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The orlicz space of the entire sequence fuzzy numbers defined by infinite matrices

N.Subramanian and C.Murugesan

Abstract—This paper is devoted to the study of the general properties of Orlicz space of entire sequence of fuzzy numbers by using infinite matrices.

Keywords—Fuzzy numbers, infinite matrix, Orlicz space, entire sequence.

I. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced Zadeh[18] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming.

In this paper we introduce and examine the concepts of Orlicz space of entire sequence of fuzzy numbers generated by infinite matrices.

Let $C(R^n) = \{A \subset R^n : A\, compact\, and\, convex\}$. The space $C(R^n)$ has linear structure induced by the operations $A+B = \{a+b : a \in A, b \in B\}$ and $\lambda A = \{\lambda a : a \in A\}$ for $A, B \in C(R^n)$ and $\lambda \in R$. The Hausdorff distance between A and B of $C(R^n)$ is defined as

$$\delta_\infty(A,B) = max\{sup_{a\in A} inf_{b\in B}\|a-b\|, sup_{b\in B} inf_{a\in A}\|a-b\|\}$$

It is well known that $(C(R^n), \delta_\infty)$ is a complete metric space.

The fuzzy number is a function X from R^n to [0,1] which is normal, fuzzy convex, upper semi-continuous and the closure of $\{x \in R^n : X(x) > 0\}$ is compact. These properties imply that for each $0 < \alpha \le 1$, the $\alpha-$level set $[X]^\alpha = \{x \in R^n : X(x) \ge \alpha\}$ is a nonempty compact convex subset of R^n, with support $X^c = \{x \in R^n : X(x) > 0\}$. Let $L(R^n)$ denote the set of all fuzzy numbers. The linear structure of $L(R^n)$ induces the addition $X+Y$ and scalar multiplication $\lambda X, \lambda \in R$, in terms of $\alpha-$ level sets, by $|X+Y|^\alpha = |X|^\alpha + |Y|^\alpha, |\lambda X|^\alpha = \lambda |X|^\alpha$ for each $0 \le \alpha \le 1$. Define, for each $1 \le q < \infty$,

$$d_q(X,Y) = \left(\int_0^1 \delta_\infty(X^\alpha, Y^\alpha)^q d\alpha\right)^{1/q}, and\, d_\infty = sup_{0\le\alpha\le1}\delta_\infty(X^\alpha, Y^\alpha),$$

where δ_∞ is the Hausdorff metric. Clearly $d_\infty(X,Y) = lim_{q\to\infty} d_q(X,Y)$ with $d_q \le d_r$, if $q \le r$ [11]. Throughout the paper, d will denote d_q with $1 \le q \le \infty$. A complex sequence, whose k^{th} terms is x_k is denoted by $\{x_k\}$ or simply x. Let ϕ be the set of all finite sequences. Let ℓ_∞, c, c_0 be the sequence spaces of bounded, convergent and null sequences $x = (x_k)$ respectively. In respect of ℓ_∞, c, c_0 we have
$\|x\| = \overset{sup}{k} |x_k|$, where $x = (x_k) \in c_0 \subset c \subset \ell_\infty$. A sequence $x = \{x_k\}$ is said to be analytic if $sup_k |x_k|^{1/k} < \infty$. The vector space of all analytic sequences will be denoted by Λ. A sequence x is called entire sequence if $lim_{k\to\infty} |x_k|^{1/k} = 0$. The vector space of all entire sequences will be denoted by Γ.
Orlicz [26] used the idea of Orlicz function to construct the space (L^M). Lindenstrauss and Tzafriri [27] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p (1 \le p < \infty)$. Subsequently different classes of sequence spaces defined by Parashar and Choudhary[28], Mursaleen et al.[29], Bektas and Altin[30], Tripathy et al.[31], Rao and subramanian[32] and many others. The Orlicz sequence spaces are the special cases of Orlicz spaces studied in Ref[33].
Recall([26],[33]) an Orlicz function is a function $M : [0,\infty) \to [o,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0, M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \le M(x)+M(y)$ then this function is called modulus function, introduced by Nakano[34] and further discussed by Ruckle[35] and Maddox[36] and many others.
An Orlicz function M is said to satisfy Δ_2- condition for all values of u, if there exists a constant $K > 0$, such that $M(2u) \le KM(u)(u \ge 0)$. The Δ_2- condition is equivalent to $M(\ell u) \le K\ell M(u)$, for all values of u and for $\ell > 1$.
Lindenstrauss and Tzafriri[27] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, for\, some\, \rho > 0\right\}. \quad (1)$$

