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Application of Generalized Stochastic Petri Nets (GSPN) in Modeling and Evaluating a Resource Sharing Flexible Manufacturing System

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***Abstract*—**In most study fields, a phenomenon may not be studied directly but it will be examined indirectly by phenomenon model. Making an accurate model of system, there is attained new information from modeled phenomenon without any charge, danger, etc… there have been developed more solutions for describing and analyzing the recent complicated systems but few of them have analyzed the performance in the range of system description. Petri nets are of limited solutions which may make such union. Petri nets are being applied in problems related to modeling and designing the systems. Theory of Petri nets allow a system to model mathematically by a Petri net and analyzing the Petri net can then determine main information of modeled system's structure and dynamic. This information can be used for assessing the performance of systems and suggesting corrections in the system. In this paper, beside the introduction of Petri nets, a real case study will be studied in order to show the application of generalized stochastic Petri nets in modeling a resource sharing production system and evaluating the efficiency of its machines and robots. The modeling tool used here is SHARP software which calculates specific indicators helping to make decision.

***Keywords*—**Flexible manufacturing system, generalized stochastic Petri nets, Markov chain, performance evaluation.

I. Introduction

FLEXIBLE manufacturing systems include a set of computerized numerical controlling machines and supporting stations which connected to an automatic transportation system and controlled by a central computer. Key elements of this producing system include:

- Automated machines with reprogramming ability;
- Automatically developing the tools for material transportation;
- Automated controlling system.

Designing the flexible producing systems and then analyzing their performance is one of the problems attended long time ago by researchers.

Using Petri net to modeling the flexible manufacturing systems provide the possibility to consider integrally the effects of different elements of producing system (elements like machinery, robots, parts, etc…) and determine the improved exploiting program in the planning phase.

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Using this method, it is also possible to not only consider the time deterministically for calculating the performance of producing unit, but also consider the changes of time in the calculations while are inevitable.

The most fundamental question is: can we use of Petri net to model a producing system. Researchers believe that as there has been allocated a time to Petri nets proportional to the time for activity of each producing unit, if time for all elements of producing unit follow normal distribution or exponential distribution (or finite time) one can model it using Petri net algorithm and as such conditions can be made in a flexible manufacturing system, so we can use of Petri net algorithm for this reason.

Petri nets were introduced for the first time by Carl Adam Petri in 1962 which described cause and effect relations without intending the time considerations. Hullti continued this study on the project for information system theory of applied data. Most developments on symbolization and recent considerations of Petri nets are of results of this project. This project indicated that how Petri net can model and analyze concurrent and consistent elements and compounds, other studies have also attracted the researchers' attention in the Masachusset Researching Institute. "Structures' determination" group, leaded by Deniss, was the origin of most studies and publications in the field of Petri nets [15].

Introducing the time concepts in Petri nets was suggested by C.Ramachandoni, P.M.Merlin, D.S.Farber and J.Sifakis years later it. First definition of Petri nets along with possible time presented by F.Symons, G. Florin, S. Natkinand M. Malloy. Thisplan discussed the possibility of relation between Petri nets and concepts of performance evaluation based on the approach for modeling the stochastic processes. In a developing approach by Malloy, there was combined probable time with deterministic null delays, so both time changes and system's logical changes were described by model, thus such modeling called General Stochastic Petri Nets [1].

This study aims to present a method by which a comprehensive and accurate plan may be obtained for a production system in the planning phase with optimal efficiency. On the other hand, as a production system is going to upgrades to a flexible manufacturing cell, the suggested flexible manufacturing cell is studying as a dynamic system, using a generalized stochastic Petri nets and then it's performance such as machinery efficiency, robots efficiency, inventory in the manufacturing process size and buffers size will be calculated and then most optimal option will be selected for implementing.

II. Structural Components of Petri Nets

Structural components of Petri nets include places, transitions and arcs. Places apply for stating the possible states of the system. Transitions apply for stating the approaches which can change the state of systems. Arcs can determine the relation between states and events. Tokens are indistinguishable tokens which located in places and used for stating the states of systems which usually called Petri net marking. If for example a place indicates a provision, it must include a token of null mark. If there is a token in the place, the provision is true otherwise it is false. If a place also defines a situation, the number of marks may be used to determine the situation.

