International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:6, No:8, 2012

Robust Fuzzy Control of Nonlinear Fuzzy Impulsive Singular Perturbed Systems with Time-varying Delay

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***Abstract*—**The problem of robust fuzzy control for a class of nonlinear fuzzy impulsive singular perturbed systems with time-varying delay is investigated by employing Lyapunov functions. The nonlinear delay system is built based on the well-known T–S fuzzy model. The so-called parallel distributed compensation idea is employed to design the state feedback controller. Sufficient conditions for global exponential stability of the closed-loop system are derived in terms of linear matrix inequalities (LMIs), which can be easily solved by LMI technique. Some simulations illustrate the effectiveness of the proposed method.

***Keywords*—**T–S fuzzy model, singular perturbed systems, time-varying delay, robust control.

I. Introduction

OVER the past few decades, fuzzy logic control of nonlinear systems has received considerable attentions because this approach is effective to obtain nonlinear control systems, especially in the incomplete knowledge of the plant or even of the precise control action appropriate to a given situation. Among various kinds of fuzzy methods, fuzzy model- based control is widely used because the design and analysis of the overall fuzzy system can be systematically performed using the well-established classical linear systems theory [1–4]. The stability analysis and controller design for nonlinear systems based on T–S fuzzy model are discussed in [1–3]. On the other hand, time delay is commonly encountered and is often the sources of instability. Recently, the robust stability analysis problems for fuzzy time-delay systems have received considerable attentions [5–7].In modern science and technology, there are natural phenomena in real world which are characterized by the fact that some systems have a lot of states, some are quick but the others are slow. It is known, for example, that many fields involving complex circuit, soft robot and communication networks. During the past few years, control of singular perturbed systems has been extensively studied due to the fact that they better describe physical systems than regular ones [8-12]. In [8], Controller design and stability analysis for fuzzy singular perturbed systems are studied. $H\infty$ and H_2 control are investigated in [9-12]. Time-delay singular perturbed systems have been investigated in [12].

Very recently, there have been growing attentions on the study of T–S fuzzy systems with impulse [13-16]. In [13], a class of nonlinear fuzzy impulsive systems is defined by extending the ordinary T–S fuzzy model and sufficient conditions for global exponential stability of the closed loop systems are derived. In [14], some criteria of uniform stability and uniform asymptotic stability for T–S fuzzy delay systems with impulse have been presented. On the other hand, there are some interesting applications on impulsive control or synchronization of chaotic systems based on T–S fuzzy model [15, 16]. In the design of controller systems, one is not only interested in global stability, but also in some other performances. Particularly, it is often desirable to have systems that converge fast enough in order to achieve fast response. Considering this, many researchers have studied the exponential stability analysis problem for impulsive systems [17], singular perturbed systems [18] and so on. To the best of our knowledge, so far, the problem of global exponential stabilization for fuzzy impulsive singular perturbed systems with time-varying delay has not been addressed in the literature, which is still open and remains unsolved. Motivated by the aforementioned discussions, we investigate the problem of robust fuzzy control for a class of singular perturbed systems with time-varying delay. The nonlinear delay system is represented by the well-known T–S fuzzy model. The so-called parallel distributed compensation (PDC) idea is employed to design the state feedback controller. Sufficient conditions for global exponential stability of the closed-loop system are derived by employing Lyapunov functions. The conditions are in terms of linear matrix inequalities (LMIs), which can be easily solved by LMI technique.The remainder of this paper is organized as follows. In Section 2, the problem to be investigated is given and some necessary definitions and useful lemmas are also presented. In Section 3, some criteria are derived to ensure the global exponential stability of the closed-loop system. In Section 4, an example is given to demonstrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 5.

II. Problem Statement and Basic Assumptions

Consider the following nonlinear system with time-varying delay represented by T–S fuzzy model.