The space ℓ_M with the norm

$$\|x\| = inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \le 1\right\} \quad (2)$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p, 1 \le p < \infty$, the space ℓ_M coincide with the classical sequence space ℓ_p. Given a sequence $x = \{x_k\}$ its n^{th} section is the sequence $x^{(n)} = \{x_1, x_2, ..., x_n, 0, 0, ...\}$

N.Subramanian is with the Department of Mathematics,SASTRA University, Tanjore-613 401, India. e-mail:(nsmaths@yahoo.com).
C.Murugesan is with the Department of Mathematics, Sathyabama University, chennai-600 119,India. e-mail: (murugaa23@sify.com).
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$\delta^{(n)} = (0, 0, ..., 1, 0, 0, ...)$, 1 in the n^{th} place and zero's else where.

II. Definitions and Prelimiaries:

Let w denote the set of all fuzzy complex sequences $x = (x_k)_{k=1}^{\infty}$, and M be an Orlicz fucntion, or a modulus function. consider
$\Gamma_M = \left\{ x \in w : lim_{k\to\infty} \left(M \left(\frac{|x_k|^{1/k}}{\rho} \right) \right) = 0\, for some\, \rho > 0 \right)$
and
$\Lambda_M = \left\{ x \in w : sup_k \left(M \left(\frac{|x_k|^{1/k}}{\rho} \right) \right) < \infty\, for some\, \rho > 0 \right)$
The space Γ_M and Λ_M is a metric space with the metric

$$d(x, y) = inf \left\{ \rho > 0 : sup_k \left(M \left(\frac{|x_k - y_k|^{1/k}}{\rho} \right) \right) \leq 1 \right\} \quad (3)$$

for all $x = \{x_k\}$ and $y = \{y_k\}$ in Γ_M.

We now give the following new definitions which will be needed in the sequel.

A. Definition

Let $X = (X_k)$ be a sequence of fuzzy numbers. The fuzzy numbers X_n denotes the value of the function at $n \in N$ and is called the n^{th} term of the sequence. We denote $w(F)$ the set of all $X = (X_k)$ sequences of fuzzy numbers.

B. Definition

Let $X = (X_k)$ be a sequence of fuzzy numbers. Then the set of all $X = (X_k)$ sequences of fuzzy numbers is said to Orlicz space of entire sequence of fuzzy numbers convergent to zero, written as $\left(M \left(\frac{|X_k|^{1/k}}{\rho} \right) \right) \to 0$ as $k \to \infty$, for some arbitrarily fixed $\rho > 0$ and is defined by $\left[d \left(M \left(\frac{|X_k|^{1/k}}{\rho} \right) \right) \to 0\, as\, k \to \infty \right]$. We denote by $\Gamma_M(F)$ the set of all Orlicz space of entire sequence of fuzzy numbers with M being a Orlicz function. The $\Gamma_M(F)$ is a metric space with the metric $\rho(X, Y) = sup_k d(X_k, Y_k) = sup_k d \left(M \left(\frac{|X_k - Y_k|^{1/k}}{\rho} \right) \right)$

C. Definition

A sequence $X = (X_k)$ of fuzzy numbers. Then the set of all $X = (X_k)$ sequences of fuzzy numbers is said to be Orlicz space of analytic if the set $\left\{ M \left(\frac{|X_k|^{1/k}}{\rho} \right) : k \in N \right\}$ of fuzzy numbers is bounded.
By Λ_M with M being a Orlicz function, we shall denote the set of all Orlicz space of analytic sequence of fuzzy numbers.

Let $A = (a_{nk})$ be an infinite matrix of fuzzy numbers and let (p_k) be a bounded sequence of positive real numbers, then $A_k(X) = \sum_{k=1}^{\infty} a_{nk} x_k$ (provided that the series converges for each $k = 1, 2, \cdots .)$ is called the $A-$ transform of X. We write $AX = A_k(X)$.