Petri net model is graphically indicated by such procedure that places indicated by circles, transitions by bar and tokens by black spots in the places [1, 2, 3, 4].

Transitions can be divided into two parts: temporal transitions and immediate transitions. Firing a transition in the Petri network model is related to the event changing the state of a real system. This changing in the state can be occurred due to one of the following reasons: either by examining the accuracy of some of conditions in the system or by ending some of activities. Regarding to the second case, transitions remarkably be used for modeling the activities. Thus, when transition is fired, then complementing the activity will be stated. Therefore, temporal concept can naturally be related to the transitions. Temporal transitions are indicated by boxes in the figure, but all events happening in a model of real system may not be related to the time consuming processes. A model of a multi processor system for example, is usually ignored of time for staring activities comparing to the time for doing other activities. Integrating non-temporal transitions in the temporal Petri net models can define a kind of transitions called “immediate”. As soon as becoming powerful, immediate transitions will be fired without delay. Immediate transitions thus are preferred than temporal transitions in the Petri nets.

III. System Dynamic

In this discussion, there will be dealt with changes and dynamics of markings in Petri nets. These are controlled by the firing transitions and cause losing or adding tokens.

Enabling Rule and Firing Rule are related to transitions. Enabling rule determine situations by which transitions are allowed to fire. Firing rule determine marking changes obtained by firing the transitions. On the other hand, enabling rule determines situations by which a transition allowed to fire and firing rule determine the transformation made by transition.

Both rules are made by arcs. In enabling rule, input and inhibitory arcs are involved, while in the firing rule, input and output arcs are involved.

Definition1. A transition is said to be enabled if and only if:

The number of tokens in the input place is greater or equal with frequency of related arc and the number of tokens in the inhibitory place is smaller than arc frequency:

$$\forall P \in^{\bullet} t, M(P) \geq I(t,p),$$
$$\forall P \in^{0} t, M(P) < H(t,p) \qquad (1)$$

As indicated in Fig. 1, transition t is enabled if and only if the number of tokens in the place P1 is greater or equal with n and the number of tokens in the place P2 is smaller of equal with M.

When transition t is fired, from a place in the set **.**t, by the number of frequency of arcs connecting it to the place t, it will clear the tokens and will add to any place in the output set of t^{O}, by the number of frequencies related to the output of transition t [1].

Definition2. Firing the transition t is enabled in the marking M, so it makes marking M’ and may be given by [1]:

$$M' = M + 0(t) - I(t) \qquad (2)$$

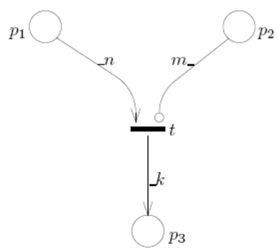

Fig. 1 Enabling and Firing Rules

IV. Situations Common in Changing the Petri Net System

A. Conflict

Petri nets are most suitable for describing the conflicts. Conflict indeed makes a choice determining with Petri net. Fig. 2 indicates that when transition t_1 is firing, a token will be located in the place P and both transitions t_3 and t_2 will be enabled. When one of them is fired, it will remove the token from place P and thus another transition will be unable.

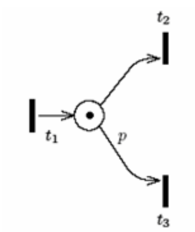

Fig. 2 An example of conflict situation in Petri nets

B. Concurrency

Concurrency is known by parallel activities.

Definition3. t_1, t_m is said to be concurrent only and only if both are enabled in marking M and not in conflict with each other.

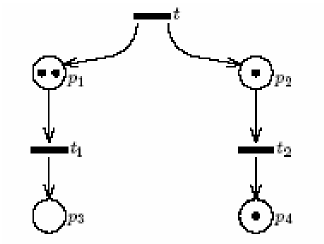

Fig. 3 An example of concurrency situation in Petri nets

C. Confusion

In marking M in where P_1,P_2 are marked, transitions of t_2,t_1 are concurrent. If first t_1 is fired, transitions t_2,t_3 will be in conflict together. While, if t_2 is fired initially, there is no conflict. Thus as indicated, if there is begun from marking P_1+P_2 and attained to marking P_4+P_3, it is not necessary to solve a conflict. This is corresponded to the order of firing two transitions t_1, t_2 concurrently. This is called confusion and makes complicated problems [1].

