

Applying Similarity Theory and Hilbert Huang Transform for Estimating the Differences of Pig's Blood Pressure Signals between Situations of Intestinal Artery Blocking and Unblocking

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Abstract—A mammal's body can be seen as a blood vessel with complex tunnels. When heart pumps blood periodically, blood runs through blood vessels and rebounds from walls of blood vessels. Blood pressure signals can be measured with complex but periodic patterns. When an artery is clamped during a surgical operation, the spectrum of blood pressure signals will be different from that of normal situation. In this investigation, intestinal artery clamping operations were conducted to a pig for simulating the situation of intestinal blocking during a surgical operation. Similarity theory is a convenient and easy tool to prove that patterns of blood pressure signals of intestinal artery blocking and unblocking are surely different. And, the algorithm of Hilbert Huang Transform can be applied to extract the character parameters of blood pressure pattern. In conclusion, the patterns of blood pressure signals of two different situations, intestinal artery blocking and unblocking, can be distinguished by these character parameters defined in this paper.

Keywords—Blood pressure, spectrum, intestinal artery, similarity theory and Hilbert Huang Transform.

I. INTRODUCTION

BLOOD pressure signal is a non-stationary time series. Standard spectral analysis by Fourier Transform (FT) can provides a spectrum of the signal as well as stationary signals [1]. A method for time-frequency decomposition (SDA) was presented for the analysis of cardiovascular signals, during steady state as well as under transient conditions [2]. Means of Fourier Transform for spectral analysis provides a spectral decomposition of the signal [3,4]. Most of the methods available for processing non-stationary data still depend on Fourier analysis. They are limited to linear systems only [5]. Hilbert Huang Transform (HHT) is a new method for analyzing nonlinear and non-stationary data. The key part of the method is the 'Empirical Mode Decomposition (EMD)' method with

which any complicated data set can be decomposed into a finite and often small number of 'Intrinsic Mode Function (IMF)' that admit well-behaved Hilbert transforms [5], from which the Instantaneous Frequency (IF) can be calculated. Thus, the local energy and the instantaneous frequency derived from the IMFs through the Hilbert Huang transform can give us a full energy-frequency-time distribution of the data [5]. Therefore, Hilbert-Huang transform is widely applied to analyze different nonlinear time series (i.e. earthquake [6], surface waveform of ocean [7] and bio-signals [8]). In 1998, Huang et al. applied HHT for engineering analysis of blood pressure [8].

Hence, in this investigation, a surgical operation was conduct to a healthy pig by a legal surgeon. During the operation, two different situations, intestinal artery blocking and normal situation, were simulated by clamping intestinal artery and relaxing intestinal artery. Each situation was remained for 1 minute and exchanged for twice. Totally, 4 minutes recording of blood pressure signal was taken for off-line analysis. Whole recording was divided to 4 sections, two sections of experimental recordings and two sections of contrastive recordings.

Before identifying the character parameters of blood pressure signal, similarity theory was applied for confirming that the difference between two sets of recordings with different situations surely exists. A measurement of similarity between two complex signals was proposed by Yang et al. in 2003 [9]. Heart rate recordings of different groups (i.e., healthy young, healthy elderly, congestive heart failure, and arterial fibrillation) were successfully assorted. In this paper, the similarity measurement is applied for intra-section and inter-section comparisons. Results of comparisons show that two recordings of different situations have different patterns of signals because of bigger weighted distance between similarity measurements of time series of different situations.

When the existence of difference between two sets of recordings with different situations was confirmed, HHT algorithm was applied for decomposing the sampled recordings. The first eight IMFs of recordings were extracted and energy-frequency-time distributions of IMFs were also derived. Then, the concept of gravity center was used to define the central frequency of IMF and average kinetic energy in the fixed interval as two character parameters of IMF. Finally, eight pairs of character parameters can be applied for

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expressing the pattern of recording.

