Single Image Defogging Method Using Variational Approach for Edge-Preserving Regularization

Wan-Hyun Cho, In-Seop Na, Seong-Chae Seo, Sang-Kyoon Kim, and Soon-Young Park

***Abstract*—**In this paper, we propose the variational approach to solve single image defogging problem. In the inference process of the atmospheric veil, we defined new functional for atmospheric veil that satisfy edge-preserving regularization property. By using the fundamental lemma of calculus of variations, we derive the Euler-Lagrange equation foratmospheric veil that can find the maxima of a given functional. This equation can be solved by using a gradient decent method and time parameter. Then, we can have obtained the estimated atmospheric veil, and then have conducted the image restoration by using inferred atmospheric veil. Finally we have improved the contrast of restoration image by various histogram equalization methods. The experimental results show that the proposed method achieves rather good defogging results.

***Keywords*—**Image defogging, Image restoration, Atmospheric veil, Transmission, Variational approach, Euler-Lagrange equation, Image enhancement.

I. Introduction

IMAGES of outdoor scene often contain haze, fog, or other types of atmospheric degradation caused by particles in the atmospheric medium absorbing and scattering light as it travels from the scene points to the observer. In the robust haze removal, two main characteristics that we notice in a haze image are the decrease of the visibility distance on the image, and the scene blurring due to the loss of high frequency components.

A number of methods have been proposed for haze removal from images. But most of methods used to multiple images or additional information. Recently, single image haze removal has made significant progress. The success of these methods lies in using a stronger assumption and a prior.

Tan observes that the haze-free image must have higher contrast than the haze image [1]. By combining the maximization of local contrast with the assumption that neighboring pixels suffered from the same degradation, an optional method based on MRF is used to remove haze. This method tends to produce over-enhanced images in practice.

Fattal considers the shading and transmission signals to be unrelated and uses Independent Component Analysis to estimate the transmission[2]. And then they infer the color of the whole image by MRF. The method works quite well for haze, but has difficulty with scenes involving fog, as the magnitude of the surface reflectance is much smaller than that of the air light when the fog is suitable thick.

He et al. employs a dark channel prior which assumes every local patch (15×15)in the haze-free image have at least one color component near zero [3]. This assumption is sometime violated when there is no black body in some local patches. Instead of using an MRF, a soft matting algorithm is used to refine the transmission values, which is much computational expensive.

Tarel proposed a bilateral filter to replace the optimization method, which improves the efficiency of algorithm and can be used in real-time system [4]. But the dehazing result is not so good when there are discontinuous in the depth of scene. The haze among gaps cannot be removed.

Tripathi proposed the algorithm that uses anisotropic diffusion theory to recover scenecontrast, and then they conducted a post-processing for the contrast enhancement like histogram equalization, and histogram specification [5].

In this paper, we proposed the variational approach to solve single image defogging problem.The atmospheric veil or airlight map can be directly estimatedby maximizing the energy functional that both satisfy edge-preserving regularization propertyand can adequately remove the fog. First, using the fundamental lemma of calculus of variations [6]-[10], we derive the Euler-Lagrange equation for atmospheric veil that can find the maxima of a given functional. This equation can be solved by using a gradient decent method and time parameter [7]. Then, we can have obtained the estimated atmospheric veil, and then have conducted the image restoration by using inferred atmospheric veil. Finally we have improved the contrast of restoration image by various histogram equalization methods. Block diagram of proposed fog removal algorithm is shown in Fig. 1.

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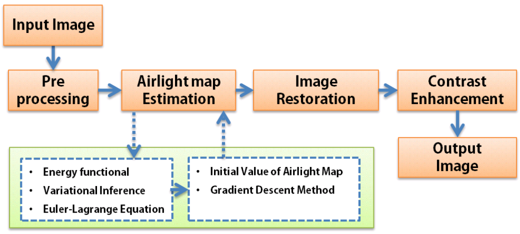

Fig.1 Block diagram of the proposed algorithm

II. Visibility Restoration Algorithm

According to the Koschmieder law, model of fog effect can be represented as

$$I(x,y) = I_0(x,y)e^{-\kappa d(x,y)} + I_\infty\left(1 - e^{-\kappa d(x,y)}\right) \quad (1)$$

where $I_0(x,y)$ is image intensity in absence of fog, κ is extinction coefficient and $d(x,y)$ is the distance of the scene point from the camera or viewer, $I_\infty(x,y)$ is global atmospheric constant or sky intensity and $I(x,y)$ is observed image intensity at pixel (x,y).

