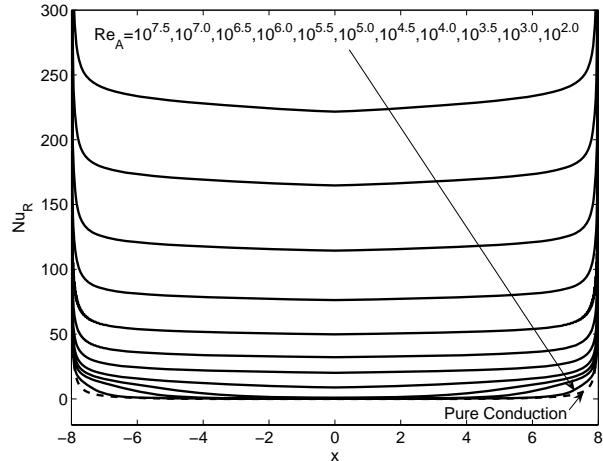


Fig. 7 Axis Direction Distribution of  $Nu_R$  on Wall (Dashed line: pure conduction, Solid lines:  $Re_R=328.4$  and  $Re_A$  from  $10^2$  to  $10^{7.5}$ )

**B.3 Averaged Nusselt Number based on various  $(Re_A, Re_R)$**

The averaged Nusselt Numbers along x direction are plotted in Fig.8 in form of  $Re_A$  and  $Re_R$ . At lower  $Re_A$  and  $Re_R$ ,  $\bar{Nu}_R$  is near to the pure conduction value (1.7031). While the value increase with both  $Re_A$  and  $Re_R$ . Those  $\bar{Nu}_R$  lines in coordinate  $(Re_A, Re_R)$  never cross each other, which shows  $Re_R$  and  $Re_A$  are the basic two independent governing parameters for heat transfer as well as the structure of the flows in oscillating pipe flows. If  $\bar{Nu}_R$  is plotted in form of  $Re$  and  $KC$ , we can see overlap of the data, which is similar as the overlap of data shown in the experimental study of [24].

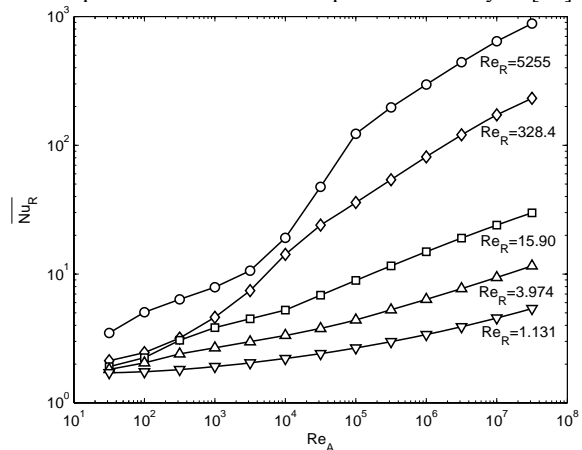


Fig. 8 Averaged Nusselt Number  $\bar{Nu}_R$  for various cases



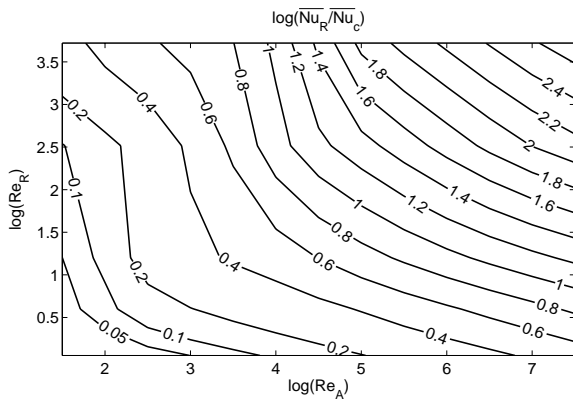


Fig. 9 Contour plot of  $\log(\overline{Nu_R} / \overline{Nu_C})$  in term of coordinates  $(\log(Re_A), \log(Re_R))$

#### B.4 Heat Transfer Enhancement

Comparing to the pure conduction  $x$  axis averaged Nusselt Number ( $\overline{Nu_C} = 1.7031$ ), the heat transfer enhancement ratio  $\overline{Nu_R} / \overline{Nu_C}$  is always greater than 1 in all ranges of  $Re_R$  and  $Re_A$ . Fig. 11 show the distribution of  $\log(\overline{Nu_R} / \overline{Nu_C})$  in term of coordinates  $(\log(Re_A), \log(Re_R))$ . From Fig. 11, we can see that the enhancement of heat transfer increased form near 1 at lower  $Re_A$  value less than  $10^3$  and lower  $Re_R$  value less than 4. The enhancement ratio is less than 10 for  $Re_A < 0.8 \times 10^4$  or  $Re_R < 10$ . It means that in laminar flow and quasi-steady turbulent regions, the heat transfer enhancement is small and in order of 1. This similarity is consistent with the previous discussion on the quasi-steady turbulent flow has similar drag formation as laminar flow in (14). For oscillating turbulent boundary layer flow region, heat transfer enhancement ratio can be in order higher than 10. At higher  $Re_A$  and  $Re_R$  region such as  $Re_R = 5255.0$  and  $Re_A = 10^{7.5}$ , the heat transfer enhancement ratio  $\overline{Nu_R} / \overline{Nu_C}$  can be as much as 500. From Fig. 9, we can see that increase of both  $Re_R$  and  $Re_A$ , i.e. increase of the pipe diameter  $R$  and the amplitude of the oscillating flow  $A$ , can definitely increase the heat transfer enhancement ratio. It is obviously that if we use larger pipe at places, where sea water oscillating amplitude as much as possible, we can get more effective heat transfer. Thus, we can remove thermal energy more quickly through sea water cooling by increase the oscillating amplitude and the pipe diameter. However the effect of the  $KC$  number on the heat transfer enhancement is not clear in Fig. 9, so in the following section, we will present a clear picture, which shows the effects of the  $KC$  numbers and  $Re$  numbers.