II. ANALYSIS ALGORITHM

A. Measurement of Similarity

Consider an blood pressure time series, $\{x_0, x_1, x_2, \dots, x_N\}$, where x_i is the voltage value of the i th sampling point of blood pressure recording. The complex time series can be simplified via mapping time series to binary sequences, where the increase and decrease of voltage values are denoted by 1 and 0. This mapping can be expressed as below:

$$I_n = \begin{cases} 0, & \text{if } x_n \leq x_{n-1}, \\ 1, & \text{if } x_n > x_{n-1}, \end{cases} \quad (1)$$

Then, we map successive binary sequence of length 8 called an 8-bit word. Each word represents a unique pattern of fluctuations in the time series. By shifting one sampling point at a time, a collection of 8-bit words over the whole time series is derived. Count the frequencies of occurrences of different words and sort them according to descending frequency. We obtain the ranks of frequency distribution. This set of ranks of words represents the statistical hierarchy of symbolic words of the original time series.

To define a measurement of similarity between two time series, a weighted distance, D_m , between two symbolic sequences, S_1 and S_2 , can be expressed as below:

$$D_m(S_1, S_2) = \frac{\sum_{k=1}^{2^m} |R_1(w_k) - R_2(w_k)| \cdot p_1(w_k) p_2(w_k)}{(2^m - 1) \sum_{k=1}^{2^m} p_1(w_k) p_2(w_k)} \quad (2)$$

Where $p_i(w_k)$, and $R_i(w_k)$ represent probability and rank of a specific word, w_k , in time series S_i , $i=1$ or 2 . Two time series with similar patterns of fluctuations have similar probabilities and ranks of words, and result a smaller distance. Contrastively, two time series with different patterns of fluctuations derives a bigger distance. That's the reason we apply similarity theory to make sure that the time series of blood pressure signals of different situations are different and those of similar situations are similar.

B. Empirical Mode Decomposition (EMD)

In this investigation, the difference between two time series of blood pressure signals with different situations was proven by similarity theory. The next target for our study is to identify what is the difference. The empirical mode decomposition is a new method for analyzing nonlinear and non-stationary data. It can be applied to decompose a nonlinear and non-stationary time series to several IMF components. Here, the character parameters of intrinsic mode function are defined and calculated as a set of character parameters of time series data.

An IMF is a function that satisfies two conditions: (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one. (2) at any point, the mean value of the envelop defined by the local maxima and the envelop defined by the local minima is zero.

So, Hilbert Transform can provide the description of the frequency content for an IMF component as part of original time series data.

The decomposition method is based on the assumptions. (1) the signal has at least two extrema- one maximum and one minimum ; (2) the characteristic time scale is defined by the time lapse between the extrema; and (3) if the data were totally devoid of extrema but contained only inflection points, then it can be differentiated once or more times to reveal the extrema. Final results can be obtained by integration(s) of components. IMF simply use the envelopes defined by the local maxima and minima separately. All the local maxima are connected by a cubic spline line of identified maxima as the upper envelop. Repeat the procedures for local minima to produce the lower envelop. The upper and lower envelopes should cover all the data between them. Their mean is designated as m_1 , and the difference between the data and m_1 is the first component, h_1 , i.e.

$$X(t) - m_1 = h_1 \quad (3)$$

The process for deriving a component by producing mean of envelopes and calculating the difference between data and m_1 is named a sifting process. The sifting process serves two purposes: to eliminate riding waves; and to make the wave-profiles more symmetric. The sifting process has to be repeated more times. In the second sifting process, h_1 is treated as the data, then

$$h_1 - m_{11} = h_{11} \quad (4)$$

The sifting process is repeated k times, until the criterion of stopping shifting is satisfied. The criterion can be accomplished by limiting the size of the standard deviation, SD, computed from the two consecutive sifting results as

$$SD = \sum_{t=0}^T \left[\frac{|(h_{1(k-1)}(t) - h_{1k}(t))|^2}{h_{1(k-1)}^2(t)} \right] \quad (5)$$

A typical value for SD can be set between 0.2 and 0.3. When the criterion is satisfied, the result after k siftings is designated as $c_1 = h_{1k}$, the first IMF component from the data. The first IMF contains the finest scale or shortest period component of the signal. The rest of data can be derived by calculating the difference between the original data and the first IMF.

$$r_1 = X(t) - c_1 \quad (6)$$

Since the residue, r_1 , still contains information of longer period components, it is treated as the new data and subjected to the same sifting process as described above. Then, n IMFs and the n -th residue can be obtained. The relationship among the original data, IMFs, and the n -th residue can be expressed below:

$$X(t) = r_n + \sum_{i=1}^n c_i \quad (7)$$

In this study, the first eight IMFs are derived for identifying the character parameters of pattern of blood pressure signals.