In (1), the first term of the right hand side is the direct attenuation and the second term is the airlight or atmospheric veil. Attenuation is an exponential decreasing function of the scene point distance $d(x,y)$. The contrast of the objects is reduced and thus its visibility in scene. The airlight adds whiteness in the scene. The airlight is an increasing function of the scene point distance $d(x,y)$. If airlight is represented as $A(x,y)$, then the formula (1) can be written as

$$I(x,y) = I_0(x,y)\left(1 - \frac{A(x,y)}{I_\infty}\right) + A(x,y) \quad (2)$$

For simulation foggy image $I(x,y)$ is normalized. Fog being pure white, this implies that sky intensity I_∞ can be set to 1. To restore the image $I_0(x,y)$, information of the airlight $A(x,y)$ is required. From (2), we assume $I_\infty = 1$, thenwe have that

$$I(x,y) = I_0(x,y)(1 - A(x,y)) + A(x,y) \quad (3)$$

This model is directly extended to a color image by applying the same model on each RGB component. However, airlight $A(x,y)$ map remains same from each color component.

A. Initial Estimation of Airlight Map

First, we need to appropriate initial values for the airlight map to use the calculus of variational approach. Natural outdoor images are usually full of shadows and colourful objects (viz. green grass, trees; red or yellow plants and blue water surface). Thus dark channel assumption of these images is valid. For fog-free image except for sky region intensity of dark channel is low and tends to zero. Hence, in order to satisfy the properties mentioned above for airlight map, we can assume the following initial values for airlight map. If input image is color image, then the initial values of airlight map $A^0(x,y)$ is assume as

$$A^0(x,y) = \beta \min_{c\in(r,g,b)} I^c(x,y),$$

where β is a constant and $0 < \beta < 1$. On the other hand, if input image is gray-scale image, then initial values can be assumed as

$$A^0(x,y) = \beta\, I(x,y).$$

B. Refinement of Airlight Map

The airlight $A(x,y)$ should be the color value of the scene point with infinite distance away from the camera. The airlight $A(x,y)$ can be estimated in the most haze-opaque region. Due to its physical properties, the atmospheric veil is subject to two constraints when the observed image $I(x,y)$ is known: it is positive $0 < A(x,y)$ and being pure white for each pixel, it cannot be higher than minimum of the components of $I(x,y)$. Thus, we compute an image $W(x,y) = \min(I(x,y))$ defined as the image of the minimal components of $I(x,y)$ for each pixel. $W(x,y)$ is the image of the whiteness within the observed image $I(x,y)$. The second constraint can thus be written as $A(x,y) \le W(x,y)$.

In this time, to get the defogging image, we need to airlight map which satisfy the conditions mentioned above. The airlight map $A(x,y)$ may be derived by a regularized solution that maximize the contrast of the resulting image assuming that the depth-map must be smooth except along edges with large depth jumps. The problem can thus be reformulated as maximizing $A(x,y)$ assuming that $A(x,y)$ is smooth on the most of time. Let us firstly the Heaviside function H, and the one-dimensional Dirac measure δ_0 define respectively by

$$H(z) = \begin{cases} 1 & \text{if } z \ge 0 \\ 0 & \text{otherwise} \end{cases}, \quad \delta_0(z) = \frac{d}{dz}H(z).$$

Then, we can express this problem as the energy functional in the following way:

$$E(A(x,y)) = \int_\Omega H\big(W(x,y) - A(x,y)\big)dxdy$$
$$+\lambda^2 \int_\Omega \varphi(\| \nabla A(x,y) \|)\, dxdy \quad (4)$$

where Ω is an open bounded set in $\Re^2$, $\| \nabla A(x,y) \|$ is the modulus of the gradient of $A(x,y)$, and λ is a regularization parameter which balances the influence between two terms of (2). The choice of the function φ as an edge-preserving regularization function is assumed to be an even function of class $\mathcal{C}^2(\Re$. Common examples of regularizations function φ with some of their properties are given in the following Table I.

