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Seat Assignment Problem Optimization

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Abstract**—**In this paper the optimality of the solution of an existing real word assignment problem known as the seat assignment problem using Seat Assignment Method (SAM) is discussed. SAM is the newly driven method from three existing methods, Hungarian Method, Northwest Corner Method and Least Cost Method in a special way that produces the easiness & fairness among all methods that solve the seat assignment problem.

Keywords**—**Assignment Problem, Hungarian Method, Least Cost Method, Northwest Corner Method, Seat Assignment Method (SAM), A Real Word Assignment Problem.

I. Introduction

FLIGHT seat assignment, scholarship type assignment and assigning students to school bus are examples of assignment problems that have special type of restrictions and constrains, for example *Workstation Assignment Problem* [1] is a specific assignment problem that has a facility constraint. Some existing methods such as Hungarian Method, Northwest Corner Method & Least Cost Method[2],[3],[5],[6]will give at least one solution but will not give guarantee of fairness in assigning the seat to the candidate. However in some cases industrial experts proposed a solution to an assignment problem to create a flexible manufacturing system (FMS) [4] and some other methods such as QuickMach [7] which solve linear assignment problems that based on the successive shortest path algorithm.

In this paper, fairness in assigning the seat to candidate is the main objective that will draw the optimality of the whole seat assignment problem using ***S***eat ***A***ssignment ***M***ethod (SAM).

Here a model description will be introduced which will contain the model setup & model readings, subsequently a seat assignment problem will be solved using SAM.

II. Model Description

A. Model Setup

To understand SAM easily,one unique example for all situations is considered that is the model of assigning huge number of students (more than 1000 students) to number of major seats or fields of study. At the beginning of each academic year, every educational organization accepts application for new students into their academic departments and majors. The educational organization provides a facility to the students to choose more than one majors, arranged according to the candidates preference in ascending order where number 1 is the most desirable field of study to the candidate and the lowest desirable has the rank *m*. The academic department has specific criteria to evaluate the applied students.

After receiving whole applications from all candidates, the department ranks all candidates according to the agreed criteria in ascending order where the highest rank candidate is given number 1 and the last candidate is given number *n* as shown in the following preference matrix:

TABLE I
Reference Matrix

Field	F_1	F_2	…	F_j	…	F_m	Required Seats
sT_1	P_{11}	P_{12}	…	P_{1j}	…	P_{1m}	1
sT_2	P_{21}	P_{22}	…	P_{2j}	…	P_{2m}	1
⋮	⋮	⋮	⁝⁝	⋮	⁝⁝	⋮	⋮
sT_i	P_{i1}	P_{i2}	…	P_{ij}	…	P_{im}	1
⋮	⋮	⋮	⁝⁝	⋮	⁝⁝	⋮	⋮
sT_n	P_{n1}	P_{n2}	…	P_{nj}	…	P_{nm}	1
Available Seats	A_1	A_2	…	A_j	…	A_m	N

F_j : is the j^{th} field of study which any student can apply to.
sTi : is the i^{th} candidate of students.
m : is the total number of major fields of study.
n : is the total number of candidates.
P_{ij} : is the preference rank for i^{th} candidate to j^{th} field of study.
A_j : is the number of seats available in the j^{th} field of study

B. Model Readings

The model readings can be extracted right after setting up the preference matrix and we are able to find the following readings:

1. The Most Desirable Field of Study

Since the candidates arrange their preference in ascending order, then finding the ***maximum*** value of ($\sum_{i=1}^{i=n} P_{ij}$ for all $P_{ij} = 1$where $j = 1,2,\ldots,m$) is granted to the most ***desirable*** field of study.

2. The Most Undesirable Field of Study

Since the candidates arrange their preference in ascending order, then finding the ***maximum*** value of ($\sum_{i=1}^{i=n} P_{ij}$ for all $P_{ij} = m$ where $j = 1,2,\ldots,m$) is granted to the most ***undesirable*** field of study.

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3. The Most Desirable and Undesirable Field of Study (Unusual Case)

In some unusual cases the same field of study becomes both most desirable and undesirable. That case is true for $j = 1,2,\ldots,m$when the maximum value of $\sum_{i=1}^{i=n} P_{ij}$ for all $P_{ij} = 1$ and $\sum_{i=1}^{i=n} P_{ij}$ for all $P_{ij} = m$ are occurs at the same value of *j*.