C. Instantaneous Frequency

Instantaneous frequency is accepted only for special 'mono-component' signals[10,11]. For an arbitrary time series, $X(t)$, We can always have its Hilbert Transform, $Y(t)$, as

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt' \quad (8)$$

Where P indicates the Cauchy principal value. This transform exists for all functions of class L^p [12]. With this definition, $X(t)$ and $Y(t)$ form the complex conjugate pair, so we can have an analytic signal, $Z(t)$, as

$$Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)} \quad (9)$$

in which, the amplitude, $a(t)$, and phase, $\theta(t)$, can be derived as

$$a(t) = \sqrt{X(t)^2 + Y(t)^2}; \theta(t) = \arctan\left(\frac{Y(t)}{X(t)}\right) \quad (10)$$

Thus, instantaneous frequency can be defined as

$$w(t) = \frac{d\theta(t)}{dt} \quad (11)$$

In principle, some limitations on the data are necessary, for the instantaneous frequency given in eq. (11) is a single value function of time. At any given time, there is only one frequency value; therefore, it can only represent one component, hence 'mono-component'. But, there is no clear definition of the 'mono-component'. 'Narrow band' was adopted as a limitation on data for the instantaneous frequency to make sense.

The instantaneous frequency and instantaneous amplitude derived from the IMFs through the Hilbert transform can give us a full energy-frequency-time distribution of the data. The local energy at any time point is the square of the instantaneous amplitude and expressed as

$$E(t) = a(t)^2 \quad (12)$$

D. Central Frequency and Averaged Local Energy

For any IMF extracted by EMD method has fluctuating instantaneous frequency and instantaneous amplitude, it is difficult to define the central frequency and average local energy of an IMF as character parameters of IMF. In this paper, the central frequency of IMF is defined as the central frequency of energy distribution and calculated by eq. (13).

$$f_{\text{central}} = \frac{\int_{t_0}^{t_0+\Delta t} E(t) \cdot w(t) dt}{\int_{t_0}^{t_0+\Delta t} E(t) dt} \quad (13)$$

Where t_0 is time of the first sampling point, and Δt is sampling time interval. In eq.(13), the denominator part is the total energy of IMF during the fixed interval. We also define the average local energy as the total energy divided by sampling time interval and expressed as

$$E_{\text{ave}} = \frac{\int_{t_0}^{t_0+\Delta t} E(t) dt}{\Delta t} \quad (14)$$

In this study, central frequency and averaged local energy are defined as two character parameters for a IMF and the first eight IMFs are extracted from a time series of blood pressure signal. Thus, there are eight sets of character parameters to determine the pattern of a time series data. Comparing patterns of two time series datum by checking their character parameters can give us more information about the differences of energy distribution and central frequency.

III. MATERIAL AND METHODOLOGY

In this investigation, the datum of time series of blood pressure signal were obtained by an experimentally surgical operation conducted to a healthy pig. During this surgical operation, pig's intestinal artery was blocked by clamping for one minute and relaxed the clamping for next minute for producing two time series datum for two different situations, intestinal artery blocking and unblocking. This procedure was repeated two times consecutively and four-minute time series data was recorded as Fig. 1.