#### B.5 Optimal Keulegan-Carpenter number and Tube Diameter

In order to show the effects of the Keulegan-Carpenter number  $KC$  and the Reynolds number  $Re$  on the heat transfer,  $\log(\overline{Nu_R})$  in form of coordinates  $(\log(KC), \log(Re))$  is shown in Fig. 10. From Fig.10 we can see that  $\log(\overline{Nu_R})$  increase dramatically with  $\log(Re)$ . Now that  $Re$  is linear to

the velocity scale  $2\pi fA$ , we should put sea water cooling heat exchangers on sea floors, where both the frequency  $f$  and the amplitude  $A$  of the ocean wave are as large as possible. The circular white balls in Fig. 10 show the positions of the maximum  $\overline{Nu_R}$  for a wide range of fixed  $Re$  number from 5 to  $10^5$ . It is clear that  $\overline{Nu_R}$  has a maximum value near the line of  $\log(KC) \approx 1.8$ , i.e. the optimal Keulegan-Carpenter number is about  $10^{1.8}$  for heat transfer. Hence, in order to enhance the heat transfer between the tube wall and the sea water, the optimal  $KC$  number is about 63. Thus, the optimal size for the heat exchanger pipes should be around  $R/A \approx 2\pi/10^{1.8}$ , i.e. the inner diameter of the heat exchanger pipe over the wave oscillating amplitude should be  $D/A \approx 4\pi/10^{1.8}$ . So the optimal ratio of the inner diameter of the heat exchanger pipe over the ocean wave oscillating amplitude is  $D/A \approx 0.2$ .

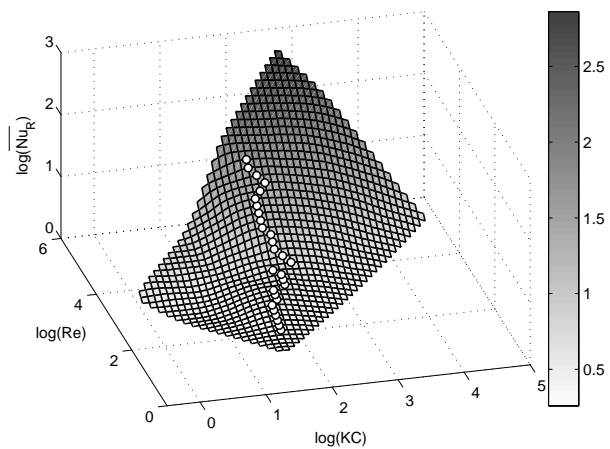


Fig. 10 Three dimensional plot of  $\log(\overline{Nu_R})$  in term of coordinates  $(\log(KC), \log(Re))$ .

## V. CONCLUSIONS

Fluid flow and heat transfer in oscillating pipe flows have been studied using RANS method with Saffman's turbulence model over wide ranges parameters to reveal the full structures of the fluid flow and heat transfer in oscillating pipe flows. For low  $Re_A$ , the flows are laminar and the present computed results are in excellent agreement with those predicted from a laminar-flow analytical solution. For laminar flow, the flow characteristics depend solely on  $Re_R$ , with  $Re_R \gg 1$  corresponding to the Stokes layer flow limit and  $Re_R \ll 1$  to the quasi-steady laminar flow limit. As  $Re_A$  increases, the high  $Re_R$  Stokes layer flow becomes unstable at approximately  $(Re_A)_{cr} = 0.8 \times 10^4$  and experiences through a turbulent boundary layer flow regime before reaching a quasi-steady turbulent flow regime as  $Re_A \rightarrow \infty$ , while the low  $Re_R$  quasi-steady laminar flow transits directly, with a delay, to the quasi-steady turbulent flow. This value of  $(Re_A)_{cr} = 0.8 \times 10^4$  agrees very well with the experimental results of Ohmi [10]. For turbulent oscillating flow, present numerical results ( $Re_R = 224.767$  and  $Re_A = 6.2 \times 10^3$ ) and the experimental data of Akhavan [11] agree very well on both amplitude and phase

angle. By the way of analyzing the overall Nusselt Number over wide ranges of the Reynolds number and Keulegan-Carpenter number, we know that  $\overline{Nu}_R$  increase dramatically with the Re number and we also obtain the optimal  $KC$  number is about  $10^{1.8}$  for a wide range of Re from 5 to  $10^5$ . So the best place to put pipe heat exchangers for the application of sea water cooling is to choose the ocean floor, where both the frequency  $f$  and the amplitude  $A$  of the ocean water waves are as large as possible. The optimal ratio of the inner diameter of the heat exchanger pipe  $D$  over the wave oscillating amplitude  $A$  is  $D/A \approx 0.2$ .

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