4. The Mean of the Preference Data Matrix

Since the built matrix is a preference matrix, then each candidate have the same number of choices or as we are calling them field of study which is resulting in:

	F_1	F_2	…	F_j	…	F_m
sT_i	P_{i1}	P_{i2}	…	P_{ij}	…	P_{im}

$\sum_{j=1}^{m} P_{ij} = \sum_{j=1}^{m} j$is always constant for values of $i = 1,2,\ldots,n$. That constant can be used to find the total sum of the preference matrix elements. Then it is true that

$$\sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{m} j = n \times \sum_{j=1}^{m} j$$

To find the preference matrix mean, then:

$$Mean = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij}}{n \times m} \rightarrow \frac{n \times \sum_{j=1}^{m} j}{n \times m} = \frac{\sum_{j=1}^{m} j}{m}$$

5. The Median of the Preference Data Matrix

To find the median, the preference matrix has to be arranged in ascending order which is resulting in:

sT_1	1	2	…	J	…	m
sT_2	1	2	…	J	…	m
⋮	⋮	⋮	⁝⁝⁝	⋮	⁝⁝⁝	⋮
sT_i	1	2	…	J	…	m
⋮	⋮	⋮	⁝⁝⁝	⋮	⁝⁝⁝	⋮
sT_n	1	2	…	J	…	m

In none matrix form the data is shown as:

$$\underbrace{1,1,\ldots}_{n}, \underbrace{2,2,\ldots}_{n}, \underbrace{\ldots,\ldots,\ldots}_{n}, \underbrace{j,j,\ldots}_{n}, \underbrace{\ldots\ldots\ldots}_{n}, \underbrace{m,m,\ldots}_{n}$$

It holds true that each element of the data is recurring *n* times. In other words, frequency of each element is exactly *n*. So, it is valid that if we find the median for any candidate, then it holds true that the median for one candidate is equal to the median for the whole data set. Consequently, the median is the middle element of the following set: $1,2,\ldots,j,\ldots,m$ and since the preference matrix elements is fixed and incremental by one with no repeating for the same candidate (row wise) then we can guarantee that *m* is the total number of elements in the row and it is also the value of the last element. Therefore, the median is simply $\frac{m+1}{2}$ which is true always regardless whether the number of elements is even or odd.

6. The Mean and Median of the Preference Data Matrix Are equal

It is always true that the mean and median of the built preference data matrix are equal. From the previous model readings mainly readings 4 and 5,

$Mean = \frac{\sum_{j=1}^{m} j}{m}$&$Median = \frac{m+1}{2}$. To prove mean and median are equal, the proveis initiated from$Mean = \frac{\sum_{j=1}^{m} j}{m}$ and we reach $mode = \frac{m+1}{2}$.

$Mean = \frac{\sum_{j=1}^{m} j}{m}$where$\sum_{j=1}^{m} j$ is known as the m^{th} partial sum of a triangle series that has *m* terms and total sum of$\frac{m\times(m+1)}{2}$. By substituting the m^{th} partial sum of the triangle series that has a sum of$\frac{m\times(m+1)}{2}$ into the main formula for the mean we get

$$mean = \frac{m \times (m+1)}{2 \times m} = \frac{m+1}{2} = median$$

III. Solving the Assignment Problem Using SAM

SAM is a mixture of three methods the Hungarian and Northwest corner& Least Cost method in a special way that produces the easiness and fairness of allmethods. Here considering the preference matrix in table I which we have got after setting up SAM model previously.

To have the final optimal solution to the assignment problem we are creating new matrix called Result matrix where R_{ij} has the values 1 or 0. If R_{ij} is 1 then i^{th} candidate is assigned to j^{th} seat.

TABLE II
Result Matrix

Field	F_1	F_2	…	F_j	…	F_m	Required Seats
sT_1	R_{11}	R_{12}	…	R_{1j}	…	R_{1m}	1
sT_2	R_{21}	R_{22}	…	R_{2j}	…	R_{2m}	1
⋮	⋮	⋮	⁝⁝⁝	⋮	⁝⁝⁝	⋮	⋮
sT_i	R_{i1}	R_{i2}	…	R_{ij}	…	R_{im}	1
⋮	⋮	⋮	⁝⁝⁝	⋮	⁝⁝⁝	⋮	⋮
sT_n	R_{n1}	R_{n2}	…	R_{nj}	…	R_{nm}	1
Available Seats	A_1	A_2	…	A_j	…	A_m	n