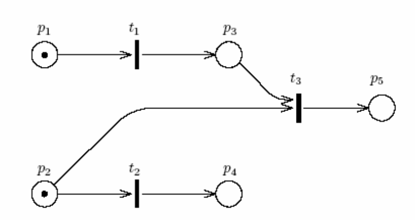

Fig. 4 An example for confusion situation in Petri networks

D. Mutual Exclusion

As models for Petri networks are used for validity and accuracy of systems, it is important knowing this subject that when two transitions are not enabled in marking together.

Definition4. For any Petri net system, transitions t_1, t_2 are in mutual exclusion if:

$$\exists M \in RS(M_0): t_1 \in E(M)\, and\, t_m \in E(M) \qquad (3)$$

V. Discrete-State Stochastic Processes Related to a Generalized Stochastic Petri Net

Regarding to the property of lack of memory in the exponential distribution function for firing delays, one can indicated that members of reachability set of a generalized stochastic Petri net system are analogous with states space of a Continuous Time Markov Chains (CTMC). Continuous time Markov chain related to a generalized stochastic Petri net system can be obtained from following rules:

1) For continuous time Markov chains, states space of S=[Si] based on a generalized stochastic Petri net equals with the reachability set: (Mi→ Si)

2) Rate of transition from state Si (related to marking Mi) to the state of Sj (related to marking Mj) is obtained from summation of firing rate of transitions which are enabled in Mi and their firing produce the marking Mj.

Algorithm for automatically producing the matrix Q (matrix for rate of state transition) for continuous time Markov chain considering w_k as the rate for firing the transition T_k and considering $E_j(M_i) = \{h : T_h \in E(M_i)^\wedge E_i[T_h > M_j]\}$ as set of transitions which their firing will take the net from marking Mi to marking M_j, is equal with:

$$q_{ij} = \{\sum_{tk \in E_j(Mi)} w_k \qquad i \neq j\} \qquad (4)$$

$$q_{ij} = \{qi \qquad i = j\}$$

Where
$$q_i = \sum w_k$$
$$T_k \in E(M_i)$$

Vector η is the steady state probabilities distribution on the markings of generalized stochastic Petri net given by:

$$\begin{cases} \eta Q = 0 \\ \eta 1^T = 0 \end{cases} \qquad (5)$$

Where o is a line vector equal with size η with all its elements equal with zero and 1^T is a column vector equal with η in size with all its elements equal with 1.

VI. Case Study

A. Acknowledging with the Producing Process of Studied Cell

Industrial Poly-Met factory is of the polymer devices manufactures in Iran. One of its products is a heater frame of GLX405 (kind of Iranian automobile).

Fig. 5 A heater Frame of GLX405

Required raw material is Polypropylene which after becoming hot, is transferred by an operator to an injection machine (M_1) and then injected to its mold. After the completion of operation, the injected pieces are transferred by an operator to a drilling Machine (M_2) and after completion of machine operation; products are moved by an operator to the next production department.

The factory manager is going to improve the production process by purchasing three new automatic robots. So it is very important to evaluate the new production process whether this cause to an increasing in efficiency or not.

The new process is modeled as a flexible manufacturing cell. The shape of related cell has indicated in Fig. 6. As shown, this cell includes two machines M_1 and M_2 with three robots R_1, R_2 and R_3.

Robot R_1 is transferring the raw material from the storing place to machine M_1 and maintains the part in a suitable place for operation in the place of machine M_1. After completing the related operation by Machine M_1 on the raw material, this robot will be freed from and can repeat the operation again.