TABLE I
SUMMARY OF SOME REGULARIZATION FUNCTIONφ

Name of function	$\varphi(t)$	Convexity	$\varphi'(t)$
Tikhonov	t^2	yes	$2t$
Total variation	t	yes	1
Perona&Malik	$-\exp(-t^2)+1$	no	$2t\exp(-t^2)$
German & McClure	$\frac{t^2}{1+t^2}$	no	$\frac{2t}{(1+t^2)^2}$
Hebert & Leahy.	$\log(1+t^2)$	no	$\frac{2t}{(1+t^2)}$

The first function is quadratic and corresponds to the Tikhonov standard regulation which is obviously not edge-preserving. The second function leads to the minimization of the total variation of $A(x,y)$, which is common regularizing criterion. The third function called anisotropic diffusion is used most frequently in the noise removal.The second and third functions define edge-preserving regularization.

Here,we should try to find an airlight A(x,y) that maximizes the energy function E(A(x,y)).This will be given as a solution of Euler-Lagrange equation for the unknown airlightA(x,y),but before this derivation, we consider slightlyregularized versions of the function H and δ_0, denoted here by H_ε and $\delta_\epsilon = H'_\epsilon$. Here, we may denote the associated regularized functional $E_\epsilon(A(x,y))$ defined by

$$E_\varepsilon(A(x,y)) = \int_\Omega H_\epsilon\big(W(x,y)-A(x,y)\big)dxdy + \lambda^2\int_\Omega \varphi(\| \nabla A(x,y) \|\, dxdy \tag{5}$$

Therefore, using the calculus of variation, the airlight $A(x,y)$ maximizing the regularized energy functional $E_\epsilon(A(x,y))$ can be derived by the following Euler-Lagrange equation [9], [10]:

$$\begin{cases} -\delta_\epsilon\big(W(x,y)-A(x,y)\big) \\ -\lambda^2 \mathrm{div}\left\{\frac{\varphi'(\|\nabla A(x,y)\|)}{\|\nabla A(x,y)\|}\nabla A(x,y)\right\} = 0, \\ \qquad (x,y)\in\Omega \\ \frac{\partial A(x,y)}{\partial \vec{n}} = 0, \qquad (x,y)\in\partial\Omega. \end{cases} \tag{6}$$

where$\vec{n}$ represents a vector normal to the boundary $\partial\Omega$ of Ω, and "div" stands for the divergence operator defined by

$$\mathrm{div}(u(x,y) = \frac{\partial u(x,y)}{\partial x} + \frac{\partial u(x,y)}{\partial y}$$

Furthermore, the numerical solution of the Euler-Lagrange equation (6) can be obtained by using a gradient-decent method, or equivalently, by using a dynamic scheme wherein t is an artificial time parameter;

$$\begin{cases} \frac{\partial A(t,x,y)}{\partial t} = \delta_\epsilon\big(W(x,y)-A(x,y)\big) \\ +\lambda^2 \mathrm{div}\left\{\frac{\varphi'(\|\nabla A(x,y)\|)}{\|\nabla A(x,y)\|}\nabla A(x,y)\right\} = 0, \\ \qquad (t,x,y)\in(0,\infty)\times\Omega \\ \frac{\partial A(x,y)}{\partial \vec{n}} = 0, \qquad (x,y)\in\partial\Omega \\ A(0,x,y) = A_0(x,y),\ (x,y)\in\partial\Omega. \end{cases} \tag{7}$$

C. NumericalApproximation by Dynamic Scheme

First, we introduce and use the following regularization of H in our experiments [7]:

$$H_\epsilon(z) = \frac{1}{2}\left(1+\frac{2}{\pi}\arctan\left(\frac{z}{\epsilon}\right)\right)$$

and its derivative is given as follows:

$$\delta_\varepsilon(z) = \frac{\epsilon}{\pi(\epsilon^2+z^2)}$$

As $\varepsilon \to \infty$, they converge respectively to H and δ_0.