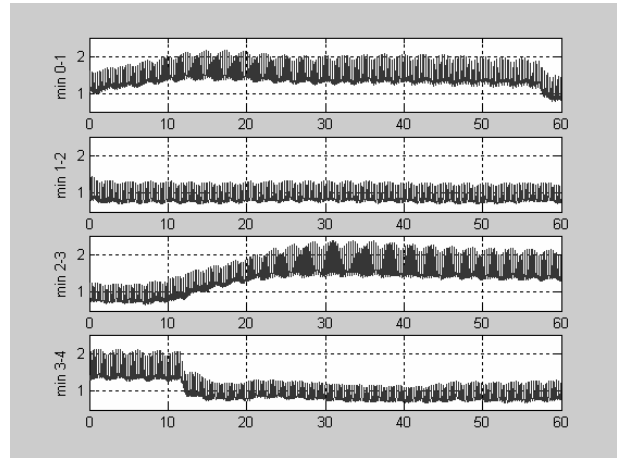


Fig. 1 The four-minute time series data of blood pressure signal

Based on the recording of blood pressure signal shown in Fig. 1, recordings of the first and the third minutes are experimental data, data recorded in the situation of intestinal artery blocking, noted as section 1 and section 3. Recordings of the second and the fourth minutes are control data, data recorded in the situation of intestinal artery relaxing, noted as section 2 and section 4. It is clear to be observed by Fig. 1 that the transition state always happens when situations exchanged. Excluding the data of transition state, we captured a twelve-second recording as a sample and took two samples from a section. Then, we have eight samples of datum, four

experimental data and four control data. The notations of samples and sampling intervals are shown in Table I.

TABLE I
NOTATIONS AND TIME INTERVALS OF SAMPLES OF BLOOD PRESSURE RECORDING. THE COLUMNS WITH GREY BACKGROUND MEAN EXPERIMENTAL DATA AND THOSE WITH WHITE BACKGROUND MEAN CONTROL DATA

| Sample ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------|------|------|------|------|------|------|------|------|
| Time | 0:18 | 0:30 | 1:10 | 1:30 | 2:28 | 2:40 | 3:48 | 3:30 |
| Interval | 0:30 | 0:42 | 1:22 | 1:42 | 2:40 | 2:52 | 4:00 | 3:42 |

IV. RESULTS

A. Results of Comparisons by the Measurement of Similarity

Each sample compared to any other sample by similarity theory. The results of comparisons can be noted by quantitative values, the distance of two measurements of similarity, shown in Table II. When the distance between two samples is small means two samples have similar patterns. Otherwise, two samples have relatively different patterns.

For Table II, it is clear that the distance between two samples both belonging to experimental data or control data is smaller than that between two samples in different situations. The value of weighted distance between two samples in similar situation is in the range from 0.0004 to 0.0073, and the value of distance between two samples of different situations is in the range from 0.026 to 0.035. By the similarity theory, the two groups of samples can be separated easily because of the different measurements of similarity.

B. Results of HHT Analysis

The samples of data are non-linear and non-stationary time series and were treated by HHT method. In the procedure of HHT, eight IMFs were extracted from original data. The IMFs of experimental data and control data are shown as Fig. 2(a) and Fig 2(b). The IMF 4 and IMF 5 have clearly different patterns between experimental data and control data. For IMF 4, the waveform is conspicuous for experimental data but plain for control data. For IMF 5, the amplitude of experimental data is larger than that of contrastive data.

Each IMF can derive time-IF distributions, shown in Fig. 3,

time-amplitude distributions, shown in Fig. 4, and time-energy distributions, shown in Fig. 5. The sample 2 presents the characteristics of experimental data and the sample 4 presents the characteristics of control data.