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Plant Rule i

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } M_1^i \text{ and}, \cdots, \text{and } z_g(t) \text{ is } M_g^i\\&\text{THEN } E\dot{x}(t) = A_i x(t) + B_i(t-\tau(t)) + C_i u(t),\ t \neq t_k,\\&\qquad \Delta x = \tilde{G}_{ki} x(t_k^-),\ k = 1,2,\cdots,\\&x(t) = \phi(t),\ t \in [t_0 - \tau_0, t_0]\\&x(t_0^+) = x_0, t_0 \geq 0, i = 1,2,\cdots,q,\end{aligned} \tag{1}$$

where M_j^i is the fuzzy set, q the number of rules, $z(t) = [z_1(t), z_2(t), \cdots, z_g(t)]^T$ the premise variable. $E = \begin{bmatrix} I_{n\times n} & 0 \\ 0 & \varepsilon I_{m\times m}\end{bmatrix}$,perturbed parameter, $x(t) \in R^n$ the state vector, $u(t) \in R^m$ the control input, $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, and $C_i \in R^{n\times m}$ the constant matrices, $0 \leq \tau(t) \leq \tau_0$ the unknown bounded time-varying delay in the state and $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t^+) = \lim_{h\to 0^+} x(t+h)$ and $x(t^-) = \lim_{h\to 0^-} x(t+h)$, Without loss of generality, we assume that $\lim_{t\to t_k^+} x(t) = x(t_k)$, which means that the solution $x(t)$ is right continuous at time t_k , $\tilde{G}_{ki} \in R^{n\times n}$ is constant coefficients, the impulsive time instants $\{t_k\}$ satisfy $0 \leq t_0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$. We assume that there exists a constant L >1, such that $t_k - t_{k-1} \geq L\tau_0$.

Remark 1: In (1), if $u(t) = 0$, then the nonlinear system reduces to

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } M_1^i \text{ and}, \cdots, \text{and } z_g(t) \text{ is } M_g^i\\&\text{THEN } E\dot{x}(t) = A_i x(t) + B_i(t-\tau(t)),\ t \neq t_k,\\&\qquad \Delta x = \tilde{G}_{ki} x(t_k^-),\ k = 1,2,\cdots,\\&x(t) = \phi(t),\ t \in [t_0 - \tau_0, t_0]\\&x(t_0^+) = x_0, t_0 \geq 0, i = 1,2,\cdots,q,\end{aligned} \tag{2}$$

which is called the unforced fuzzy impulsive system with time-varying delay.

In (1), if $\tau(t) = 0$, $\tilde{G}_{ki} = 0$, i =1, 2, . . ., k=1, 2, . . ., then the nonlinear system reduces to

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } M_1^i \text{ and}, \cdots, \text{and } z_g(t) \text{ is } M_g^i\\&\text{THEN } E\dot{x}(t) = A_i x(t) + C_i u(t),\\&\qquad i = 1,2,\cdots,q,\end{aligned}$$

which is a typical continuous T–S fuzzy model. Stability of this T–S fuzzy model has been extensively investigated [8–10].

In (1), if $\tilde{G}_{ki} = 0$, k=1, 2, . . ., then the nonlinear system reduces to

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } M_1^i \text{ and}, \cdots, \text{and } z_g(t) \text{ is } M_g^i\\&\text{THEN } E\dot{x}(t) = A_i x(t) + B_i(t-\tau(t)) + C_i u(t),\\&\qquad x(t) = \phi(t),\ t \in [t_0 - \tau_0, t_0]\\&\qquad i = 1,2,\cdots,q,\end{aligned}$$

which is a typical continuous T–S fuzzy time-delay model. Stability of this T–S fuzzy model has been extensively investigated [11-12].

By using the fuzzy inference method with a singleton fuzzification, product inference and centre average defuzzification, the overall fuzzy model is of the following form

$$\begin{aligned}E\dot{x}(t) &= \sum_{i=1}^{q} h_i(z(t))[A_i x(t) + B_i x(t-\tau(t)) + C_i u(t)], t \neq t_k,\\\Delta x(t) &= \sum_{i=1}^{q} h_i(z(t))\tilde{G}_{ki} x(t_k^-), k = 1,2,\cdots,\end{aligned} \tag{3}$$

where

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{q} w_i(z(t))},\ w_i(z(t)) = \prod_{j=1}^{g} M_j^i(z_j(t))$$

We assume that $w_i(z(t)) \geq 0$ and $\sum_{i=1}^{q} w_i(z(t)) > 0$. It is clear that

$$h_i(z(t)) \geq 0,\quad \sum_{i=1}^{q} h_i(z(t)) = 1$$

The control objective is to design a state feedback fuzzy controller such that the closed-loop system is exponential stable, that is to say, there exist $N > 0$, $\gamma > 0$ such that

$$\| x(t) \| \leq N \| \bar{\phi} \| e^{-\gamma(t-t_0)} \to 0\ ,\ t \to +\infty \tag{4}$$

where $\| \bar{\phi} \| = \sup_{t_0-\tau_0\leq t\leq t_0} \|\phi(t)\|$.