D. Definition

Let $X = (X_k)$ be a sequence of fuzzy numbers. Then we define
$\Gamma_M(F, A, p) = \left\{ X \in w(F) : \left[d \left(M \left(\frac{|A_k(X)|^{1/k}}{\rho} \right) \right) \right]^{p_k} \right\} \to 0\, as\, k \to \infty$
$\Lambda_M(F, A, p) = \left\{ X \in w(F) : sup_k \left[d \left(M \left(\frac{|A_k(X)|^{1/k}}{\rho} \right) \right) \right]^{p_k} \right\} < \infty$
If $A = I$, the unit matrix, then we get
$\Gamma_M(F, A, p) = \Gamma_M(F, p) = \left\{ X \in w(F) : \left[d \left(M \left(\frac{|X_k|^{1/k}}{\rho} \right) \right) \right]^{p_k} \to 0\, as\, k \to \infty \right\}$
$\Lambda_M(F, A, p) = \Lambda_M(F, p) = \left\{ X \in w(F) : sup_k \left[d \left(M \left(\frac{|X_k|^{1/k}}{\rho} \right) \right) \right]^{p_k} < \infty \right\}$
If A is an infinite matrix as above $p_k = p$ for all k, then we get
$\Gamma_M(A, p) = (\Gamma_M)_A(F) = \{ X \in w(F) : AX \in \Gamma_M(F) \}$
$\Lambda_M(A, p) = (\Lambda_M)_A(F) = \{ X \in w(F) : AX \in \Lambda_M(F) \}$.
Suppose that p_k is a constant for all k, then $\Gamma_M(F, A, p) = \Gamma_M(F, A)$. A metric d on $L(R)$ is said to be translation invariant if $d(X + Z, Y + Z) = d(X, Y)$ for $X, Y, Z \in L(R)$.

In this paper we study of the spaces $\Gamma_M(F)$ and $\Lambda_M(F)$ to $\Gamma_M(F, A, p)$ to $\Lambda_M(F, A, p)$ respectively, by applying the infinite matrix $A = (a_{nk})\,(n, k = 1, 2, 3, \cdots)$.

III. Main Results

A. Proposition

If d is a translation invariant metric on $L(R)$ then,
(i)$d(X + Y, 0) \leq d(X, 0) + d(Y, 0)$ (ii)$d(\lambda X, 0) \leq |\lambda| d(X, 0), |\lambda| > 1$. If d is a translation invariant, we have the following straight forward results.

B. Proposition

Let $X = (X_k)$ and $Y = (Y_k)$ be a sequence of fuzzy numbers and if M is a Orlicz function, then $\Gamma_M(A, p)$ is linear set over the set of complex numbers C.
Proof: It is easy. Therefore omit the proof.

IV. Inclusion Relations

A. Proposition

If $X = (X_k)$ be a sequence of fuzzy numbers. Let $0 \leq p_k \leq q_k$ and let $\left\{ \frac{q_k}{p_k} \right\}$ be bounded. Then $\Gamma_M(A, q) \subset \Gamma_M(A, p)$.
Proof: The proof is clear.

B. Proposition

Let $X = (X_k)$ be a sequence of fuzzy numbers.
a) Let $0 < inf p_k \leq p_k \leq 1$. Then $\Gamma_M(A, p) \subset \Gamma_M(A)$
b) Let $1 \leq p_k \leq sup p_k < \infty$. Then $\Gamma_M(A) \subset \Gamma_M(A, p)$.
Proof:(a) Let $X \in \Gamma_M(A, p)$. Then

$$\left[d \left(M \left(\frac{|A_k(X)|^{1/k}}{\rho} \right) \right) \right]^{p_k} \to 0\, as\, k \to \infty \quad (4)$$

Since $0 < inf p_k \leq p_k \leq 1$.

$$\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right] \leq \left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} \quad (5)$$

From (4) and (5) it follows that $X \in \Gamma_M(A)$. Thus $\Gamma_M(A,p) \subset \Gamma_M(A)$. We have thus proven (a)

Proof: (b) Let $p_k \geq 1$ for each k and $supp_k < \infty$. Let $X \in \Gamma_M(A)$.

$$\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} \to 0 \, as \, k \to \infty \quad (6)$$

Since $1 \leq p_k \leq supp_k < \infty$. we have

$$\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} \leq \left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right] \quad (7)$$

Hence $\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} \to 0 \, as \, k \to \infty$ [by using eq(7)]. Therefore $X \in \Gamma_M(A,p)$. This completes the proof.

C. Proposition

If $X = (X_k)$ be a sequence of fuzzy numbers. Let $0 < p_k \leq q_k < \infty$ for each k. Then $\Gamma_M(A,p) \subseteq \Gamma_M(A,q)$.