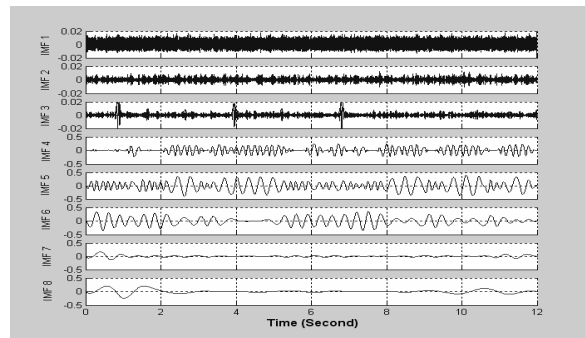


Fig. 2(a) The first eight IMFs of Sample 2

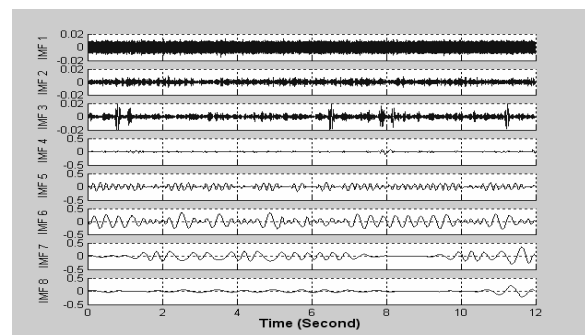


Fig. 2(b) The first eight IMFs of Sample 4

Fig. 3 shows the time-amplitude distributions of sample 2 and sample 4. The distributions of IMF 4 and 5 have clearly difference between experimental and control data. We also can observe there are many differences between the time-IF distributions and time-energy distributions of those two groups of data. But, it is hard to describe what the difference is on these figures. Thus, the central frequency and averaged local energy are defined for identifying the characteristics of IMFs. These parameters of IMFs of sample time series were calculated and shown in Table III.

TABLE II
THE RESULTS OF COMPARISONS AMONG THE SAMPLES OF BLOOD PRESSURE SIGNAL. IN THIS TABLE, COLUMNS WITH LIGHTLY GREY BACKGROUND MEAN THE WEIGHTED DISTANCES RESULTED BY TWO SAMPLES IN SIMILAR SITUATION AND COLUMNS WITH DARK GRAY BACKGROUND MEAN SAMPLES OF EXPERIMENTAL DATA

| | | ID of sample | | | | | | | |
|--------------|---|--------------|----------|----------|----------|----------|----------|----------|----------|
| Distance | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| ID of sample | 1 | 0. | 0.000435 | 0.027042 | 0.028088 | 0.005991 | 0.004708 | 0.029681 | 0.030757 |
| | 2 | | 0 | 0.026157 | 0.027025 | 0.006095 | 0.004935 | 0.028702 | 0.029320 |
| | 3 | | | 0 | 0.004260 | 0.029561 | 0.027888 | 0.005557 | 0.007269 |
| | 4 | | | | 0 | 0.031891 | 0.029229 | 0.002698 | 0.004382 |
| | 5 | | | | | 0 | 0.002017 | 0.033445 | 0.034771 |
| | 6 | | | | | | 0 | 0.030957 | 0.031755 |
| | 7 | | | | | | | 0 | 0.005794 |
| | 8 | | | | | | | | 0 |