Based on the so-called PDC idea, the state feedback fuzzy controller is designed as follows

Plant Rule i:

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } M_1^i \text{ and}, \cdots, \text{and } z_g(t) \text{ is } M_g^i\\&\text{THEN } u(t) = F_i x(t),\\&\quad i = 1,2,\cdots,q,\end{aligned} \tag{5}$$

where $F_i \in R^{m\times p}$ are constant control gains to be determined later.

By using the fuzzy inference method with a singleton fuzzification, product inference and centre average defuzzification, the overall fuzzy regulator is represented by

$$u(t) = \sum_{i=1}^{q} h_i(z(t)) F_i x(t) \tag{6}$$

The closed-loop system of (1) and (5) is

$$\begin{aligned}E\dot{x}(t) &= \sum_{i=1}^{q} h_i(z(t)) \sum_{j=1}^{q} h_j(z(t))[(A_i + C_i F_j)x(t) + B_i x(t-\tau(t))], t \neq t_k,\\\Delta x(t) &= \sum_{i=1}^{q} h_i(z(t))\tilde{G}_{ki} x(t_k^-), k = 1,2,\cdots,\end{aligned} \tag{7}$$

Before proceeding, we recall some preliminaries which will be used throughout the proofs of our main results.

Definition 1 [14]: For $(t, x) \in (t_{k-1}, t_k] \times R^n$, we define

$$D^+V(t,x) \stackrel{\Delta}{=} \lim_{h\to 0+} \sup \frac{1}{h}[V(t+h, x+hf(t,x)) - V(t,x)]$$

Lemma 1 [18]: Suppose those matrices X, Y with proper dimensions, and a positive definite matrix S , then the following condition holds

$$X^T Y + Y^T X \le X^T S^{-1} X + Y^T S Y$$

Lemma 2 (Halanay Lemma, [19]): Let m(t) be a scalar positive function and assume that the following condition holds

$$D^+ m(t) \le -a m(t) + b \overline{m}(t), \quad t \ge t_0$$

where constants $a > b > 0$. Then, there exits $\alpha > 0$, such that for all $t \ge t_0$

$$m(t) \le \overline{m}(t_0) e^{-\alpha(t-t_0)}$$

here, $\overline{m}(t) = \sup_{t-\tau_0 \le s \le t} \{m(s)\}$ and $\alpha > 0$ satisfy $\alpha - a + b e^{\alpha\tau_0} = 0$.

Lemma 3 [20]: Suppose that matrices $M_i \in R^{m\times n}, i = 1,2,\cdots,r,$ and a positive semi-definite matrix $P \in R^{m\times n}$ are given. If $\sum_{i=1}^r P_i = 1$ and $0 \le P_i \le 1$, then

$$(\sum_{i=1}^r P_i M_i)^T P (\sum_{i=1}^r P_i M_i) \le \sum_{i=1}^r P_i M_i^T P M_i$$

III. Design of Controller and Stability Analysis

Now, we present the design of controller and stability analysis for the nonlinear fuzzy impulsive system (1) with timevarying delay.

Theorem 1: If there exist symmetric and positive definite matrix X, and some matrices Y_i, such that the following LMIs hold

$$XE^T = EX \ge 0 \tag{8}$$

$$\begin{bmatrix} XA_i^T + A_i^T X + Y_j^T C_i^T + C_i Y_j + aXE^T & B_i X \\ XB_i^T & -bXE^T \end{bmatrix} < 0 \tag{9}$$

$$i = 1,2,\cdots q, \quad j = 1,2,\cdots q$$

$$\begin{bmatrix} 2XE^T + 2X\tilde{G}_{ki}^T E^T + 2E\tilde{G}_{ki} X - 2\lambda_k XE^T & X\tilde{G}_{ki}^T & X\tilde{G}_{ki}^T E^T \\ \tilde{G}_{ki} X & -X & 0 \\ E\tilde{G}_{ki} X & 0 & -X \end{bmatrix} < 0 \tag{10}$$