Part pulling out of the machine M_1 (semi-manufactured part) may pass one of the following routes based on accessing to the machine M_2 or not accessing to it: either machine M_2 is ready to accept the semi-manufactured machine, thus robot R_2 can bring the semi-manufactured part near the machine M_2 or machine M_2 is occupied with conducting the operation on another semi-manufactured part, then part must be transferred to the buffer B_1. This transition is done by robot R_2 and then robot R_2 will be freed from. After machine M_2 being released, semi-manufactured part will be transferred to machine M_2 by robot R_2 from buffer B_1 to be operated.

To maintain the semi-manufactured part in the place of machine M_2 and in a suitable situation Robot R_3 is used to operate on it and after operation by machine M_2 on the semi-manufactured part, robot R_3 will pull out the product from machine M_2 and transfer it to the buffer B_2 to be transferred to the next cell.

It is indicated that robot R_2 is a shared source between machine M_1, machine M_2 and buffer B_1, while such situation lacks in robots R_1 and R_3. Based upon this case, there are two different policies for using robots R_1 and R_3 comparing with robot R_2: robot R_1 is used for replacing the raw materials to machine M_1 and maintaining the raw materials in the suitable situation in the place of machine M_1 and robot R_3 is also used for maintaining the semi-manufactured part in its suitable situation in the place of machine M_2 and then transferring the product from machine M_2 to buffer B_2.

This is while robot R_2 is used to transition semi-manufactured part from machine M_1 to the place of machine M_2 as well as transferring it from machine M_1 to buffer B_1 and transferring it from buffer B_1 to machine M_2. Robot R_2 is freed after transferring the part to the machine M_2.

B. Suggested Model for Flexible Produced Cell using Generalized Stochastic Petri Net

Petri net system of production system studied is indicated in Fig. 7. Above mentioned model includes 18 places and 14 transitions. Details of each place and transition have indicated in the following tables:

TABLE I
PROPERTIES OF PLACES

rank	Place name	Descriptions
1	P_0	Presence of raw materials in the warehouse
2	P_1	Readiness of robot R_1 to replace raw materials from warehouse to machine M_1
3	P_2	Readiness of machine M_1 to inject raw material
4	P_3	Entrance of raw materials to machine M_1
5	P_4	Presence of semi-conducted part (but not determined whether goes to M_2 or B_1)
6	P_5	Readiness of robot R_2
7	P_6	Readiness of machine M_2 for receiving the semi-manufactured part
8	P_7	Presence of semi-manufactured part to be transferred to machine M_2
9	P_8	Accessibility of semi-manufactured part to be delivered to machine M_2 from M_1
10	P_9	Presence of semi-manufactured part to be transferred to buffer B_1
11	P_{10}	Not completeness of semi-manufactured asset in the buffer B_1
12	P_{11}	Accessibility of semi-manufactured part for being delivered to machine M2 from buffer B_1

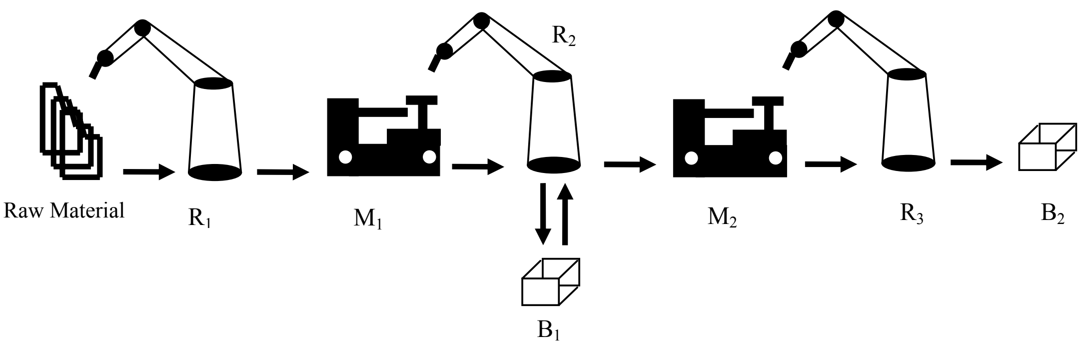

Fig. 6 Suggested Flexible Manufacturing Cell

13	P_{12}	Accessibility of semi-manufactured part in the place of machine M_2
14	P_{13}	Accessibility of product to being transferred to buffer B_2
15	P_{14}	Presence of product to being delivered to buffer B_2
16	P_{15}	Readiness of robot R_3
17	P_{16}	Readiness of product to being transferred to the next produced cell
18	P_{17}	Presence of semi-manufactured cell to being transferred to buffer B_1