Proof: Let $X \in \Gamma_M(A,p)$

$$\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} \to 0 \, as \, k \to \infty \quad (8)$$

This implies that $\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right] \leq 1$ for sufficiently large k. Since M is non-decreasing, we get

$$\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{q_k} \leq \left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} \quad (9)$$

$\Rightarrow \left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{q_k} \to 0 \, as \, k \to \infty$ by using eq(8). We get $X \in \Gamma_M(A,q)$. Hence $\Gamma_M(A,p) \subseteq \Gamma_M(A,q)$. This completes the proof.

D. Proposition

If $liminf_k\left(\frac{p_k}{q_k}\right) > 0$ then $\Gamma_M(A,q) \subset \Gamma_M(A,p)$

Proof: Suppose that $liminf_k\left(\frac{p_k}{q_k}\right)$ holds. Let $X \in \Gamma_M(A,q)$. Then there is $\beta > 0$ such that $p_k > \beta q_k$ for large k. $\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} \leq \left[\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{q_k}\right]^{\beta}$ Since $\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} \leq 1$ for each $k, X \in \Gamma_M(A,p)$. This completes the proof.

V. PARNORMED SPACES

If E is a linear space over the filed C, then a paranorm on E is a function $g : E \to R$ which satisfies the following axioms; for $X, Y \in E$,
(P.1)$g(\theta) = 0$, (P.2)$g(X) \geq 0$ for all $X \in E$ (P.3)$g(-X) = g(X)$ for all $X \in E$, (P.4)$g(X+Y) \leq g(X) + g(Y)$ for all $X, Y \in E$, (P.5)If (λ_n) is a sequence of scalars with $\lambda_n \to \lambda \, (n \to \infty)$ and (X_n) is a sequence of the elements of E with $X_n \to X$ imply $\lambda_n X_n \to \lambda X$, where $\lambda_n \lambda \in C$ and $X_n, X \in E$; In other words $|\lambda_n - \lambda| \to 0, g(X_n - X) \to 0$ imply $g(\lambda_n X_n - \lambda X) \to 0 \, (n \to \infty)$. A paranormed space is a linear space E with a paranorm g and is written as (E, g)

A. Theorem

If $X = (X_k)$ be a sequence of fuzzy numbers. Then $\Gamma_M(A,p)$ complete with respect to the topology generated by the paranorm h defined by $h(X) = sup_k\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k}, where \, d \, is \, translation \, invariant$

Proof: Clearly $h(\theta) = 0, h(-X) = h(X)$. It can also be seen easily that $h(X+Y) \leq h(X) + h(Y)$ for $X = (X_k), Y = (Y_k) \in \Gamma_M(A,p)$, since d is a translation invariant. Now for any scalar λ, we have $|\lambda|^{p_k} < max\{1, |\lambda|\}$, so that $h(\lambda X) < max\{1, |\lambda|\}, h(X)$ on $\Gamma_M(A,p)$. Hence $\lambda \to 0, X \to \theta$ implies $\lambda X \to \theta$, and also $X \to \theta, \lambda$ fixed implies $\lambda X \to \theta$. Now let $\lambda \to 0, X$ fixed. For $|\lambda| < 1$ we have

$$\left[d\left(M\left(\frac{|A_k(X)|^{1/k}}{\rho}\right)\right)\right]^{p_k} < \epsilon \text{ for } n > N(\epsilon)$$

Also, for $1 \leq k \leq N$, since $\left[d\left(M\left(\frac{|A_k(X)|^{1/i}}{\rho}\right)\right)\right]^{p_k} < \epsilon$, there exists m such that $\left(\sum_{i=m}^{\infty}\left[d\left(M\left(\frac{|\lambda a_{k,i} X_i|^{1/i}}{\rho}\right)\right)\right]^{p_i}\right) < \epsilon$. Taking λ small enough then we have $\left(\sum_{i=m}^{\infty}\left[d\left(M\left(\frac{|\lambda a_{k,i} X_i|^{1/i}}{\rho}\right)\right)\right]^{p_i}\right) < 2\epsilon$, for all i. Hence $h(\lambda X) \to 0 \, as \, \lambda \to 0$. Therefore h is a paranorm on $\Gamma_M(A,p)$.