$$i = 1,2,\ldots,q, \quad k = 1,2,\ldots,$$

Where $a > b > 0$ and $\alpha > 0$ is the unique positive root of the following equation

$$\alpha - a + b e^{\alpha\tau_0} = 0$$

The parameters λ_k are specified by the designer, where $e^{\alpha\tau_0} \le \lambda_k \le e^{\alpha(t_k - t_{k-1})/L}, \quad k = 1,2,\ldots$, $P = X^{-1}$, Then the T–S fuzzy system (1) with time-varying delay is global exponential stable via the state feedback fuzzy controller (5). In this case, the control feedback gains are $F_i = Y_i X^{-1}$.

Proof: Consider the Lyapunov function candidate $V(x(t)) = x^T E^T P x$, taking the Dini derivative of $V(x(t))$, for $t \in [t_{k-1}, t_k)$, we obtain

$$\begin{aligned} D^+V(t) &= \dot{x}^T(t) E^T P x(t) + x^T(t) P E \dot{x}(t) \\ &= \sum_{i=1}^q \sum_{j=1}^q h_i(z(t)) h_j(z(t)) \{x^T(t)[A_i^T P + PA_i + (C_i F_j)^T P + PC_i F_j] x(t) \\ &\quad + x^T(t) PB_i x(t-\tau(t)) + x^T(t-\tau(t)) B_i^T P x(t)\} \\ &= -a x^T(t) E^T P x(t) + b x^T(t-\tau(t)) E^T P x(t-\tau(t)) \\ &\quad + \sum_{i=1}^q \sum_{j=1}^q h_i(z(t)) h_j(z(t)) \{x^T(t)[A_i^T P + PA_i + (C_i F_j)^T P \\ &\quad + PC_i F_j + aE^T P] x(t) + x^T(t) PB_i x(t-\tau(t)) + x^T(t-\tau(t)) B_i^T P x(t) \\ &\quad - b x^T(t-\tau(t)) E^T P x(t-\tau(t))\} \end{aligned} \tag{11}$$

From (11), we have

$$\begin{aligned} D^+V(t) &= \dot{x}^T(t) E^T P x(t) + x^T(t) P E \dot{x}(t) \\ &= -a x^T(t) E^T P x(t) + b x^T(t-\tau(t)) E^T P x(t-\tau(t)) \\ &\quad + \sum_{i=1}^q \sum_{j=1}^q h_i(z(t)) h_j(z(t)) \{x^T(t)[A_i^T P + PA_i + (C_i F_j)^T P \\ &\quad + PC_i F_j + aE^T P] x(t) \\ &\quad + x^T(t) PB_i x(t-\tau(t)) + x^T(t-\tau(t)) B_i^T P x(t) \\ &\quad - b x^T(t-\tau(t)) E^T P x(t-\tau(t))\} \\ &= -a x^T(t) E^T P x(t) + b x^T(t-\tau(t)) E^T P x(t-\tau(t)) \\ &\quad + \sum_{i=1}^q \sum_{j=1}^q h_i(z(t)) h_j(z(t)) \begin{pmatrix} x(t) \\ x(t-\tau(t)) \end{pmatrix}^T \\ &\quad \begin{bmatrix} A_i^T P + PA_i + (C_i F_j)^T P + PC_i F_j + aE^T P & PB_i \\ B_i^T P & -bE^T P \end{bmatrix} \begin{pmatrix} x(t) \\ x(t-\tau(t)) \end{pmatrix} \end{aligned} \tag{12}$$

Pro- and Post- multiplying both sides of (9) by $\begin{bmatrix} X^{-1} & 0 \\ 0 & X^{-1} \end{bmatrix}$, we have

$$\begin{bmatrix} A_i^T P + PA_i + (C_i F_j)^T P + PC_i F_j + aE^T P & PB_i \\ B_i^T P & -bE^T P \end{bmatrix} < 0 \tag{13}$$

from (12) and (13), we have

$$D^+V(x(t)) \le -aV(x(t)) + b\overline{V}(x(t)) \tag{14}$$

Where $\overline{V}(x(t)) = \sup_{t-\tau_0 \le s \le t} \{V(x(s))\}$.