TABLE II
PROPERTIES OF TEMPORAL TRANSITION

rank	Place name	descriptions
1	T_0	Transferred raw material to machine M_1
2	T_1	Operation on raw material using machine M_1
3	T_2	Transferred semi-manufactured part from machine M_1 to buffer B_1
4	T_3	Transferred semi-manufactured part from machine M_1 to machine M_2
5	T_4	Delivering semi-manufactured part to machine M_2 from machine M_1
6	T_5	Delivering the semi-manufactured part to buffer B_1
7	T_6	Delivering semi-manufactured part to machine M_2 from buffer B_1
8	T_7	Operation on the semi-manufactured part by machine M_2
9	T_8	Transferred product from machine $M_2$by robot R_3
10	T_{11}	Transferred product to machine M_2 from buffer B_1

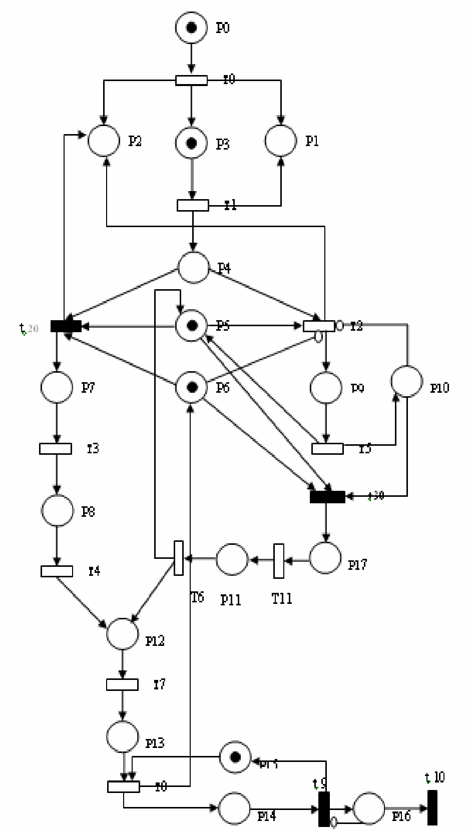

Fig. 7 Petri net system of studied product

TABLE III
PROPERTIES OF IMMEDIATE TRANSITION

rank	Place name	descriptions
1	T_{20}	transferred semi-manufactured part from machine M_1 to machine M_2
2	T_{30}	transferred semi-manufactured part to machine M_2 from buffer B_1
3	T_9	Delivering the product to buffer B_2
4	T_{10}	transferred the product to the next produced cell

C. Evaluating the Efficiency of Studied Flexible Produced System

To evaluate the efficiency of studied flexible produced system ii is used an approach of generalized stochastic processes Petri net. In this approach there is used both temporal transitions and immediate transitions. Thus, model is able to describe complement of operations by temporal transitions and systems' logical behavior by immediate transitions. Properties of temporal and immediate transitions are as follows:

TABLE IV
RATE OF TEMPORAL TRANSITIONS FIRING

Rank	Transition name	Period proportional to transferring based on min	Rate of transition firing based on min
1	T_0	5	0.2
2	T_1	50	0.02
3	T_2	5	0.02
4	T_3	5	0.02
5	T_4	5	0.02
6	T_5	5	0.02
7	T_6	5	0.02
8	T_7	45	0.022
9	T_8	5	0.02
10	T_{11}	15	0.067

TABLE V
LEVELS FOR PRIORITY OF IMMEDIATE TRANSITIONS

Rank	Place name	Level of priority
1	T_{20}	4
2	T_{30}	3
3	T_9	1
4	T_{10}	2

To determine the cause for allocating different levels of priority to immediate transitions it is better to determine conflict transitions and this include: ECS= $\{t_{20}, t_{30}, t_2\}$.

