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Research of Linear Camera Calibration Based on Planar Pattern

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***Abstract*—**An important step in three-dimensional reconstruction and computer vision is camera calibration, whose objective is to estimate the intrinsic and extrinsic parameters of each camera. In this paper, two linear methods based on the different planes are given. In both methods, the general plane is used to replace the calibration object with very good precision. In the first method, after controlling the camera to undergo five times' translation movements and taking pictures of the orthogonal planes, a set of linear constraints of the camera intrinsic parameters is then derived by means of homography matrix. The second method is to get all camera parameters by taking only one picture of a given radius circle. experiments on simulated data and real images,indicate that our method is reasonable and is a good supplement to camera calibration.

***Keywords*—**camera calibration, 3D reconstruction, computer vision

I. Introduction

Camera calibration is a necessary and very important step in three-dimensional reconstruction in order to extract metric information from 2D images. One of the tasks of computer vision is to calculate the geometry information of the image by the camera and then reconstruct and recognize the object based on this information. The relation between the points on the object surface(3D) and the corresponding points on the images(2D) lies on the geometrical model of the camera. The parameters of the geometrical model are the different combination of the camera parameters. The process to get these parameters by the calculation and the experiments is camera calibration [1].

Much work has been done. The methods of camera calibration mainly include: the traditional methods, the self-calibration methods and the active motion based methods.

Traditional method is also called photogrammetric calibration[2-3]. Calibration is performed by observing a calibration object, as shown in figure 1, whose geometry in 3D space is known with very good precision. Using image process and mathematical translation, all camera parameters will be gotten and calibration can be done very efficiently. This method is under the limited conditions, such as calibration object with given figure and size. So traditional calibration requires an expensive calibration apparatus and an elaborate setup.

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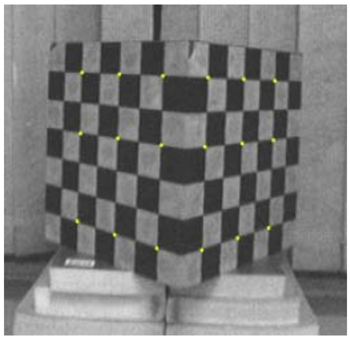

Fig. 1. The calibration object

Manybank and Faugeras first proposed the self-calibration based on Kruppa equation [3]. Many researchers put forward some similar methods recently [4-8]. All of these methods are based on the absolute conic and absolute quadric, which need to work out the non-linear equations of several values. Though this method is flexible, it has large computational complexity. The greatest shortcoming of self-calibration is the lack of robustness. For solving these problems, some researchers advance the active motion based camera calibration.

In active methods, camera calibration is undergoing with the known motion of the camera [9]. The known motion information includes the quantitative and qualitative information. Up to now, active methods focus on linearly working out the model parameters with minimum limitations to the motion of the camera. However, minimum limitation doesn't means without any restriction. If there are no limitations, the calibration goes back to solve the nonlinear optimization and this method is the same with the self-calibration.

In this paper, two different linear camera calibrations are put forward. In both methods, the general plane is used to replace the calibration object with very good precision. With the characters of the orthogonal planes, the homography matrix in the first method defines the constraint equation of the intrinsic parameters. In second method, we get the all camera parameters by taking only one picture of a given radius circle at the optional direction. It uses the coordinates of the special points and ellipse equation to define the constrain equations of the parameters of the camera.

II. Camera Model and Basic Equations

The camera parameters are always relative to some kind of the geometric model. The pinhole is the basic model of the camera calibration, which is showed in figure 2. Camera calibration is the method of parameterizing and acquiring with accurate geometric knowledge of the camera system.In general, camera calibration should calculate the following parameters:

1. Intrinsic parameter, which describe the optic and geometric characters of camera, such as the focus, the scale factor and lens distortion;
2. Extrinsic parameters, which describe the camera coordination relative to the word coordination system, such as the rotation and translation.