Since $a > b > 0$, by Lemma 2 and (14), there exits a constant $\alpha > 0$, such that for all $t \in [t_{k-1}, t_k), \ k = 1,2,\ldots$

$$V(x(t)) \le \overline{V}(x(t_{k-1})) e^{-\alpha(t-t_{k-1})} \tag{15}$$

where α satisfies $\alpha - a + b e^{\alpha\tau_0} = 0$.

On the other hand, when $t = t_k$, by Lemma 3, we have

$$\begin{aligned} V(x(t_k)) &= x^T(t_k) E^T P x(t_k) = x^T(t_k^-)(I + \sum_{i=1}^q h_i(z(t_k))\tilde{G}_{ki})^T \\ &\quad E^T P (I + \sum_{j=1}^q h_j(z(t_k))\tilde{G}_{kj}) x(t_k^-) \\ &\le \sum_{i=1}^q h_i(z(t_k)) x^T(t_k^-)(I + \tilde{G}_{ki})^T E^T P (I + \tilde{G}_{ki}) x(t_k^-) \\ &= \sum_{i=1}^q h_i(z(t_k)) x^T(t_k^-)(E^T P + \tilde{G}_{ki}^T E^T P + E^T P \tilde{G}_{ki} + \tilde{G}_{ki}^T E^T P \tilde{G}_{ki}) x(t_k^-). \end{aligned} \tag{16}$$

Taking $e^{\alpha\tau_0} < \lambda_k \le e^{\alpha(t_k - t_{k-1})/L}$, we have

$$V(x(t_k)) \le \sum_{i=1}^{q} h_i(z(t_k))[x^T(t_k^-)(E^T P + \tilde{G}_{ki}^T E^T P + E^T P\tilde{G}_{ki} + \tilde{G}_{ki}^T E^T P\tilde{G}_{ki} - \lambda_k E^T P]x(t_k^-) + x^T(t_k^-)\lambda_k E^T P x(t_k^-) \quad (17)$$

Pro- and Post- multiplying both sides of (10) by $\begin{bmatrix} X^{-1} & 0 & 0 \\ 0 & X^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix}$, yields

$$\begin{bmatrix} 2E^T P + 2\tilde{G}_{ki}^T E^T P + 2PE\tilde{G}_{ki} - 2\lambda_k E^T P & \tilde{G}_{ki}^T P & \tilde{G}_{ki}^T E^T \\ P\tilde{G}_{ki} & -P & 0 \\ E\tilde{G}_{ki} & 0 & -P^{-1} \end{bmatrix} < 0 \quad (18)$$

Applying Schur complement Lemma to (18), we obtain

$$2E^T P + 2\tilde{G}_{ki}^T E^T P + 2PE\tilde{G}_{ki} + \tilde{G}_{ki}^T P P^{-1} P\tilde{G}_{ki} + \tilde{G}_{ki}^T E^T PE\tilde{G}_{ki} - 2\lambda_k E^T P < 0 \quad (19)$$

Applying Lemma 1, we get

$$2E^T P + 2\tilde{G}_{ki}^T E^T P + 2PE\tilde{G}_{ki} + \tilde{G}_{ki}^T PE\tilde{G}_{ki} + \tilde{G}_{ki}^T E^T P\tilde{G}_{ki} - 2\lambda_k E^T P < 0 \quad (20)$$

From (8) and (20), we can see

$$E^T P + \tilde{G}_{ki}^T E^T P + E^T P\tilde{G}_{ki} + \tilde{G}_{ki}^T E^T P\tilde{G}_{ki} - \lambda_k E^T P < 0 \quad (21)$$

Then, from (17) and (21), we have

$$V(x(t_k)) \le \lambda_k V(x(t_k^-)) \quad (22)$$

By mathematical induction, one can show that

$$V(x(t)) \le \lambda_1\lambda_2 \cdots \lambda_{k-1}\lambda_k \lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t-t_0)}, \quad (23)$$

$t \in [t_{k-1}, t_k), \quad k = 1,2,\cdots$

Indeed, when $k = 1$

since $\|x(t)\| = \|\phi(t)\| \le \|\bar{\phi}\|$, $t \in [t_0 - \tau_0, t_0]$,

we have

$$V(x(t)) \le \lambda_{\max}(E^T P)\|x(t)\|^2 \le \lambda_{\max}(E^T P)\|\bar{\phi}\|^2,$$

$t \in [t_0 - \tau_0, t_0]$

Hence

$$\bar{V}(x(t_0)) \le \lambda_{\max}(E^T P)\|\bar{\phi}\|^2,$$

$$V(x(t)) \le \bar{V}(x(t_0))e^{-\alpha(t-t_0)} \le \lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t-t_0)},$$

$t \in [t_0, t_1)$

Thus, (23) holds when $k = 1$

Next, assume that (18) holds for $k \le m, \quad m \ge 1$. Then, we need to show (22) still holds when $k = m+1$.