To remove this conflict there is used from allocating the different levels of priority and inhibitor arcs such that presence of inhibitor arcs from place p6 to temporal transition T_6, it will practically remove the conflict of T_2 with T_{20} and T_{30} because these three transitions are common in the places of P_4, P_5 and P_6 but presence of inhibitory arc from transition T_2 means that for T_2 being enabled, it is necessary there is no token in the place of P_6; in this case, transitions T_{20} and T_{30} will be unable and thus conflict between T_2 and transitions of T_{20} and T_{30} will be removed.

In the process for providing the semi-manufactured parts also to operate by machine M_2 on it, one can use from semi-manufactured part or buffer B_1 or machine M_1 (transitions T_3 and T_{11}). Thus, there is conflict between these two transitions. In addition, because both of them include temporal transitions, so any transition with short period, will be fired earlier and results in another transition be unable forever. On the other hand, these two transitions are enabled together and if each one has short period, will be fired and make another transition unable. To remove this problem, there has been used from immediate transitions with different priority levels. To allocate priority there are two different approaches:

1) Economy

Semi-manufactured part will be transferred from machine M_1 not from buffer B_1. Thus, it will avoid machine M_1 to be idle.

2) Reducing the inventory while manufacturing

Semi-manufactured part will be transferred from buffer B_1 not from machine M_1. In this case, although machine M_1 is idle, but in contrast the size of inventory will be reduced while manufacturing. This approach is used when there is less space. Priority level of T_{10} has always priority on T_9. Its reason is that product will immediately be transferred to the next producing cell and remaining process will not be ceased due to its shortage.

There has been used first approach for studied flexible produced cell. To evaluate the efficiency of related system, it is necessary to initially examine possible states of the system. As mentioned previously, accessing set (set for possible states) must be limited. Accessing set for above mentioned system has been attained and indicated in Table VI. To determine the rate of system transformation, Fig. 8 indicates accessing graph. To calculate limit states, there is used Sharp software. In this study, the aim is calculating the efficiency of machines M_1 and M_2 and robots R_1, R_2 and R_3.

