Debye Layer Confinement of Nucleons in Nuclei by Laser Ablated Plasma

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Abstract—Following the laser ablation studies leading to a theory of nuclei confinement by a Debye layer mechanism, we present here numerical evaluations for the known stable nuclei where the Coulomb repulsion is included as a rather minor component especially for lager nuclei. In this research paper the required physical conditions for the formation and stability of nuclei particularly endothermic nuclei with mass number greater than to which is an open astrophysical question have been investigated. Using the Debye layer mechanism, nuclear surface energy, Fermi energy and coulomb repulsion energy it is possible to find conditions under which the process of nucleation is permitted in early universe. Our numerical calculations indicate that about 200 second after the big bang at temperature of about 100 KeV and subrelativistic region with nucleon density nearly equal to normal nuclear density namely, $\sim 10^{38}\, cm$ all endothermic and exothermic nuclei have been formed.

Keywords—Endothermic nuclear synthesis, Fermi energy, Surface tension, Debye length.

I. INTRODUCTION

FOR many years astrophysicists have been wondering how endothermic synthesis of nuclei heavier than Iron could be produced resulting in the existing abundance in our Universe. The exothermic nucleation process of elements, lighter than Iron, could take place via fusion reactions [1]. During the last 15 years research papers have been published addressing this question with new approach [2-6] following the theory of nonlinear forces and double layers in laser produced plasmas [7-10]. The first step from these classical plasma conditions to the degenerate electrons in a metal by changing temperature into Fermi energy for the Debye length resulted in surface energy and the work function of metals. This quantum theory of surface tension led to values agreeing with measurements [11] differing from the usual explanation of surface tension [12]

Following these results, this was generalized to the case of protons and neutrons instead of electrons with their Fermi energy instead of the temperature in the Debye layer. A first convincing semi-empirical result is when taking the averaged density of nucleons in nuclei of $n = 2 \times 10^{38}\, cm$, one arrives at a Debye length of the value of the measured [6] 3 *fm*

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decay of the nuclear density at their surface as measured by Hahn et al [13] and Hofstadter et al [14]. When dropping the empirical value of n and calculating the surface energy form the Debye layer, it was noted [2-5] that the compression of the nucleons is about compensated by the surface energy just about at the density of the nuclei. In this case the compression energy was just only that of the Fermi energy of the nucleons. We present here the inclusion of the Coulomb repulsion energy of the nuclei and take into account some updating of the preceding formulations. Since the number of proton charges has to be taken individually, we have to follow up the energy values for each nucleus as given in the following separately for the subrelativistic and for the relativistic range of the Fermi energy. The calculations are done in such a way that the compensation of the interior energy of the unexcited nucleus (enthalpy) with the surface energy is the starting point and the resulting density n of the nucleus is calculated.

II. THE SURFACE TENSION AND ENERGY IN A DEBYE LAYER OF NUCLEI

The thickness of the double layer formed in the homogeneous plasma expanding into vacuum [3,8] is given by the Debye length [12]:

$$\lambda_D = [KT/(4\pi e^2 n_e)]^{\frac{1}{2}} \tag{1}$$

Where K is the Boltzman constant, T is the plasma electron temperature, e and n_e are electron charge and density of plasma respectively. The surface tension is defined as the electric field energy within the volume V (given by layer surface, S times Debye length λ_D) per unit area of layer [8]:

$$\sigma_e = 0.27T^2/(8\pi e^2 \lambda_D) \tag{2}$$

The surface tension for nuclei is given by replacing temperature by Fermi energy of protons and neutrons [5]. Therefore we have

$$\sigma_n = 0.27E_F^2/(8\pi e^2 \lambda_D) \tag{3}$$

The Fermi energy is generally given [9]

$$E_F = \frac{(3/\pi)^{2/3}}{4}\frac{\hbar^2 n^{2/3}}{2m}\left\{\left(\frac{\lambda_c}{2}\right)[n + 1/(\lambda_c/2)]^{1/3}\right\}^{-1} \tag{4}$$