By (15) and (22) and the above induction assumption, one has

$$V(x(t_k)) \le \lambda_k V(x(t_k^-)) \le \lambda_1\lambda_2 \cdots \lambda_{k-1}\lambda_k \lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t-t_0)},$$

for $t \in [t_k, t_{k+1})$

$$V(x(t)) \le \bar{V}(x(t_k))e^{-\alpha(t-t_k)} \le \max_{t_k-\tau_0 \le t \le t_k}\{V(x(t))\}e^{-\alpha(t-t_k)}$$

$$= \max_{t_k-\tau_0 \le t \le t_k}\{\sup_{t_k-\tau_0 \le t \le t_k} V(x(t)), V(x(t_k))\}e^{-\alpha(t-t_k)}$$

$$\le \max_{t_k-\tau_0 \le t \le t_k}\{\lambda_1\lambda_2 \cdots \lambda_{k-1}\lambda_k\lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t_k-\tau_0-t_0)}, \lambda_1\lambda_2 \cdots \lambda_{k-1}\lambda_k\lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t_k-t_0)}\}e^{-\alpha(t-t_k)}$$

$$\le \max_{t_k-\tau_0 \le t \le t_k}\{\lambda_1\lambda_2 \cdots \lambda_{k-1}\lambda_k\lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t_k-t_0)}, \lambda_1\lambda_2 \cdots \lambda_{k-1}\lambda_k\lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t_k-t_0)}\}e^{-\alpha(t-t_k)}$$

$$\le \lambda_1\lambda_2 \cdots \lambda_{k-1}\lambda_k\lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t_k-t_0)}e^{-\alpha(t-t_k)}$$

Therefore

$$V(x(t)) \le \lambda_1\lambda_2 \cdots \lambda_{k-1}\lambda_k\lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha(t-t_0)},$$

Since $e^{\alpha\tau_0} < \lambda_k \le e^{\alpha(t_k-t_{k-1})/L}$, thus

$0 < \lambda_k e^{\alpha(t_k-t_{k-1})/L} \le 1$.

Therefore for $t \in [t_k, t_{k+1})$

$$\lambda_{\min}(E^T P)\|x(t)\|^2 \le V(x(t))$$
$$\le e^{\alpha\tau_0(L-1)/L}\lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha t(L-1)/L}e^{-\alpha(t-t_{k-1})/L}$$
$$\le e^{\alpha\tau_0(L-1)/L}\lambda_{\max}(E^T P)\|\bar{\phi}\|^2 e^{-\alpha t(L-1)/L},$$

Let $M = \sqrt{\lambda_{\max}(E^T P)/\lambda_{\min}(E^T P)}\|\bar{\phi}\|$ and $\gamma = \alpha(L-1)/L$,

It is easy to see that

$$\|x(t)\| \le M\|\bar{\phi}\|e^{-\gamma(t-t_0)}, \quad t \ge t_0$$

Therefore the T-S fuzzy singular perturbed system (1) with time-varying delay is global exponential stable via the state feedback fuzzy controller (5).

Remark 2: Theorem 1 provides sufficient conditions for the global exponential stability of the T–S fuzzy singular perturbed system (1) with time-varying delay. The conditions in Theorem 1 are all in terms of LMIs, which can be efficiently verified via solving the LMIs numerically by interior point. And the feedback gains can also be obtained via solving the LMIs.