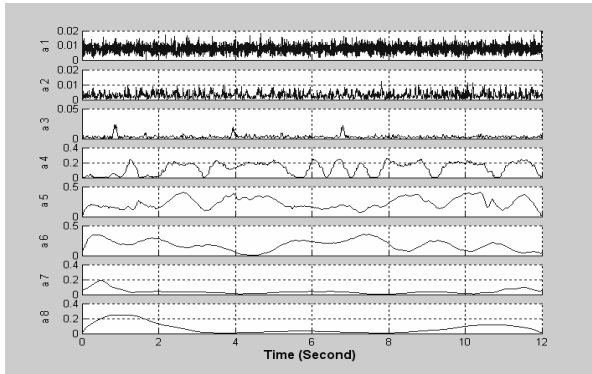


Fig. 3(a) Time-amplitude distribution of Sample 2

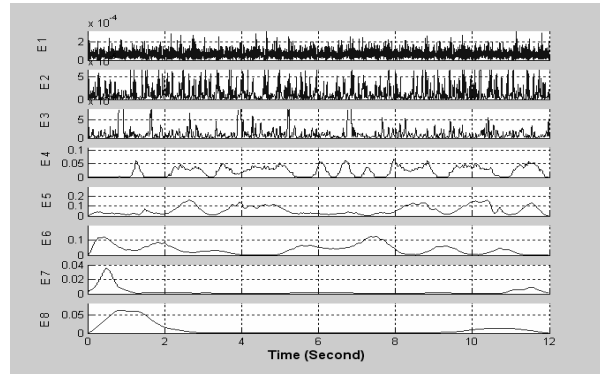


Fig. 5(a) Time-energy distribution of Sample 2

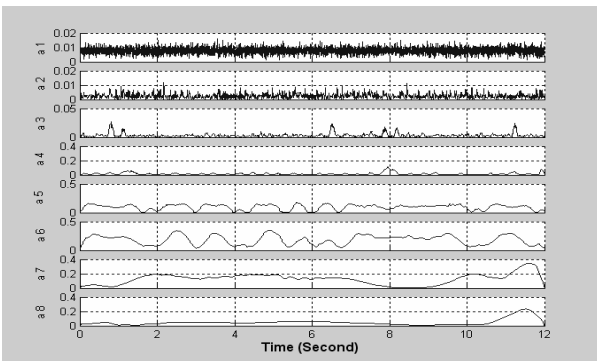


Fig. 3(b) Time-amplitude distribution of Sample 4

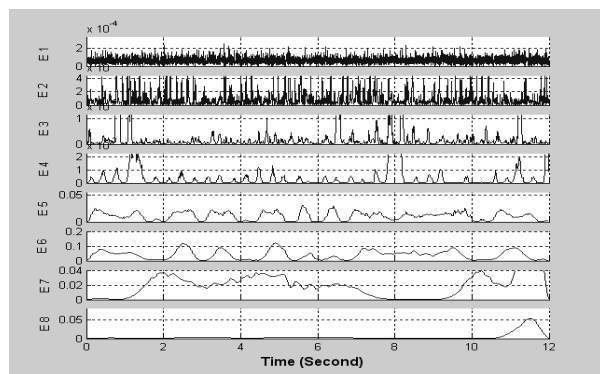


Fig. 5(b) Time-energy distribution of Sample 4

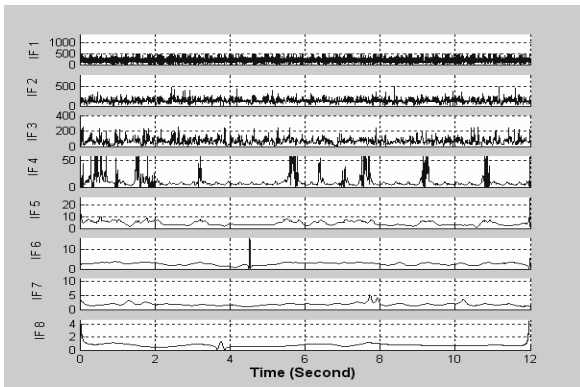


Fig. 4(a) Time-IF distribution of Sample 2

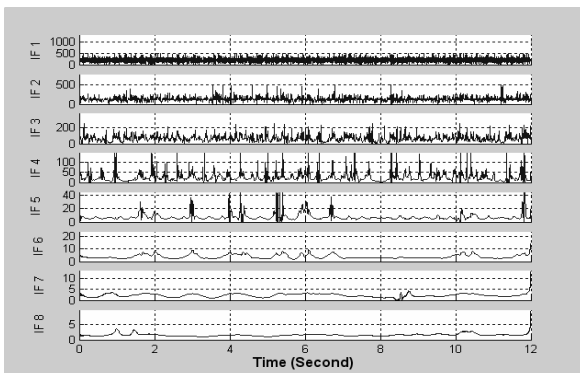


Fig. 4(b) Time-IF distribution of Sample 4

The results shown in Table III explicate the differences between the data of experimental and control groups by central frequencies and averaged local energy of IMFs. They have totally different central frequencies and averaged local energies in IMF 4 and IMF 5. Especially, the fourth central frequencies of control data are bigger than those of experimental data and the averaged local energies of IMF 4 for experimental data are ten times of those of control data.

In this study, the essential frequency of the blood pressure signal is around 3 Hz for a pig. The central frequencies of the first three IMFs are much higher than the essential frequency and their averaged local energies are relatively small. They contribute little information for distinguishing the difference between experimental and control data. Moreover, the central frequencies of the IMF 4, IMF 5 and IMF 6 are located in the interval from 3 Hz to 6 Hz. These frequencies represent the first and second harmonic frequencies of blood pressure signals. Based on the results of analysis by HHT, there are significantly differences between samples of two groups. Thus, central frequency and averaged local energy can be applied as indicators for verifying the characteristics of IMF.