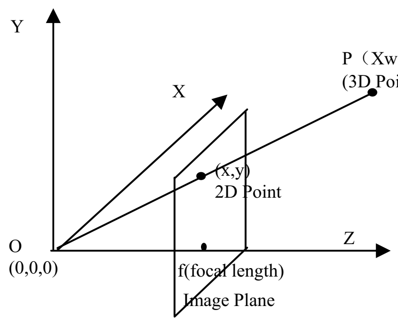

Fig. 2. The model of pinhole

Usually, the camera intrinsic parameters matrix is expressed as following:

$$\mathbf{K}=\begin{bmatrix}1/d_x & s & u_0\\ 0 & 1/d_y & v_0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}f & 0 & 0\\ 0 & f & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}a_x & s & u_0\\ 0 & a_y & v_0\\ 0 & 0 & 1\end{bmatrix} \quad (1)$$

Then the relation between the coordinates of the point $\mathbf{X}$=(X_w, Y_w, Z_w, 1) in world coordination system and its corresponding point **U**=(u, v, 1) in projected image is:

$$\lambda\mathbf{U}=\mathbf{K}(\mathbf{R}\quad\mathbf{T})\mathbf{X}=\mathbf{MX} \quad (2)$$

Where $a_x=f/d_x$, $a_y=f/d_y$, (u_0,v_0) delegates the intersection point of the optical axis and the image plane, that is, the coordinates of the principal point. Each pixel's sizes in x-axis and y-axis in the image are d_x, d_y respectively, and s means the distortion factor. λ is an arbitrary scale factor and means that equation 2 would have the exclusive meanings without the non-zero constant factor. (**R** ,**T)** called the extrinsic parameters, is the rotation and translation matrixes which relates the camera coordination system to the world coordination system.

III. Solving Strategy of the Two Proposed Calibration

A. Camera Calibration based on the Orthogonal Planes

When the points in the 3D word coordinate system are in the same plane π, the homography matrix can exclusively determine the relation of the corresponding points in the two projected images [10-11].

$$\mathbf{U}_2\approx\mathbf{HU}_1 \quad (3)$$

Where, ≈ indicates having the same meanings without the non-zero constant. When the plane π has no crossing point with the optics center of the camera, the homography matrix rank is three. And the corresponding point of a certain point can be uniquely attained by equation 3. The equation of the plane π is expressed as **nX**=d, where n is the unit normal vector of the plane π, and d denotes the distances from the origin of coordinates to the plane π, then, there are some equations as following.

$$\mathbf{U}_1=\lambda_1\mathbf{KX} \quad (4)$$

$$\begin{aligned}\mathbf{U}_2&=\lambda_2\mathbf{KX}'=\lambda_2\mathbf{K}(\mathbf{R}+\mathbf{T})\mathbf{X}\\&=\lambda_2\mathbf{K}(\mathbf{RX}+\frac{1}{d}\mathbf{TnX})=\frac{\lambda_1}{\lambda_2}\mathbf{K}(\mathbf{R}+\frac{1}{d}\mathbf{Tn})\mathbf{K}^{-1}\mathbf{U}_1\end{aligned} \quad (5)$$

Consequently, the homography matrix can be represented as following.

$$\mathbf{H}=\sigma\mathbf{K}(\mathbf{R}+\frac{1}{d}\mathbf{Tn})\mathbf{K}^{-1} \quad (6)$$

The matrix having difference of a non-zero constant with homography matrix is also called homography matrix. Especially, the matrix in equation 6 without σ is standard homography matrix.