where n is the nucleon density and m is the nucleon mass and λ_C is the Compton wavelength (h/mc) for nucleons. This

energy splits into two branches, relativistic and subrelativistic,

$$E_{F,sub} = \frac{\left(\frac{3}{\pi}\right)^{\frac{2}{3}}}{4} \frac{h^2 n^{\frac{2}{3}}}{2m} \qquad (5)$$

$$E_{F,rel} = \frac{\left(\frac{3}{\pi}\right)^{\frac{1}{3}}}{4} hcn^{1/3} \qquad (6)$$

The surface energy of the nucleus [5] is give

$$E_{surf} = 0.27 \frac{[3A(4\pi)^{4/3}]^{4/3} E_F^{5/2}}{\pi^{4/3} 2^{5/3} n^{4/3}} \qquad (7)$$

In order to compare the surface energy and the Fermi energy of the nucleus, their ratio is considered, giving,

$$\gamma = \frac{E_{surf,sub}}{AE_{F,sub}} = 0.27 \frac{3^{5/3} hn^{1/6}}{2^{19/3} \pi^{1/2} m^{1/2} A^{1/3}} \qquad (8)$$

$$\gamma' = \frac{E_{surf,rel}}{AE_{F,rel}} = 0.27 \frac{3^{5/3}}{2^{7/3} c^{1/3} A^{1/3}} \qquad (9)$$

where $\alpha = \frac{2\pi e^2}{hc}$ is the fine structure constant.

From equation (8) we conclude that if the density is too low, then the nucleus confinement by the Debye layer is not possible. The nucleus will be stable only if the ratio in (8) is equal to one. At relativistic densities above that of sub-relativistic case, we have

$$\frac{E_{surf,rel}}{AE_{F,rel}} = \frac{6.814}{A^{1/3}} \qquad (10)$$

which does not depend upon mass and density of the nucleon.

A complete nucleon density analysis for all nuclei with $A>60$ that satisfy the stability condition as stated in previous section is presented here. In order to include coulomb repulsion correction, we simply add the coulomb energy to the total Fermi energy in the denominator of equation (8-10). The following quantities are defined

$$\gamma = \frac{E_{surf,sub}}{AE_{F,sub}} \qquad (11)$$

$$\gamma' = \frac{E_{surf,rel}}{AE_{F,rel}} \qquad (12)$$

$$\beta = \frac{E_{surf,sub}}{(E_c + AE_{F,sub})} \qquad (13)$$

$$\beta' = \frac{E_{surf,rel}}{(E_c + AE_{F,rel})} \qquad (14)$$

where the coulomb energy is given by

$$E_c (MeV) = 0.72 \frac{z^2}{A^{1/3}} \qquad (15)$$

The nuclei stability condition that we are seeking is $\beta=1$ for sub-relativistic and $\beta'=1$ for relativistic branches.

Debye layer was approximately compensating the inner energy of nuclei for a confinement of the nucleons. The calculations with inclusion of the Coulomb force arrive at a modification of this result as shown in Table 1. Values for $A\leq60$ for the stable nuclei are calculated using β-values equal unity, to result in the nuclear density and the corresponding value of $A>60$ for the stable nuclei.

It is interesting that the resulting nuclei from hydrogen until iron arrive at lower nuclear density than the well known, $n_0 = 2 \times 10^{38} cm^{-3}$, approximate value of all nuclei. This fact indicates that either the Debye layer model is not applicable for explaining the nucleon confinement for the nuclei lighter than iron, or there are further repulsive forces acting in these nuclei.

The further point of interest is that at the heavy nuclei up to curium, the subrelativistic -value results in the same value as the relativistic branch. This is important as the nuclear density of $n^* = 3 \times 10^{38} cm$ is just the value where the subrelativistic Fermi energy changes into the relativistic branch. At higher nuclear densities, there is no nucleation possible at thermal equilibrium and only a homogeneous soup of particles is possible as a quark-gluon plasma or as assumed in a neutron star with such high density as clarified before [2-6].