V. CONCLUSION

Table II shows the results of analysis by similarity theory. Similarity theory is a powerful tool for verifying the difference of fluctuation pattern between two time series. But, it can't be applied to identify in what situation the time series is recorded.

TABLE III

CENTRAL FREQUENCIES AND AVERAGED LOCAL ENERGIES OF THE FIRST EIGHT IMFS FOR EIGHT SAMPLES. IN THIS TABLE, COLUMNS WITH GREY BACKGROUND MEAN DATA OF EXPERIMENTAL DATA AND THOSE WITH WHITE BACKGROUND MEAN DATA OF CONTROL DATA

| | | Sample ID | | | | | | | |
|-------|--------------|-----------|----------|----------|----------|----------|----------|----------|----------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| IMF 1 | Central freq | 224.769 | 224.457 | 217.839 | 216.882 | 232.714 | 231.014 | 217.434 | 218.549 |
| | Averaged LE | 0.000065 | 0.000063 | 0.000063 | 0.000064 | 0.000062 | 0.000061 | 0.000065 | 0.000063 |
| IMF 2 | Central freq | 141.535 | 146.686 | 132.693 | 133.067 | 148.768 | 154.006 | 125.246 | 137.380 |
| | Averaged LE | 0.000012 | 0.000013 | 0.000010 | 0.000009 | 0.000016 | 0.000015 | 0.000009 | 0.000010 |
| IMF 3 | Central freq | 58.398 | 40.147 | 34.789 | 31.275 | 26.277 | 48.620 | 25.438 | 43.652 |
| | Averaged LE | 0.000010 | 0.000016 | 0.000018 | 0.000023 | 0.000041 | 0.000013 | 0.000033 | 0.000011 |
| IMF 4 | Central freq | 6.297 | 6.038 | 7.464 | 9.843 | 6.591 | 6.622 | 7.024 | 15.227 |
| | Averaged LE | 0.015552 | 0.022150 | 0.001275 | 0.000456 | 0.017626 | 0.014590 | 0.001591 | 0.000266 |
| IMF 5 | Central freq | 3.905 | 3.578 | 5.870 | 5.788 | 3.743 | 4.181 | 4.974 | 6.289 |
| | Averaged LE | 0.058352 | 0.059783 | 0.013181 | 0.011054 | 0.094856 | 0.064959 | 0.015907 | 0.005998 |
| IMF 6 | Central freq | 3.015 | 2.937 | 3.030658 | 3.006 | 3.071 | 3.245 | 3.106 | 3.159 |
| | Averaged LE | 0.046340 | 0.039263 | 0.049859 | 0.040046 | 0.077276 | 0.075204 | 0.044932 | 0.030376 |
| IMF 7 | Central freq | 1.921 | 1.688 | 1.888 | 2.293 | 1.718 | 0.891 | 1.216 | 2.755 |
| | Averaged LE | 0.004723 | 0.002606 | 0.018962 | 0.021455 | 0.007839 | 0.011667 | 0.002624 | 0.009451 |
| IMF 8 | Central freq | 0.565 | 0.826 | 0.859 | 1.499 | 0.639 | 0.504 | 0.523 | 1.161 |
| | Averaged LE | 0.031976 | 0.009403 | 0.019732 | 0.004196 | 0.030159 | 0.036562 | 0.009057 | 0.003496 |

EMD provides another powerful tool for decomposing a complex time series to limited number of IMFs. Each IMF represents fluctuation of a character band. In this study, IMF 4, IMF 5 and IMF 6 present the most important fluctuation patterns of blood pressure signal. On the other hand, the phenomenon of mode shifting can be observed in Fig. 2. We can figure out that there two or three different patterns of fluctuations in an IMF. The patterns of fluctuation in some segments are different from those in most segments of the same IMF. Those patterns are similar to the pattern of the last or next IMF. The phenomenon of mode mixing should be solved in the following research.

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