When the camera only has the translation movements, that is, **R**=**I**. Then, we can get:

$$\mathbf{H}=\sigma\mathbf{K}(\mathbf{I}+\frac{1}{d}\mathbf{Tn})\mathbf{K}^{-1} \quad (7)$$

As for a pair of orthogonal planes, π1: $\mathbf{n_1X}=d_1$, π2: $\mathbf{n_2X}=d_2$, because the orthogonality of these two planes, then we can easy attain that $\mathbf{n_1n_2^T}=0$. The two homography matrixes: $\mathbf{H_1}$, $\mathbf{H_2}$, of the two orthogonal planes: π1, π2, can be determined by the camera's translation movement (**I, T**) respectively.

$$\begin{cases}\mathbf{H_1}-\sigma_1\mathbf{I}=\frac{1}{d_1}\sigma_1\mathbf{KTn_1K^{-1}}\\ \mathbf{H_2}-\sigma_2\mathbf{I}=\frac{1}{d_2}\sigma_2\mathbf{KTn_2K^{-1}}\end{cases}\Rightarrow\begin{cases}\mathbf{K^{-1}(H_1}-\sigma_1\mathbf{I)K}=\frac{1}{d_1}\sigma_1\mathbf{Tn_1}\\ \mathbf{K^{-1}(H_2}-\sigma_2\mathbf{I)K}=\frac{1}{d_2}\sigma_2\mathbf{Tn_2}\end{cases} \quad (8)$$

According to $\mathbf{n_1n_2^T}=0$, we can get:

$$(\mathbf{H_1}-\sigma_1\mathbf{I})\mathbf{KK^T}(\mathbf{H_2^T}-\sigma_1\mathbf{I})=0 \quad (9)$$

Let's $\mathbf{C=KK^T}$, then:

$$(\mathbf{H_1}-\sigma_1\mathbf{I})\mathbf{C}(\mathbf{H_2^T}-\sigma_1\mathbf{I})=0 \quad (10)$$

Then we get:

$$\mathbf{C}=\begin{bmatrix}c_1 & c_2 & c_3\\ c_2 & c_4 & c_5\\ c_3 & c_5 & 1\end{bmatrix}$$

Since rank ($\mathbf{KTnK}^{-1}$)=1, we can get rank($\mathbf{H}$-σ$\mathbf{I}$)=1. There is only one linear constrain of $\mathbf{C}$. It needs five constraint equations to determine $\mathbf{C}$ at least. Therefore, the camera should perform five times' translation movements. Then we can get the intrinsic matrix $\mathbf{K}$ by Cholesky factorization.

By above steps, the homography matrix $\mathbf{H}$ can be determined after the camera's translation movement every time. Generally speaking, the homography matrix $\mathbf{H}_{3\times3}$ can be obtained through four pairs of corresponding points in different images. At the same time, because of rank ($\mathbf{H}$-σ$\mathbf{I}$)=1, then as for the entire second determinant of ($\mathbf{H}$-σ$\mathbf{I}$), we can get det $(\mathbf{H}-\sigma\mathbf{I})_{2\times2}=0$. Then we can work out the the unique value of σ.

The process of determining the corresponding relation of the points in two images is important. It needs to match the characters of the different images. Character match is the focus and difficult in computer vision. At the same time, it also needs to determine the corresponding relation of the points in the 3D plane and those in 2D images. When we select many points, the process will be complex and difficult. Simultaneity, in traditional methods with calibration object, the points projected by the 3D points do not always correspond with one pixel in 2D images. To achieve precision, we need correspond them with the pixels, even sub-pixels.

To avoid the above problems, we design a plane that has five lines: four parallel lines and a line that cuts the four parallels at four points. We use Hough Translation to get the equations of the lines and can further attain the coordinate of the intersections. Because the camera only has the translation movement, under the condition of properly arranging the position of the five lines, the relative position of the four intersections will not be changed. So we can get the corresponding relations of the four crossing points without matching. At the same time, this method gets the coordinates of the intersections by the lines equations, which avoiding corresponding these points with pixels or sub-pixels in 2D images.

According to the above analysis, the steps of the calibration based on the orthogonal planes are as following:

1. Control the camera to perform five times' translation movements;
2. Determine the homography matrixes $\mathbf{H_1}$, $\mathbf{H_2}$ of the planes π_1, π_2 after the camera's translation movement every time and calculate the σ_i;
3. According to equation 10, we can get the linear constrain equations of the intrinsic parameters of the camera.