III. CONCLUSION

By using the equilibrium between the coulomb force and quantum force on one hand and the nuclear surface energy on the other hand, we found the nucleon density in big bang and also in neutron stars at which free nucleons form the existing bound state nuclei in the universe. It is noticed that such condition occur for both endothermic and exothermic nuclei at subrelativistic region which corresponds to Boltzmann distribution as well as to the universal abundance distribution.

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TABLE I
SAMPLE OF DENSITY PROFILE FOR ELEMENTS OF PERIODIC TABLE (FOR B=1).

Nucleus	Z	A	$n(cm^{-3})$	$E^{sub}_F(MeV)$	$E^{rel}_F(MeV)$	$E^{sub}_{surf}(MeV)$	$E^{rel}_{surf}(MeV)$	$E_c(MeV)$	β	β'
Ar	18	38	1.055E+38	44.342	142.039	911.725	5227.044	69.359	1.000	1.888
K	19	39	1.126E+38	46.310	145.156	979.391	5435.074	76.613	1.000	1.870
Ca	20	40	1.2E+38	48.317	148.269	1050.321	5646.119	84.177	1.000	1.851
Cr	24	50	1.735E+38	61.780	167.657	1657.145	7408.485	112.525	1.000	1.721
Mn	25	55	1.991E+38	67.716	175.528	1980.458	8265.107	118.280	1.000	1.671
Fe	26	56	2.078E+38	69.675	178.048	2077.117	8485.097	127.165	1.000	1.660
Co	27	59	2.265E+38	73.795	183.236	2310.765	9041.497	134.771	1.000	1.632
Ni	28	58	2.266E+38	73.816	183.263	2285.421	8940.358	145.767	1.000	1.637
Cu	29	63	2.561E+38	80.091	190.894	2674.234	9840.409	152.113	1.000	1.596
Zn	30	64	2.665E+38	82.245	193.444	2793.607	10077.094	161.933	1.000	1.586
Ga	31	69	2.99E+38	88.803	201.008	3232.881	11009.630	168.626	1.000	1.550
As	33	75	3.448E+38	97.654	210.787	3848.675	12205.236	185.848	1.000	1.509
Rb	37	85	4.311E+38	113.335	227.081	5039.553	14292.939	224.086	1.000	1.447
Zr	40	90	4.85E+38	122.595	236.176	5775.339	15442.769	256.955	1.000	1.419
Mo	42	92	5.1E+38	126.773	240.166	6111.269	15935.470	281.225	1.000	1.407
Ag	47	107	6.646E+38	151.247	262.327	8427.275	19249.770	334.877	1.000	1.340
I	53	127	8.968E+38	184.690	289.882	12126.808	23846.095	402.194	1.000	1.268
Au	79	197	2.039E+39	319.343	381.178	32219.801	42017.752	771.940	1.000	1.097
Hg	80	196	2.026E+39	317.984	380.366	31939.977	41786.255	792.951	1.000	1.098
Pb	82	204	2.185E+39	334.411	390.067	34934.993	44010.217	822.058	1.000	1.084
Bi*	83	209	2.277E+39	343.733	395.467	36744.956	45345.562	835.460	1.000	1.076
U*	92	235	2.84E+39	398.283	425.692	47764.827	52779.784	987.123	1.000	1.035
U*	92	238	2.914E+39	405.172	429.357	49214.183	53686.374	982.958	1.000	1.031
Pu	94	244	3.045E+39	417.225	435.697	51905.533	55390.919	1017.679	1.000	1.022
Am	95	243	3.031E+39	415.946	435.028	51565.213	55154.686	1040.870	1.000	1.023
Cm	96	247	3.132E+39	425.135	439.808	53573.142	56370.879	1057.130	1.000	1.018