Evaluating Spectral Relationships between Signals by Removing the Contribution of a Common, Periodic Source – A Partial Coherence-based Approach

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Abstract—Partial coherence between two signals removing the contribution of a periodic, deterministic signal is proposed for evaluating the interrelationship in multivariate systems. The estimator expression was derived and shown to be independent of such periodic signal. Simulations were used for obtaining its critical value, which were found to be the same as those for Gaussian signals, as well as for evaluating the technique. An Illustration with eletroencephalografic (EEG) signals during photic stimulation is also provided. The application of the proposed technique in both simulation and real EEG data indicate that it seems to be very specific in removing the contribution of periodic sources. The estimate independence of the periodic signal may widen partial coherence application to signal analysis, since it could be used together with simple coherence to test for contamination in signals by a common, periodic noise source.

Keywords—Partial coherence, periodic input, spectral analysis, statistical signal processing.

I. INTRODUCTION

THE coherence function is commonly used for evaluating the linear dependence between two random signals. It is a frequency domain, measuring technique, which is analogous to the correlation coefficient. The squared-modulus of the coherence function is often called just coherence [1].

Coherence is not affected by a time delay between the signals investigated. This feature, together with its frequency selectivity, turn coherence very useful for evaluating the relationship between signals arisen as output of many linear systems of interest. That is why it has been widely applied to data from such diverse areas as biomedical, radar and communication engineering, as well as oceanography, atmospheric science, and acoustics [2].

The extension of coherence definition to the multivariate case leads to multiple and partial coherences. While the first is

the fraction of the power accounted for in the output via linear relationships with all the input signals, the latter is the coherence between two signals after removing the linear contribution from a set of other signals. Multiple and partial coherences provide thus a more complete evaluation of the interrelationship in multivariate systems.

In some engineering applications, such as in transmission lines, engines and electric machines, the input signal may be periodic. Another example occurs in evoked responses to sensory stimulation, which are used in the clinical practice as an exam in which a fixed, periodic stimulus is applied to the human sensory system, eliciting a response in the otherwise random electrical brain activity of the electroencephalogram (EEG). For such kind of systems one may be interested in evaluating the relationship between two output locations, which is not accounted for by the input.

For the above cases, a technique that could allow measuring the degree of dependence between two signals removing the contribution of a common, periodic source would be very useful. This is exploited in the present work, where the expression of partial coherence between two random signals removing the contribution from a periodic one is obtained. The proposed technique is evaluated in simulated signals, which were also used for obtaining the critical value. An illustration example with EEG signals during stroboscopic (flash) stimulation is also provided.

II. THEORETICAL BACKGROUND

Consider the N-input-one-output linear system of Fig. 1, where $x_j[k]$ ($j=1,2,\ldots,N$) and $y[k]$ are discrete time signals. The augmented spectral matrix [3] for such system is defined as:

$$
\mathbf{A}_{\text{spec}} = \begin{bmatrix}
\hat{S}_{yy}(f) & \hat{S}_{yx1}(f) & \hat{S}_{yx2}(f) & \cdots & \hat{S}_{yxN}(f) \\
\hat{S}_{x1y}(f) & \hat{S}_{x1x1}(f) & \hat{S}_{x1x2}(f) & \cdots & \hat{S}_{x1xN}(f) \\
\hat{S}_{x2y}(f) & \hat{S}_{x2x1}(f) & \hat{S}_{x2x2}(f) & \cdots & \hat{S}_{x2xN}(f) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\hat{S}_{xNy}(f) & \hat{S}_{xNx1}(f) & \hat{S}_{xNx2}(f) & \cdots & \hat{S}_{xNxN}(f)
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\boldsymbol{\Sigma}_{yy} & \boldsymbol{\Sigma}_{y1} \\
\boldsymbol{\Sigma}_{1y} & \boldsymbol{\Sigma}_{11}
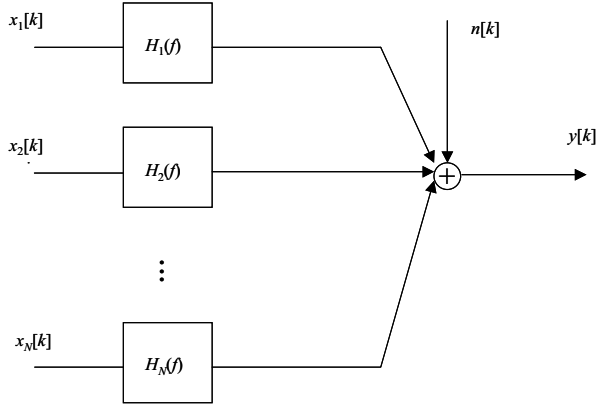\end{bmatrix}
$$