Corollary 1: If there exist symmetric and positive definite matrix P, such that the following LMIs hold

$$E^T P = PE \ge 0 \quad (24)$$

$$\begin{bmatrix} A_i^T P + PA_i + aE^T P & PB_i \\ B_i^T P & -bE^T P \end{bmatrix} < 0 \quad (25)$$

$i = 1,2,\cdots q,$

$$E^T P + \tilde{G}_{ki}^T E^T P + E^T P\tilde{G}_{ki} + \tilde{G}_{ki}^T E^T P\tilde{G}_{ki} - \lambda_k E^T P < 0 \quad (26)$$

$i = 1,2,\ldots,q$, $k = 1,2,\ldots,$

where $a > b > 0$ and $\alpha > 0$ is the unique positive root of the following equation

$$\alpha - a + be^{\alpha\tau_0} = 0$$

The parameters λ_k are specified by the designer, where $e^{\alpha\tau_0} \le \lambda_k \le e^{\alpha(t_k-t_{k-1})/L}, \quad k = 1,2,\ldots$ Then the T–S fuzzy singular perturbed system (1) with time-varying delay is global exponential stable when $u(t) = 0$.

IV. Simulations

Consider a continuous stirred tank reactor nonlinear system [17]. As in [17], the system model is given by the following equations

$$\dot{x}_1 = \frac{-1}{\lambda} x_1(t) + D_\sigma(1 - x_1(t)) \times \exp\left(\frac{x_2(t)}{(1 + x_2(t)/\gamma_0)}\right) + \left(\frac{1}{\lambda} - 1\right)x_1(t - \tau(t))$$

$$\dot{x}_2 = \left(\frac{1}{\lambda} + \beta\right)x_2(t) + HD_\sigma(1 - x_1(t)) \times \exp\left(\frac{x_2(t)}{(1 + x_2(t)/\gamma_0)}\right) + \left(\frac{1}{\lambda} - 1\right)x_2(t - \tau(t)) + \beta u(t)$$

$$x_i = \phi_i(t), \quad t \in [-\tau_0, 0], \quad i = 1, 2 \qquad (27)$$

Where $\gamma_0 = 20, H = 8, D_\sigma = 0.072, \lambda = 0.8$ and $\beta = 0.3$.

The state $x_1(t)$ corresponds to the conversion rate of the reactor $0 \le x_1(t) \le 1$ and $x_2(t)$ is the dimensionless temperature. Assume that only the temperature can be measured on line, that is $x(t) = [x_1(t), x_2(t)]^T$.

Considering the impulsive effect and the different variance ratio of the system states (27) and using the T-S modeling approach developed in [17], we can obtain the following T-S fuzzy model with impulse to represent system (27).

Plant Rule i

IF $x_2(t)$ is M_2^i

THEN $E\dot{x}(t) = A_i x(t) + B_i(t - \tau(t)) + C_i u(t), t \ne t_k$,

$\Delta x(t_k) = \tilde{G}_{ki} x(t), \; k = 1, 2, \cdots,$

$x(t) = \phi(t), \; t \in [-\tau_0, 0], i = 1, 2, \cdots, q,$

$$A_1 = \begin{bmatrix} -1.4274 & 0.0757 \\ -1.4189 & -0.9442 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2.0508 & 0.3958 \\ -6.4066 & 1.6268 \end{bmatrix}, \; A_3 = \begin{bmatrix} -4.5279 & 0.3167 \\ -26.2228 & 0.9387 \end{bmatrix}$$

$$B_1 = B_2 = B_3 = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}, \; C_1 = C_2 = C_3 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},$$

$$\phi(t) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}, \; E = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix},$$

The membership functions are selected as follows

$$M_2^1(x_2(t)) = \begin{cases} 1, & x_2 \le 0.8862 \\ 1 - \dfrac{x_2 - 0.8862}{2.7520 - 0.8862}, & 0.8862 < x_2 < 2.7520 \\ 0, & x_2 \ge 2.7520 \end{cases}$$

$$M_2^2(x_2(t)) = \begin{cases} 1 - M_2^3(x_2(t)), & x_2 \ge 2.7520 \\ 1 - M_2^3(x_2(t)), & x_2 \ge 2.7520 \end{cases}$$

$$M_2^3(x_2(t)) = \begin{cases} 0, & x_2 \le 2.7520 \\ \dfrac{x_2 - 2.7520}{4.7052 - 2.7520}, & 2.7520 < x_2 < 4.7052 \\ 1, & x_2 \ge 4.7052 \end{cases}$$

The time delay is chosen to be $\tau(t) = 0.7\sin^2(t + \pi/3)$. The impulsive matrices are as follows:

for $m \in N, i = 1,2,3,$

if $k = 4m$, then $\tilde{G}_{ki} = diag\{0.1, 0.1\}$;

if $k = 4m + 1$, then $\tilde{G}_{ki} = diag\{-0.1, -0.1\}$;

if $k = 4m + 2$ then $\tilde{G}_{ki} = diag\{0.1, -0.1\}$;

if $k = 4m + 3$ then $\tilde{G}_{ki} = diag\{-0.1, 0.1\}$.