We can attain $\mathbf{C}$ by working out the above equations and then further get the intrinsic parameters matrix $\mathbf{K}$ by Cholesky factorization.

B. Circle based Camera Calibration

The first calibration method replaces the calibration object with general planes. The calibration process is simplified to a great extent by avoiding matching the corresponding points in 2D images. However, this method confines the camera to have five times' translation moments, and we only attain camera intrinsic parameters o.

In this paper, another one is brought forward: the linear calibration method base on circle. This method inherits the first method's advantages, which using a simple plane instead of the calibration object with very good precision. In addition, it is to get all camera parameters by taking only one picture of a known radius circle with two lines crossing the center of it at any direction. The method uses Hough Transform to attain the equations of the circle and the lines and farther gets the corresponding point information by the intersections between the circle and the two lines.

Without loss of generality, we suppose Z_w of the points on the planes is equal 1, and then according to equation 2, we can get:

$$\lambda Y = \mathbf{K}(\mathbf{r}_1\ \mathbf{r}_2\ \mathbf{r}_3\quad \mathbf{T})\begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} = \mathbf{K}(\mathbf{r}_1\ \mathbf{r}_2\ \mathbf{r}_3\quad \mathbf{T})\begin{pmatrix} X_w \\ Y_w \\ 1 \\ 1 \end{pmatrix} = \mathbf{MX} \quad (11)$$

In world coordination system, the circle with the center (X_0, Y_0, Z_0, 1) and radius ρ can be expressed as the curve of intersection of the sphere and a plane, where the sphere is with center (X_0, Y_0, Z_0,1) and radius ρ, the plane is Z= Z_0. Then the equation of the circle is expressed as:

$$\begin{cases} \mathbf{X}^{\mathrm{T}}\mathbf{QX} = 0 \\ Z = Z_0 \end{cases} \quad (12)$$

where $\mathbf{X}=(X_w, Y_w, Z_0, 1)^{\mathrm{T}}$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ -X_0 & -Y_0 & -Z_0 & X_0^2 + Y_0^2 + Z_0^2 - \rho^2 \end{bmatrix} \quad (13)$$

Then the points in the circle after projection can satisfy the following equation: $\mathbf{u}^{\mathrm{T}}\mathbf{Pu}=0$，where

$$\mathbf{Q} \approx \mathbf{M}^{\mathrm{T}}\mathbf{PM} \quad (14)$$

The sign ≈ has the same denotation as that of equation 3, and means to have the exclusive meanings without the non-zero constant factor.

Usually, the circle in the plane will be projected an ellipse on the image. The equation of the ellipse can be directly attained using Hough Translation. In other words, we can directly get the matrix $\mathbf{P}$.

$$Au^2 + 2Buv + Cv^2 + 2Du + 2Ev + F = 0 \quad (15)$$

$$\begin{pmatrix} u & v & 1 \end{pmatrix}\begin{bmatrix} A & B & D \\ B & C & E \\ D & E & F \end{bmatrix}\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = 0, \quad \mathbf{P} = \begin{bmatrix} A & B & D \\ B & C & E \\ D & E & F \end{bmatrix} \quad (16)$$

We suppose that the coordinate of the circle center in world coordination system is (0, 0, 1), and then we can get:

$$\mathbf{Q}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 0 & 0 & -1 & 1-\rho^2\end{bmatrix} \tag{17}$$

Then according to equation 14, we can get:

$$\mathbf{Q}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 0 & 0 & -1 & 1-\rho^2\end{bmatrix}=\mathbf{M}^{\mathrm{T}}\mathbf{P}\mathbf{M}=\mathbf{M}^{\mathrm{T}}\begin{bmatrix}A & B & D\\ B & C & E\\ D & E & F\end{bmatrix}\mathbf{M} \tag{18}$$