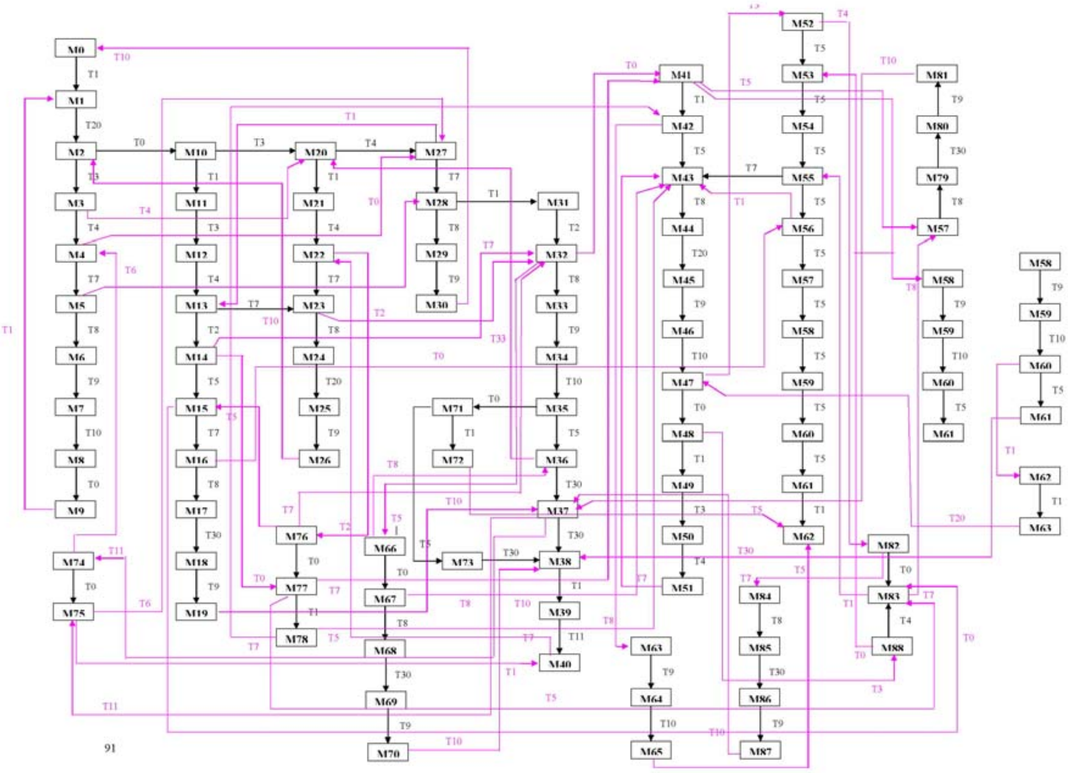

Fig. 8 Reachability graph of suggested flexible manufacturing cell

In related system, there is several factors influence on the system performance:

- Rate of firing attributed to each temporal transition;
- Priority level attributed to each immediate transition;
- The initial marking system;
- Size of buffers B_1 and B_2.

In this study first three factors have been valued and considered as known and there is studied the effect of changes in fourth factor on the efficiency of machines and robots. The rate of firing the temporal transitions has been indicated in table as well as priority level of immediate transitions. Initial marking has been assumed as following: $M_0=P_0+P_3+P_5+P_6+P_{15}$. The size of buffers B_1 and B_2, n and m, have initially considered equal with 1 and thus the amount of mean limit probabilities of the number of tokens will be calculated for each place and its results are as in Table VII.

TABLE VIII
AMOUNT OF STEADY STATE OF MEAN NUMBER OF TOKEN IN PLACE P_{10} BASED ON M=3

Amounts of n	Probabilities of Mean Number of Token in Place P10
1	0.363382522
2	0.752247715
3	1.11667184
5	1.76002002
8	2.5163808
9	2.72668649
10	2.90998133
12	3.21796736

TABLE VI
REACH ABILITY SET OF SUGGESTED FLEXIBLE MANUFACTURING CELL

Marking No.	P0	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17
0	1			1		1	1									1		
1	1	1			1	1	1									1		
2	1	1	1					1								1		
3	1	1	1						1							1		
4	1	1	1			1							1			1		
5	1	1	1			1								1		1		
6	1	1	1			1	1								1			
7	1	1	1			1	1									1	1	
8	1	1	1			1	1									1		
9	1			1		1	1									1		
10	1			1				1								1		
11	1	1			1			1								1		
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82	1	1	1			1					1		1			1		
83	1			1		1					1		1			1		
84	1	1	1			1					1			1		1		
85	1	1	1			1	1				1				1			
86	1	1	1												1			1
87	1	1	1													1	1	1
88	1	1	1						1		1					1		

TABLE VII
AMOUNTS OF STEADY STATE OF MEAN FOR NUMBER OF TOKENS IN EACH PLACE BASED ON N=1 AND M=1

Place name	Limit Possibilities of Mean for Number of Tokens in Each Place
P_0	1
P_1	0.383086974
P_2	0.616913026
P_3	0.616913026
P_4	0.321395671
P_5	0.766258576
P_6	0.163023275
P_7	0.074391686
P_8	0.0371745840
P_9	0.0245167182
P_{10}	0.363414407
P_{11}	0.0245167187
P_{12}	0.561830029
P_{13}	0.0669219896
P_{14}	0.124828685
P_{15}	0.875171315
P_{16}	0.0493530425
P_{17}	0.0731842350

Steady state of mean number of token for example in the place P_1 is equal with 0.383086974; it means robot R_1 is free in 38.31% of working time and are occupied in 61.69% working time. For example limit probability of mean number of token is also 0.766258576 in the place of P_5 and are occupied in 23.37% of working time.

As indicated, the mean number of token in the place P0 is 1; this is because in this model it is assumed that raw material is always accessible with no loss.

Now the optimal amount of buffer B_1 size is determined regarding to two amounts for buffer B_2 (m=3 and m=5). For this reason, there have been calculated amounts of limit probability for mean number of token in P_0 for different amounts of n and then there will be dealt with analyzing resulted data. For m=3, results are as Table VIII.