(1)

Fig. 1 *N*-input-one-output multivariate linear system. Input $x_j[k]$ is filtered through $H_j(f)$ to result in response $v_j[k]$, which is added to the other responses and the background, Gaussian noise $n[k]$ to result in the output measured signal $y[k]$

where $\hat{S}_{pq}(f)$ is the cross-spectrum estimate between signals $p[k]$ and $q[k]$ (or the auto-spectrum estimate if $p=q$). The conditional spectral matrix (a complex analog to the multivariate regression analysis equation provided in [4]) may be expressed using the sub-matrices from $\mathbf{A}_{\text{spec}}$ as:

$$\mathbf{C}_{\text{spec}} = \mathbf{\Sigma}_{yy} - \mathbf{\Sigma}_{y1}\mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{1y} \qquad (2)$$

According to [3] the partial coherence between the output $y[k]$ and the input $x_1[k]$ removing the contribution of the other N-1 inputs can be estimated with the following expression involving matrix $\mathbf{C}_{\text{spec}}$:

$$\hat{\gamma}^2_{x1y\bullet x2,x3,\cdots,xN}(f) = \frac{\left|\mathbf{C}_{\text{spec}}(1,2)\right|^2}{\mathbf{C}_{\text{spec}}(1,1)\cdot\mathbf{C}_{\text{spec}}(2,2)} \qquad (3)$$

where $\mathbf{C}_{\text{spec}}(i,j)$ is the ith row, jth column element of $\mathbf{C}_{\text{spec}}$.

Now consider the case of the one-input-two-output linear system of Fig. 2, where $x[k]$ is a periodic, deterministic system.
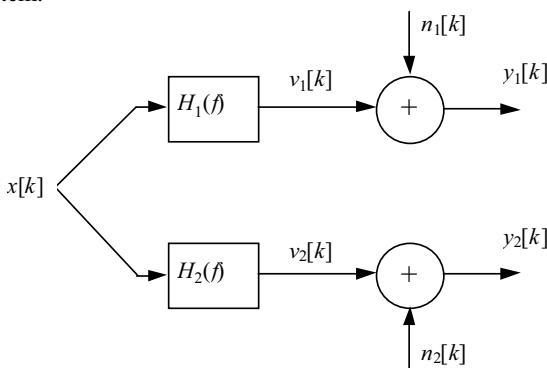

Fig. 2 One-input-two-output linear system model used for deriving the partial coherence estimate between $y_1[k]$ and $y_2[k]$, removing the contribution from $x[k]$ when this latter is a periodic, deterministic signal. The background activities $n_1[k]$ and $n_2[k]$ are zero-mean, Gaussian signals uncorrelated with each other and with input $x[k]$

If the windowing is applied as to keep the same number of

cycles in each data segment, then the Fourier transform of $x[k]$ will have the same value in all windows, say X. For this particular case, if when aims at obtaining the partial coherence between the outputs removing the effect of the input signal, the augmented spectral matrix becomes:

$$\mathbf{A}_{\text{spec}}\Big|_{x[k]\,\text{periodic}} = \begin{bmatrix} \sum_{i=1}^{M}\left|Y_{1i}(f)\right|^2 & \sum_{i=1}^{M}Y_{1i}^{*}(f)Y_{2i}(f) & X\sum_{i=1}^{M}Y_{1i}^{*}(f) \\ \sum_{i=1}^{M}Y_{2i}^{*}(f)Y_{1i}(f) & \sum_{i=1}^{M}\left|Y_{1i}(f)\right|^2 & X\sum_{i=1}^{M}Y_{2i}^{*}(f) \\ X^{*}\sum_{i=1}^{M}Y_{1i}(f) & X^{*}\sum_{i=1}^{M}Y_{2i}(f) & M\left|X\right|^2 \end{bmatrix} = $$
$$= \begin{bmatrix} \mathbf{\Sigma}_{yy} & \mathbf{\Sigma}_{y1} \\ \mathbf{\Sigma}_{1y} & \mathbf{\Sigma}_{11} \end{bmatrix} \qquad (4)$$