Usually, after the projection, the center of ellipse isn't the center of the circle [10]. Using two lines through the center of the circle, we can easily determine the projection position (u_0, v_0) in the image of the circle center. Then according to equation 11, we can get:

$$\lambda_1\begin{pmatrix}u_0\\ v_0\\ 1\end{pmatrix}=\mathbf{M}\begin{pmatrix}0\\ 0\\ 1\\ 1\end{pmatrix} \tag{19}$$

There are six intrinsic parameters and six extrinsic parameters in matrix **M**. According to equation 16 and 19, we can get the twelve linear constraint equations. By working out these equations, we can get the matrix **M**. Then further figure out the intrinsic and extrinsic parameters of the camera..

IV. Calibration Experiments

The calibration method based on the circle has smaller calculation complexity than the first method. It doesn't need the additional parameters, such as homography matrix **H.** Therefore, the second calibration method has smaller relative error in theory than the first method. In this paper, we mainly make experiments and analysis of the calibration method based on the circle.

A. Simulation Experiment

Firstly, we use synthetic data to evaluate the performance of our algorithms in presence of noise. By calculating the intrinsic parameters at different noise levels, we can get the stability of our algorithms.

We use the FUJIFILM-FinePix6900ZOOM camera to take a picture of a circle with 70mm radius as shown in figure 3.

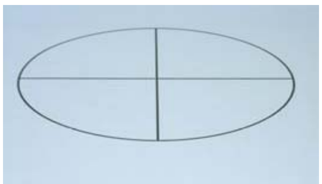

Fig. 3. The picture of a circle with 70mm radius

TABLE I
THE RESULTS OF INTRINSIC PARAMETERS IN DIFFERENT NOISE LEVELS

	a_x	a_y	u_0	v_0
0.0	8947.59267	4189.74577	463.638666	831.16547
0.1	8934.561	4178.76354	464.568974	832.15789
0.2	8957.5927	4219.74782	476.458934	839.15684
0.5	8972.37537	4232.1288	489.458964	848.59756
0.8	8997.59426	4259.7584	493.589746	853.21459
1.0	9016.45732	4281.42679	503.458923	855.14895

In table 1, Gaussian noise is added in the projected image. We vary the level from 0 to 1.0 of the variance of Gaussian noise, and its mean is zero. The results show that the error increases with the noise level and the largest relative error is 0.8627%.

B. Experiment using Real Image

The candidate camera is FUJIFILM-FinePix6900ZOOM. We take a picture of the circle with 70 mm radiuses. The image resolution is 2048×1536. Firstly, we calculate the matrix **M** shown in below.

$$\mathbf{M}=\begin{bmatrix}12.6\lambda & 0 & 6.5289597\lambda & 989.47104\lambda\\ 0 & 5.9\lambda & 11.70447219\lambda & 914.295527\lambda\\ 0 & 0 & 0.014082\lambda & 0.985917\lambda\end{bmatrix}$$

where λ =1/0.014082.

The camera to be calibrated is a super CCD camera with 1/1.7 inch's area. The diameter of its single pixel reaches 2.94366μm. According to the formula a_x=f/d_x=|m1×m2| in [2], we can get f=26.338671. The relative error of f is 0.0914%.

V. Conclusion

In this paper, two linear calibration methods based on different planes are given. The first one is based on the orthogonal planes. By controlling the camera to do five times' translation movements, we use homography matrixes and the orthogonality of the planes to build the constraint equations. Then we can attain the intrinsic matrix **K** by Cholesky factorization. To reduce errors and simplifies calibration process, we deduce the second method, which only takes one picture of the circle at optimal direction and needs not the camera to do extra movements in the whole calibration process as that of the first method. Both computer simulations and real data to be used to test the feasibility and correctness of the second calibration method, and achieves accuracy and reliable results. In theory, these two methods are good supplement to the camera calibration and would have applied practice in computer vision.

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