Figures of above mentioned table mean that for example if the size of buffer B_1 be equal with 1, averagely 0.36 of semi-manufactured part will be existed in this buffer.

Now, the amount of m is assumed equal with 5 and above mentioned calculations will be repeated:

TABLE IX
AMOUNT OF STEADY STATE OF MEAN NUMBER OF TOKEN IN PLACE P_{10} BASED ON M=5

Amounts of n	Probabilities of Mean Number of Token in Place P10
1	0.363382522
2	0.752247715
3	1.11667184
5	1.76002002
8	2.5163808
9	2.72668649
10	2.90998133
12	3.21796736

It can be indicated that the result has been obtained based on m=5 has no difference with results form m=3. This is logical answer, because the size of buffer B_2 has no effect on the size of buffer B_1 and its effect on the place P_{16} and it will affect on the performance of next produced cell.

Diagram for steady state of mean number of token in place P_{10} based on the size of buffer B_1 is indicated as following:

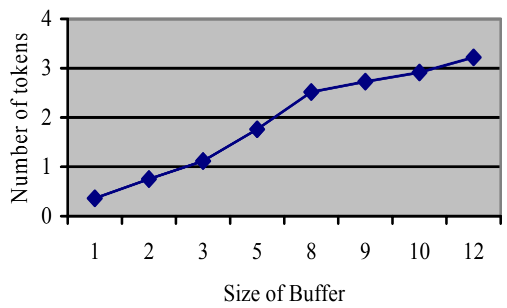

Fig. 9 Mean number of semi-manufacturing parts existing in the buffer B_1 based on the size of buffer

Considering to above mentioned diagram, one can see that the slop of diagram is most in points 3, 5 and 8 so these three points will be examined. If the size of buffer B_1 is equal with 3, the mean number of semi-manufacturing parts in buffer equals with 1.11667184. If the size of buffer B_1 is equal with 5, the mean number of semi-manufacturing parts in buffer equals with 1.76002002. If the size of buffer B_1 is equal with 8 mean number of semi-manufacturing parts in buffer equals with 2.5163808. it is indicated that increasing the buffer from 3 to 5, mean number of semi-manufacturing parts existing in the buffer will be increased to the amount of 0.643 and if this amount increases from 5 to 8, mean number of semi-manufacturing parts existing in the buffer will be increased to the amount of 0.756.

To determine the optimal size of buffer B_1 regarding to the limited size of accessing space and by increasing the size of buffer, the mean number of semi-manufacturing parts will not be increased considerably, so the optimal size of buffer B_1 has been considered equal with 3.

Now, regarding the optimal amount n=3 for buffer B_1, one can calculate the efficiency of existing machines. It must be mentioned that presence of token in places of P_1, P_2, P_5, P_6 and P_{15} indicates accessibility of robot R_1, machine M_1, machine M_2, Robot R_2 and robot R_3 respectively. Lack of token then in such places means they are idle.

TABLE X
AMOUNT OF STEADY STATE OF MEAN NUMBER OF TOKEN IN EACH PLACE BASED ON M=3

Place name	Limit probability for voiding the place
P1	0.649388960
P2	0.93506114
P3	0.268615865
P4	0.88673137
P5	0.589388960

VII. CONCLUSION

As indicated above, in the studied flexible manufactured cell, aiming to reducing the used space, the amount of inventory in the manufacturing flow and inventory of product will be determined by efficiency of machinery and robots.

The importance of above mentioned study is that in the designing phase, all possible states of studied system and influence of each element on system has been examined and finally the most optimal option for conduction may be selected. Thus, in the planning phase, dynamic of system has also been considered and the probability of anticipating the different states resulting in different functions of system will be obtained and it is also possible to prevent undesirable states.

The main application of this algorithm is for systems where the time of its elements follows normal distribution or exponential distribution; for example, in designing the telecommunication circuits, computer networks and automated producing systems.

Limitations for applying this method include lack of software for simulation of Petri algorithm. Based on extended number of possible states of a system (number of possible states in mentioned model in this study is 88), calculation is very time consuming; therefore, it is needed a software for simulation and calculation of possible different states of system; but unfortunately there are limited software designed based on this algorithm.

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