where $Y_{ji}(f)$ (j=1,2) is the ith-window Fourier Transform of $y_j[k]$, M is the number of segments used in the estimation and '*' superscript denotes complex conjugate. Here, the averaging factor in the spectral estimates – usually $1/(MT)$, with T denoting the window duration – is omitted, since it would cancel in coherence estimation. Furthermore, the conditional spectral matrix will be independent of signal $x[k]$, since X clearly cancels when $\mathbf{\Sigma}_{y1}\mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{1y}$ is obtained. Thus, such matrix becomes:

$$\mathbf{C}_{\text{spec}}\Big|_{x[k]\,\text{periodic}} = $$
$$\begin{bmatrix} \sum_{i=1}^{M}\left|Y_{1i}(f)\right|^2 - \frac{1}{M}\left|\sum_{i=1}^{M}Y_{1i}(f)\right|^2 & \sum_{i=1}^{M}Y_{1i}^{*}(f)Y_{2i}(f) - \frac{1}{M}\sum_{i=1}^{M}Y_{1i}^{*}(f)\sum_{i=1}^{M}Y_{2i}(f) \\ \sum_{i=1}^{M}Y_{2i}^{*}(f)Y_{1i}(f) - \frac{1}{M}\sum_{i=1}^{M}Y_{2i}^{*}(f)\sum_{i=1}^{M}Y_{1i}(f) & \sum_{i=1}^{M}\left|Y_{2i}(f)\right|^2 - \frac{1}{M}\left|\sum_{i=1}^{M}Y(f)\right|^2 \end{bmatrix} $$
$$(5)$$

Finally, according to (3), the partial coherence estimate between $y_1[k]$ and $y_2[k]$ removing the contribution of the input, periodic signal $x[k]$ may be expressed as:

$$\hat{\gamma}^2_{y1y2\bullet x}(f)\Big|_{x[k]\,\text{periodic}} = \hat{\kappa}^2_{y1y2\bullet}(f) = $$
$$\frac{\left|\sum_{i=1}^{M}Y_{1i}^{*}(f)Y_{2i}(f) - \frac{1}{M}\sum_{i=1}^{M}Y_{1i}^{*}(f)\sum_{i=1}^{M}Y_{2i}(f)\right|^2}{\left[\sum_{i=1}^{M}\left|Y_{1i}(f)\right|^2 - \frac{1}{M}\left|\sum_{i=1}^{M}Y_{1i}(f)\right|^2\right]\cdot\left[\sum_{i=1}^{M}\left|Y_{2i}(f)\right|^2 - \frac{1}{M}\left|\sum_{i=1}^{M}Y_{2i}(f)\right|^2\right]} \qquad (6)$$

The numerator of (6) may be also obtained as the product of the nonprincipal diagonal elements of matrix $\mathbf{C}_{\text{spec}}$ in (5). Such partial coherence is here denoted as $\hat{\kappa}^2_{y1y2\bullet}(f)$ in order to distinguish it from the generic $\hat{\gamma}^2_{y1y2\bullet x}(f)$. It is interesting to note that this multivariate coherence estimate is independent of the periodic signal (x-index is intentionally removed from the estimate symbol to emphasize this aspect). The coherence estimate between a periodic signal and a random one is known to be independent of the first [5]. This result is hence a two-variate generalization of (simple) coherence estimate independence of the input signal when such is periodic.

III. METHODS

Based on the Model of Fig. 2, a Monte Carlo simulation was carried out in order to determine critical values for $\hat{\kappa}^2_{y1y2\bullet}(f)$ under the null hypothesis of no periodic component in both $y_1[k]$ and $y_2[k]$. This leads to uncorrelated Gaussian signals. $\hat{\kappa}^2_{y1y2\bullet}(f)$ was then calculated according to (6) from pairs of independent, Gaussian distributed signals divided into M segments, which was varied from 4 to 100. Critical values were then obtained as the 95th percentiles of the resultant sample distribution.

The technique was next applied to simulated signals. Thus, $x[k]$ (Fig. 2) was generated as a 8 Hz- unit impulse train and both $H_1(f)$ and $H_2(f)$ were set constant (=1). The background noise terms $n_1[k]$ and $n_2[k]$ were obtained as the sum of two parcels – an identical activity (band-filtered white noise within 9-13 Hz) and another uncorrelated activity (unit-variance Gaussian signals). Partial coherence estimate $\hat{\kappa}^2_{y1y2\bullet}(f)$ was also applied in EEG data acquired during rhythmic photic stimulation (flash) at 6 Hz. In this case, $y_1[k]$ was the signal from electrode O_1 and $y_2[k]$, that from electrode O_2. Such electrodes are symmetrically located above the visual cortex, where the evoked responses are expected to be stronger. Simple coherence was estimated for both simulation and EEG applications using the MATAB *cohere.m* function. In all cases, the number of segments used (M) was equal to 12.