SAM starts from the upper left corner (Northwest corner) of the preference matrix and do the following:

1. Determine capacity for the j^{th} field $= A_j$ for all $j = 1\ to\ m$
2. Initialize $R_{ij} = 0$ for all *i* and *j*.
3. Set $i = 1$, $j = 1$ and go to 9
4. Update $i = i + 1$
5. If $i > n$, go to 13
6. Set $j = 1$ and go to 9
7. Update $j = j + 1$
8. If $j > m$, go to 4
9. Is P_{ij} the minimum value row wise in the preference matrix? If yes go to 10, if no go to 7.
10. Is $A_j > 0$, If yes go to 11, if no go to 7 (seat availability constraint)
11. $A_j = A_j - 1$
12. Update $R_{ij} = 1$ and go to 4
13. End

SAM can use the least cost method with some modifications and with help of newly created formula. The least cost method uses the cost value to give priority to each candidate subjected to the chosen seat availability. Using SAM will give more intelligence of fairness using modified formula for cost that brings three main parts

1. Seat field priority P_{ij} that was chosen by the candidate.
2. Number of fields*m*.
3. Seat field availability A_j.

The following Table IIIis the setup for SAM after arranging seat candidates according to agreed criteria after laying SAM in format of Least Cost Method in the following tableau:

TABLE III
FORMATTED SAM'S TABLEAU

Field	F_1	F_2	…	F_j	…	F_m	Required Seats
sT_1	P_{11} C_{11} 1 1	P_{12} C_{12} 1 2	…	P_{1j} C_{1j} 1 *j*	…	P_{1m} C_{1m} 1 *m*	1
sT_2	P_{21} C_{21} 2 1	P_{22} C_{22} 2 2	…	P_{2j} C_{2j} 2 *j*	…	P_{2m} C_{2m} 2 *m*	1
⋮	⋮	⋮	⋱	⋮	⋱	⋮	⋮
sT_i	P_{i1} C_{i1} *i* 1	P_{i2} C_{i2} *I* 2	…	**P_{ij} C_{ij} *i* *j***	…	P_{im} C_{im} *i* *m*	1
⋮	⋮	⋮	⋱	⋮	⋱	⋮	⋮
sT_n	P_{n1} C_{n1} *n* 1	P_{n2} C_{n2} *N* 2	…	P_{nj} C_{nj} *n* *j*	…	P_{nm} C_{nm} *n* *m*	1
Available Seats	A_1	A_2	…	A_j	…	A_m	*n*

where the new closed formula forthe corner cost is:

$$\boldsymbol{C_{ij} = P_{ij} + m \times (i - 1)}.$$

The resulting C_{ij}gives the 100% of assurance of fairness in assigning the seats to the candidates.

Hence achieving the fairness in seat assignment problem by least cost method, the SAM setup is very handy to use with the help of newly driver formula $\boldsymbol{C_{ij} = P_{ij} + m \times (i - 1)}$.

To show the advantages & performance of SAM, infuture paper an application will be introduced which will help an organization to ensure a very fair solution of seat assignment problem.

IV. CASE STUDY

This case study is used to illustrate by numbers and figures the effectiveness of SAM. The case is done on small size set of date with 20 candidates and 5 types of seat. College of Science in University x has to admit students on orientation year first and then give the student chance to choose one of the 5 offered majors (fields of study).

After collecting students' preferences, the college administrator arrange the students' requests in ascending order according to their GPA. By that way he builds the preference matrix as it is shown in table IV.

TABLE IV
REFERENCE MATRIX FOR STUDENTS IN COLLEGE OF SCIENCE

Field	F_1	F_2	F_3	F_4	F_5	Required Seats
sT_1	3	2	4	1	5	1
sT_2	5	3	1	2	4	1
sT_3	4	1	5	3	2	1
sT_4	5	1	2	3	4	1
sT_5	2	4	3	5	1	1
sT_6	3	1	2	5	4	1
sT_7	1	5	2	4	3	1
sT_8	4	1	2	5	3	1
sT_9	5	2	3	1	4	1
sT_{10}	3	5	4	2	1	1
sT_{11}	2	3	1	5	4	1
sT_{12}	5	1	4	2	3	1
sT_{13}	1	5	4	3	2	1
sT_{14}	3	2	5	4	1	1
sT_{15}	3	5	1	4	2	1
sT_{16}	3	4	5	2	1	1
sT_{17}	4	1	5	2	3	1
sT_{18}	3	5	1	4	2	1
sT_{19}	5	2	4	3	1	1
sT_{20}	1	3	4	2	5	1
Available Seats	6	2	4	3	5	20