To show the completeness , let $\left(X^{(i)}\right)$ be a Cauchy sequence in $\Gamma_M(A,p)$. Then for a given $\epsilon > 0$ there is $r \in N$ such that $\left[d\left(M\left(\frac{\left|A_k\left(X^{(i)} - X^{(j)}\right)\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} < \epsilon \, for \, all \, i, j > r$. Since d is a translation invariant, The above equation implies that $\left(\sum_s a_{ks} \, d\left(M\left(\frac{\left|X_k^{(i)} - X_k^{(j)}\right|^{1/k}}{\rho}\right)\right)\right) < \epsilon \, for \, all \, i, j > r. \, and \, each \, k$. Hence $d\left(M\left(\frac{\left|X_k^{(i)} - X_k^{(j)}\right|^{1/k}}{\rho}\right)\right) < \epsilon \, for \, all \, i, j > r$. Therefore $\left(X^{(i)}\right)$ is a Cauchy sequece in $L(R)$. Since $L(R)$ is complete, $lim_{j \to \infty} X_k^j = X_k$, say. Fixing $r_0 \geq r$ and letting $j \to \infty$, we obtain the above equation that

$$\left(\sum_s a_{ks} \, d\left(M\left(\frac{\left|X_k^{(i)} - X_k\right|^{1/k}}{\rho}\right)\right)\right) < \epsilon \, for \, all \, r_0 > r, \quad (10)$$

(i.e)$d\left(\sum_s a_{ks}\left(M\left(\frac{\left|X_k^{(i)} - X_k\right|^{1/k}}{\rho}\right)\right)\right) < \epsilon \, for \, all \, r_0 > r$, since d is a translation invariant. Hence $\left[d\left(M\left(\frac{\left|A_k\left(X^{(i)} - X\right)\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} < \epsilon$. (i.e) $X^{(i)} \to X$ in $\Gamma_M(A,p)$. It is easy to see that $X \in \Gamma_M(A,p)$. Hence $\Gamma_M(A,p)$. is complete. This completes the proof.

Similarly we can prove the following:

B. Theorem

If $X = (X_k)$ be a sequence of fuzzy numbers then $\Lambda_M(A, p)$ is a complete paranormed space with the paranorm given by (11) if $inf p_k > 0$

VI. Conclusion

Classical ideas of the Orlicz space of entire sequences connected with fuzzy numbers of infinite matrices.