IV. RESULTS

Fig. 3 shows the simulated critical values (L=1000 iterations) for $\hat{\kappa}^2_{y1y2\bullet}(f)$, together with the theoretical ones for partial coherence in Gaussian signals (obtained using the expression provided in [5] and carrying out the correction [3] of reducing the number of degrees of freedom by the number of signals removed, which is equal to one in this case). It can be noted random fluctuations of the first around the latter. However, such oscillatory behaviour decreases as L is increased, leading both critical values to be virtually identical (result not shown). This result would be a two-variate generalization of the well-known invariance of coherence estimate statistics (and hence of the critical values) with respect to second signal, provided the first is Gaussian and coherence is zero [6]. According to it, critical values are found as:

$$\hat{\kappa}^2_{y1y2\bullet} crit = \text{beta}_{crit}(1, M-2) \qquad (7)$$

where $\text{beta}_{crit}(1, M-2)$ is the critical value of the standard form of the beta distribution [7] with parameters $p=1$ and $q=M-2$ for a given significance level.

Fig. 4 displays the critical values for both simple and partial coherences. It can be noted that the latter is always greater than the first (which is an expected result due to the reduce in the degrees of freedom in the estimation), but the difference decreases as the number of data segments used increases.
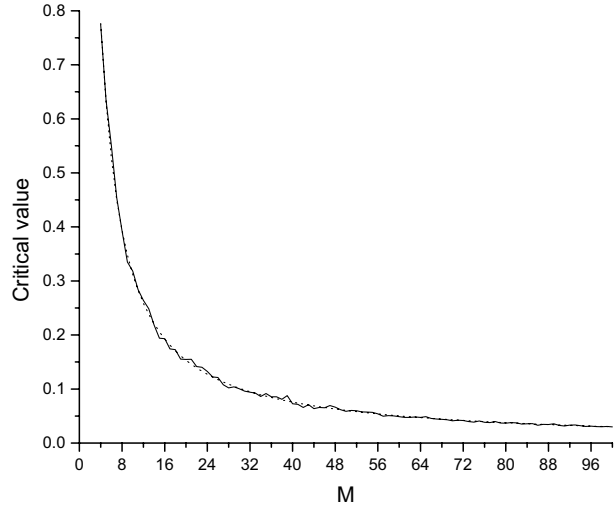

Fig. 3 Simulated (L=1000 iterations) critical values of $\hat{\kappa}^2_{y1y2\bullet}(f)$ for different number of data segments (M) used in the estimation (continuous line). Theoretical partial coherence critical values for Gaussian signal are also shown (dotted line) to allow comparison
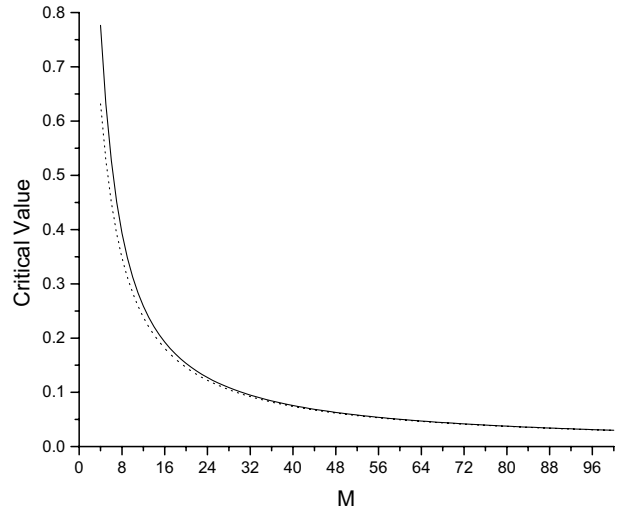

Fig. 4 Critical values of $\hat{\kappa}^2_{y1y2\bullet}(f)$ (continuous line) and of simple coherence (dotted line) for different number of data segments (M) used in the estimation