After setting up the model, SAM tableau is built as shown:

TABLE VII
FORMATTED SAM'S TABLEAU FOR COLLEGE OF SCIENCE STUDENTS

	F_1		F_2		F_3		F_4		F_5		Required Seats
sT_1	3	3	2	2	4	4	1	1	5	5	1
	1	1	1	2	1	3	1	4	1	5	
sT_2	5	10	3	8	1	6	2	7	4	9	1
	2	1	2	2	2	3	2	4	2	5	
sT_3	4	14	1	11	5	15	3	13	2	12	1
	3	1	3	2	3	3	3	4	3	5	
sT_4	5	20	1	16	2	17	3	18	4	19	1
	4	1	4	2	4	3	4	4	4	5	
sT_5	2	22	4	24	3	23	5	25	1	21	1
	5	1	5	2	5	3	5	4	5	5	
sT_6	3	28	1	26	2	27	5	30	4	29	1
	6	1	6	2	6	3	6	4	6	5	
sT_7	1	31	5	35	2	32	4	34	3	33	1
	7	1	7	2	7	3	7	4	7	5	
sT_8	4	39	1	36	2	37	5	40	3	38	1
	8	1	8	2	8	3	8	4	8	5	
sT_9	5	45	2	42	3	43	1	41	4	44	1
	9	1	9	2	9	3	9	4	9	5	
sT_{10}	3	48	5	50	4	49	2	47	1	46	1
	10	1	10	2	10	3	10	4	10	5	
sT_{11}	2	52	3	53	1	51	5	55	4	54	1
	11	1	11	2	11	3	11	4	11	5	
sT_{12}	5	60	1	56	4	59	2	57	3	58	1
	12	1	12	2	12	3	12	4	12	5	
sT_{13}	1	61	5	65	4	64	3	63	2	62	1
	13	1	13	2	13	3	13	4	13	5	
sT_{14}	3	68	2	67	5	70	4	69	1	66	1
	14	1	14	2	14	3	14	4	14	5	
sT_{15}	3	73	5	75	1	71	4	74	2	72	1
	15	1	15	2	15	3	15	4	15	5	
sT_{16}	3	78	4	79	5	80	2	77	1	76	1
	16	1	16	2	16	3	16	4	16	5	
sT_{17}	4	84	1	81	5	85	2	82	3	83	1
	17	1	17	2	17	3	17	4	17	5	
sT_{18}	3	88	5	90	1	86	4	89	2	87	1
	18	1	18	2	18	3	18	4	18	5	
sT_{19}	5	95	2	92	4	94	3	93	1	91	1
	19	1	19	2	19	3	19	4	19	5	
sT_{20}	1	96	3	98	4	99	2	97	5	100	1
	20	1	20	2	20	3	20	4	20	5	
Available Seats	6		2		4		3		5		20

By using SAM as new added layer to the Least Cost Method, it is easy and fair to use it as optimization technique. TableVIIshows the optimal and fair solution to the problem using SAM for Least Cost Method after applying the original Least Cost Method to the modified table VI which has the SAM's formulais buried in the cost cell for the least cost tableau. The direct assignment result is shown in Table VII

TABLE VII
SAM'S SOLUTION WITH LEST COST METHOD

Student #	Field	Student #	Field
sT_1	F_4	sT_{11}	F_3
sT_2	F_3	sT_{12}	F_4
sT_3	F_2	sT_{13}	F_5
sT_4	F_2	sT_{14}	F_5
sT_5	F_5	sT_{15}	F_5
sT_6	F3	sT_{16}	F_1
sT_7	F_1	sT_{17}	F_1
sT_8	F_3	sT_{18}	F_1
sT_9	F_4	sT_{19}	F_1
sT_{10}	F_5	sT_{20}	F_1

V. CONCLUSION

Fairness in seat assignment is proposed based on the Seat Assignment Method (SAM), which gives the optimal solution with fairness. To setup the SAM model in the least cost method gives the intelligence of fairness for finding the solution of seat assignment problems, Therefore SAM can be used in real word seat assignment problem for optimal solution.This approach can also be extended efficiently and effectively to get much fairness in Seat Assignment Problem.

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