To discretize the equation in $A(t,x,y)$, we use a finite difference implicit scheme [7]-[8]. We recall first the usual notations: let h be the space step, Δt be the time step, and $(x_i,y_j) = (ih, jh)$ be the grid points, for $1 \le i,j \le M$. Let $A^n_{i,j} = A(n\Delta t, x_i, y_j)$ be an approximation of $A(t,x,y)$, with $n \ge 0, A^0 = A_0$. The finite differences are

$$\Delta^x_- A_{i,j} = A_{i,j} - A_{i-1,j},\ \Delta^x_+ A_{i,j} = A_{i+1,j} - A_{i,j},$$
$$\Delta^y_- A_{i,j} = A_{i,j} - A_{i,j-1},\ \Delta^y_+ A_{i,j} = A_{i,j+1} - A_{i,j}.$$

The algorithm is as follows: knowing A^n, we first compute $\delta^*_\epsilon\big(A^n(x,y)\big) = -\delta_\epsilon\big(W(x,y)-A^n(x,y)\big)$. Then, we compute $A^{n+1}$by the following discretization and linearization of (7) in A(x,y);

$$A^{n+1}_{i,j} = A^n_{i,j} + \Delta t\cdot\eta,$$

$$\eta = \delta^*_\epsilon\left(A^n_{i,j}(x,y)\right) + \Bigg(\frac{\varphi'\Big(\sqrt{\frac{\left(\Delta^x_+ A^n_{i,j}\right)^2}{h^2} + \frac{\left(\Delta^y_+ A^n_{i,j}\right)^2}{h^2}}}{h^2}\Bigg)$$
$$\times\left\{\Delta^x_-\left(\frac{\Delta^x_+ A^n_{i,j}}{\sqrt{\left(\Delta^x_+ A^n_{i,j}\right)^2/h^2 + \left(A^n_{i,j+1}-A^n_{i,j-1}\right)^2/(2h)^2}}\right) + \Delta^y_-\left(\left(\frac{\Delta^y_+ A^n_{i,j}}{\sqrt{\left(A^n_{i+1,j}-A^n_{i-1,j}\right)^2/(2h)^2 + \left(\Delta^y_+ A^n_{i,j}\right)^2/(h)^2}}\right)\right)\right\} \tag{8}$$

This linear system is solved by an iterative method. Finally, the principle steps of our algorithm are given as follows:

- Initialize A^0 by A_0, n=0.
- Compute $\delta_\varepsilon(A^n(x,y))$.
- Solve the PDE in $A(x,y)$ form (7) to obtain A^{n+1}.
- Check whether the solution is stationary. If not, $n = n + 1$ and repeat.

D. Image Restoration

Once airlight map $A(x, y)$ are estimated, then each color component of dehazing image $I_0(x, y)$ can be restored as

$$I_0(x,y) = \frac{I(x,y) - A(x,y)}{(1 - \frac{A(x,y)}{I_\infty})} \tag{9}$$

But, when we restored dehazing image using (9), we have to estimate the atmospheric light. Here, we have taken the atmospheric light I_∞ with the top 1% brightness pixel values within the upper fifth lines of the observed image.

E.PostProcessing

It is obvious that restored image has low contrast. The reason is that scene radiance is usually not as bright as the atmospheric light, thus image after haze removal looks dim. Hence, there is a requirement of contrast enhancement as a post-processing. There are many choices for the contrast enhancement like histogram equalization, histogram specification and histogram stretching. It is found that histogram equalization produces a saturated output image. Owing to large variations in image contents, a standard reference image cannot be fixed for histogram specification. Thus, to increase contrast, our algorithm uses histogram stretching.

III. Experimental Results

In order toverify the validity of our method, several comparison experiments are made. The experiments are conducted on popular images that He, Fattal, Tarel et al. have tested. Fig. 2 shows the comparison results for defogging approaches. We can conclude that our approach can restore afogged image with better color. And also we can find that our approach performs at the similar level to that of He et al., but the much faster. From the experimental results, we note that our method is able to significantly improve the quality of the defogged image.

(a)

(b)

(c)

(d)

(e)

Fig. 2 Steps in conducting our approach: (a) Input image (b) Estimated airlight map (c) Refined transmission map (d) Our recovery image (e) He et al.'s result

IV.Conclusions

Inthis paper, we presented an efficient fog removal algorithm. Proposed algorithm uses the variational approach with edge-preserving regularization for estimation of airlight map. And also this algorithm can be applied for color as well as grey image. Moreover, in order to improve the contrast of restored image, we have used various histogram equalization methods.Experimental results show that proposed algorithm performs well in comparison with other existing algorithms. Even in case of heavy fog, our algorithm performs well.

In the future, we would like to implements hardware to further improve the performance and extend it to handle videos.

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