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References

[1] S.Aytar, Statistical limit points of sequences of fuzzy numbers, *Inform. Sci.*, **165**(2004), 129-138.
[2] M.Basarir and M.Mursaleen, Some sequence spaces of fuzzy numbers generated by infinite matrices, *J. Fuzzy Math.*, **11(3)**(2003), 757-764.
[3] T.Bilgin, $\Delta-$statistical and strong $\Delta-$Cesáro convergence of sequences of fuzzy numbers,*Math. Commun.*, **8**(2003), 95-100.
[4] P.Diamondand P.Kloeden, Metric spaces of fuzzy sets, *Fuzzy sets Syst.*, **35**(1990), 241-249.
[5] Jin-xuan Fang and Huan Huang, On the level convergence of a sequence of fuzzy numbers, *Fuzzy Sets Syst.*, **147**(2004), 417-435.
[6] H.Fast, Surla convergence statistique, *Colloq. Math.*, **1951**, 241-244.
[7] J.Fridy, On the statistical convergence, *Analysis*, **5**(1985), 301-313.
[8] L.Leindler, Über die Vallee-Pousinsche Summierbarkeit Allgemeiner Orthogonalreihen, *Acta Math Acad. Sci. Hungar.*, **16**(1965), 375-387.
[9] J.S.Kwon, On Statistical and P-Cesàro convergence of fuzzy numbers, *Korean J. Compu. Appl. math.* , **7(1)**(2000), 195-203.
[10] M.Matloka, Sequences of fuzzy numbers, *Busefal*, **28**(1986), 28-37.
[11] M.Mursaleen and M.Basarir, On some new sequence spaces of fuzzy numbers, *Indian J. Pure Appl. Math.*, **34(9)** (2003), 1351-1357.
[12] S.Nanda, On sequences of fuzzy numbers, *Fuzzy Sets Syst.* , **33**(1989), 123-126.
[13] I.Niven and H.S.Zuckerman, An Introduction to the Theory of Numbers, *fourth ed.,John Wiley and Sons, New York*, **1980**.
[14] F.Nuray, Lacunary statistical convergence of sequences of fuzzy numbers, *Fuzzy Sets Syst.*, **99**(1998), 353-356.
[15] E.Savas, On strongly $\lambda-$ summable sequences of fuzzy numbers, *Inform. Sci.*, **125**(2000), 181-186.
[16] I.J.Schoenberg, The integrability of certain functions and related summability methods, *Amer. Math. Monthly*, **66**(1959), 361-375.
[17] Congxin Wu and Guixiang Wang, Convergence of sequences of fuzzy numbers and fixed point theorems for increasing fuzzy mappings and application, *Fuzzy Sets Syst.*, **130**(2002), 383-390.
[18] L.A.Zadeh, Fuzzy sets, *Inform Control*, **8**(1965), 338-353.
[19] H.Kizmaz, On certain sequence spaces, *Canad Math. Bull.* , **24(2)**(1981), 169-176.
[20] M.Et and R.Colak, On some generalized difference sequence spaces, *Soochow J. Math.*, **21(4)**(1995), 377-386.
[21] M.Et, On some topological properties of generalized difference sequence spaces, *Int. J. Math. Math. Sci.*, **24(11)**(2000), 785-791.
[22] M.Et and F.Nuray, Δ^m- Statistical Convergence, *Indian J. Pure Appl. Math.*, **32(6)** (2001), 961-969.
[23] R.Colak, M.Et and E.Malkowsky, Some topics of sequence spaces, *Lecture Notes in Mathematics, Firat University Press, Elazig, Turkey*, **2004**.
[24] M.Isik, On statistical convergence of generalized difference sequences, *Soochow J. Math.*, **30(2)**(2004), 197-205.
[25] Y.Altin and M.Et, Generalized difference sequence spaces defined by a modulus function in a locally convex space, *Soochow J. Math.* , **31(2)**(2005), 233-243.
[26] W.Orlicz, Über Raume (L^M) *Bull. Int. Acad. Polon. Sci. A*, (1936), 93-107.
[27] J.Lindenstrauss and L.Tzafriri, On Orlicz sequence spaces, *Israel J. Math.*, **10** (1971), 379-390.
[28] S.D.Parashar and B.Choudhary, Sequence spaces defined by Orlicz functions, *Indian J. Pure Appl. Math.* , **25(4)**(1994), 419-428.
[29] M.Mursaleen,M.A.Khan and Qamaruddin, Difference sequence spaces defined by Orlicz functions, *Demonstratio Math.* , **Vol. XXXII** (1999), 145-150.
[30] C.Bektas and Y.Altin, The sequence space $\ell_M(p, q, s)$ on seminormed spaces, *Indian J. Pure Appl. Math.*, **34(4)** (2003), 529-534.
[31] B.C.Tripathy,M.Etand Y.Altin, Generalized difference sequence spaces defined by Orlicz function in a locally convex space, *J. Analysis and Applications*, **1(3)**(2003), 175-192.
[32] K.Chandrasekhara Rao and N.Subramanian, The Orlicz space of entire sequences, *Int. J. Math. Math. Sci.*, **68**(2004), 3755-3764.
[33] M.A.Krasnoselskii and Y.B.Rutickii, Convex functions and Orlicz spaces, *Gorningen, Netherlands*, **1961**.
[34] Nakano, Concave modulars, *J. Math. Soc. Japan*, **5**(1953), 29-49.
[35] W.H.Ruckle, FK spaces in which the sequence of coordinate vectors is bounded, *Canad. J. Math.*, **25**(1973), 973-978.
[36] I.J.Maddox, Sequence spaces defined by a modulus, *Math. Proc. Cambridge Philos. Soc*, **100(1)** (1986), 161-166.
[37] H.I.Brown, The summability field of a perfect $\ell - \ell$ method of summation, *J. Anal. Math.*, **20**(1967), 281-287.
[38] A.Wilansky, Summability through Functional Analysis, *North-Holland Mathematical Studies, North-Holland Publishing, Amsterdam*, **Vol.85**(1984).
[39] P.K.Kamthan and M.Gupta, Sequence spaces and series. Lecture Notes in Pure and Applied Mathematics, *Marcel Dekker Inc. New York*, **65**(1981).
[40] P.K.Kamthan, Bases in a certain class of Frechet space, *Tamkang J. Math.*, **1976**, 41-49.
[41] C.Goffman and G.Pedrick, First Course in Functional Analysis, *Prentice Hall India, New Delhi*, **1974**.