The design parameters are chosen as follows:

$\Delta t_k = t_k - t_{k-1} = 1, k = 1, 2, \cdots; \tau_0 = 0.8, L = 1.1,$

$a = 1, b = 0.5, \alpha = 0.34242962$ is the unique positive root of the equation

$$\alpha - a + be^{\alpha\tau_0} = 0;$$

$$e^{\alpha\tau_0} \le \lambda_k = 1.3610 = e^{0.9\alpha} \le e^{\alpha\Delta t_k / L}, \; k = 1, 2, \cdots$$

Using MATLAB LMI toolbox, when $\varepsilon \in [1/432 \quad 432]$, The LMIs (8)-(10) hold.

A. Let $\varepsilon = 0.8$, we obtain that

$F_1 = [-3.7530, -44.9239], F_2 = [-3.3831, -48.5588]$ and $F_3 = [-0.7792, -47.2840]$, Simulation results are shown in Fig.1 and Fig.2 under initial condition:

$$x(t) = \phi(t) = [2 \quad -3]^T, t \in [-\tau_0 \quad 0].$$

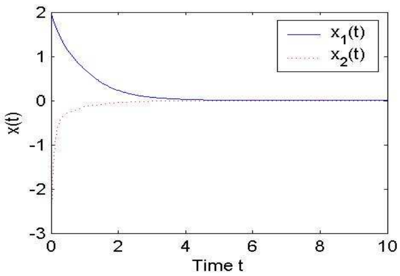

Fig.1 Responses of system states

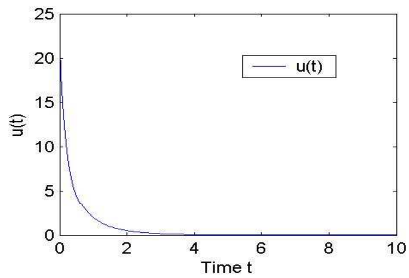

Fig. 2 Control input u

B. Let $\varepsilon = 0.08$, we obtain that

$F_1 = [-2.8225, -26.3952], F_2 = [-2.4825, -30.3895]$ and $F_3 = [-1.1772, -34.2972]$, Simulation results are shown in Fig.3 and Fig.4 under initial condition:

$$x(t) = \phi(t) = [2 \quad -3]^T, t \in [-\tau_0 \quad 0].$$

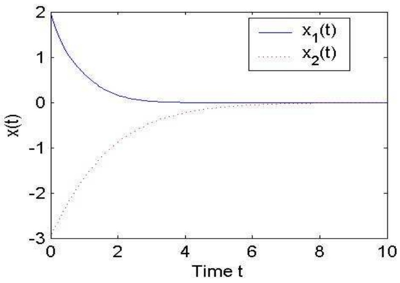

Fig. 3 Responses of system states

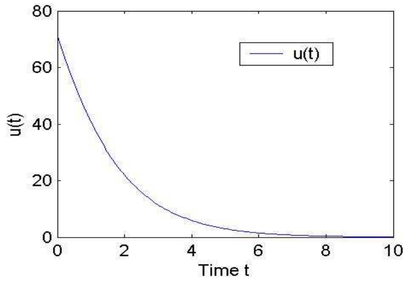

Fig. 4 Control input u

V. CONCLUSIONS

We have investigated the problem of robust fuzzy control for a class of nonlinear systems with time-varying delay. Based on Lyapunov method and LMI technique, some criteria have been proposed to guarantee the global exponential stability of the closed-loop system. Numerical simulations have been included to demonstrate the effectiveness of the proposed controller.

ACKNOWLEDGMENT

This work is partially supported by the Natural Science Foundation of Jiangsu Higher Education Institutions of China (No.08KJD510008,No.09KJB510018,No.10KJB510026) and the Qin Lan Project of the Jiangsu Higher Education Institutions of China.

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