The results with simulated signals are displayed in Fig. 5. It is interesting to note that major differences between $\hat{\kappa}^2_{y1y2\bullet}(f)$ and simple coherence occur in the frequency of the input signal (8 Hz) and its harmonics. $\hat{\kappa}^2_{y1y2\bullet}(f)$ clearly removes the peaks due to the periodic input signal. It however preserves the wide peak that occurs within 9-13 Hz, which is due to the nonsynchronized, background activity in both signals. Such results indicate the specificity of the proposed technique, since it seems to remove only parcels due to a common source that is periodic.
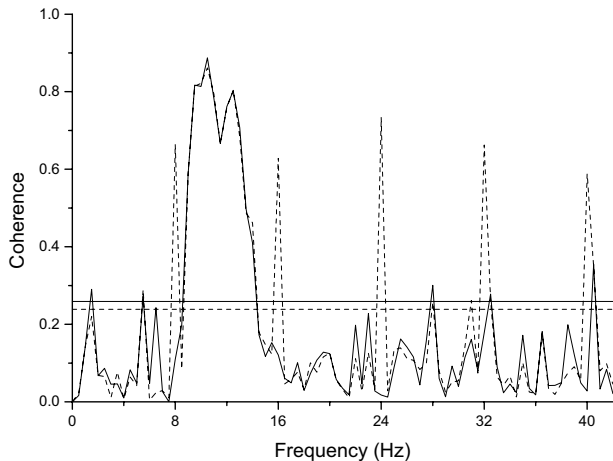
Fig. 5 Partial coherence $\hat{\kappa}^2_{y1y2\bullet}(f)$ (continuous line) and simple coherence estimates (dashed line) between the simulated signals $y_1[k]$ and $y_2[k]$. The respective critical values are indicated in the horizontal lines and have been taken from Fig. 4

The results with real EEG data (Fig. 6) agree well with those from simulation. It is interesting to note that the attenuation in coherence peak at 12 Hz with $\hat{\kappa}^2_{y1y2\bullet}(f)$ is not so pronounced as in the remaining harmonics of the stimulation frequency (6 Hz). This may be due to the background alpha rhythm, which seems to have not been entrained by the stimulation.
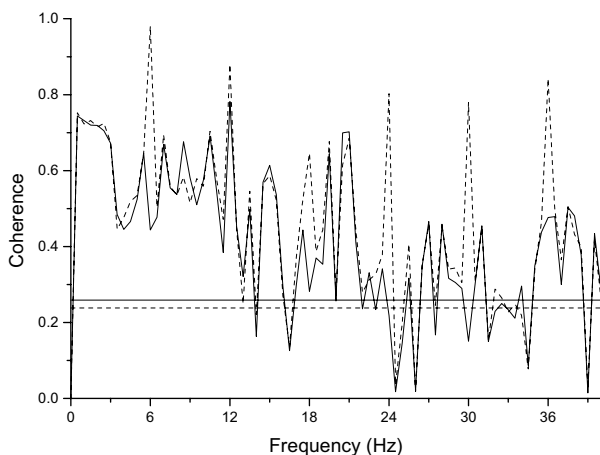

Fig. 6 Partial coherence $\hat{\kappa}^2_{y1y2\bullet}(f)$ (continuous line) and simple coherence estimates (dashed line) between the EEG signals $y_1[k]=O_1$ and $y_2[k]=O_2$ acquired during photic stimulation at 6 Hz. The respective critical values are indicated in the horizontal lines and have been taken from Fig. 4

V. CONCLUSION

In the present work, the expression of partial coherence estimate between two signals removing the contribution from a periodic one is derived. The application of the proposed technique in both simulation and real EEG data indicate that

$\hat{\kappa}^2_{y1y2\bullet}(f)$ seems to be very specific in removing the contribution of periodic sources

As can be noticed in expression (6), $\hat{\kappa}^2_{y1y2\bullet}(f)$ is independent of the periodic signal. This result is relevant, especially for transient-periodic, input signals, whose amplitude may decrease very fast. Thus, $\hat{\kappa}^2_{y1y2\bullet}(f)$ could be an interesting option since it would reduce random error introduction during the acquisition of the input signal. In addition, since its critical values were found to be the same as those for partial coherence in Gaussian signals, they can be readily obtained according to (7).

Furthermore, such input signal may be also unknown, and hence using $\hat{\kappa}^2_{y1y2\bullet}(f)$ may widen the application of partial coherence to signal analysis, since it could be used together with simple coherence to test for contamination in signals by a common, periodic noise source.

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