International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:6, No:10, 2012

Optimization of Parametric Studies Using Strategies of Sampling Techniques

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***Abstract*—**To improve the efficiency of parametric studies or tests planning the method is proposed, that takes into account all input parameters, but only a few simulation runs are performed to assess the relative importance of each input parameter. For K input parameters with N input values the total number of possible combinations of input values equals NK. To limit the number of runs, only some (totally N) of possible combinations are taken into account. The sampling procedure Updated Latin Hypercube Sampling is used to choose the optimal combinations. To measure the relative importance of each input parameter, the Spearman rank correlation coefficient is proposed. The sensitivity and the influence of all parameters are analyzed within one procedure and the key parameters with the largest influence are immediately identified.

***Keywords*—**Concrete, pavement, simulation, reliability, Latin Hypercube Sampling, parametric studies.

I. INTRODUCTION

AS modeling efforts expand to a broader spectrum of problems, a judicious selection of procedures is required. Among others, the parametric studies and test planning should be optimized. Generally, all input parameters that may be important for the process being modeled or tested should be taken into account. Frequently, the numbering of parameters is very large. The combination of many parameters results in a time consuming process because a lot of calculations or tests must be performed.

To improve the efficiency of parametric studies and test planning it is proposed to take into account all parameters, but to perform only a few runs to assess the relative importance of each parameter first. The appropriate method must be used to choose the desired combinations of input values for individual runs. Among others, sampling procedure Updated Latin Hypercube Sampling [1] seems to be the most appropriate one.

The efficiency of the proposed method is demonstrated by the illustrative parametric study of the deflections of concrete pavement.

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II. PROPOSED METHOD

All input parameters that may be important for the process being modeled or tested should be taken into account. All possible input values of all parameters must be combined with each other and the simulation runs or tests with the combinations of input values must be performed.

When there are several parameters, some of them are usually more influential on the output than others. Up until now, in order to simplify the parametric studies or tests, only some parameters were taken into account, believed they are the most important ones and all the others have no or minor influence on the output. However, it is usually not possible to determine, a priori, the most important parameters having dominant influence.

Therefore, it may be sensible to take into account all parameters, to perform only a few runs, and to assess the relative importance of each parameter first. Then it doesn't matter which parameters are the most important, because they are all taken into account. After this (if needed), all necessary calculations or tests can be refined taking into account only the parameters with large influence.

For K parameters X_k, k = 1, 2…K with N input values S_{kn}, n = 1, 2…N, the total number of possible combinations of input values equals N^K. To limit the number of calculations or tests, only some of possible combinations (totally N) will be taken into account. In order to choose the optimal combinations, every value of each parameter must have the same possibility of appearing in combination coupled with each value of each other parameter. Therefore, random combinations of the input values of parameters are required. If there are only two parameters this method of sampling is known in sample surveys as a "Latin Square". Because we are using more than two parameters, a different, more general method should be used.

III. METHOD OF GENERATING OPTIMAL COMBINATIONS

The method used for generating optimal combinations of input values used in N simulation runs or tests is based on the sampling technique Updated Latin Hypercube Sampling used in reliability analyses. In the case of parametric studies or tests planning, the method is not used to generate random variables but for planning the optimal strategy resulting in a limited number of necessary simulation runs or tests. The proposed method uses the same theoretical background as Latin Hypercube Sampling [2].

Suppose that the range of each parameter X_k is partitioned into N disjunct intervals of the equal length. Each interval is represented by the input value S_{kn} taken at the centroid of the n-th interval. The input value S_{kn} is defined as

$$S_{kn} = MIN_k + (m_{kn} - 0{,}5) \, . \, (MAX_k - MIN_k)/N \tag{1}$$

where m_{kn} is the rank number of the interval, and MAX_k and MIN_k are the upper and the lower range limits of parameter X_k. The input value S_{kn} is used just once during the parametric study or test and so there are N input values S_{kn} for each parameter X_k that will be used in N combinations in N simulation runs or tests.

The selection of input values S_{kn} to be sampled for a particular combination must be randomized. N values of each parameter are associated with a sequence of integers (rank numbers of intervals) representing random permutations of integers 1, 2,…N. They are ordered in the table of random permutations of rank numbers that has N rows and K columns. The rank numbers of intervals used in the n-th combination are represented by the n-th row in the table.

Tables used in Latin Hypercube Sampling are generated randomly. The Updated Latin Hypercube Sampling uses the improved strategy of generating combinations based on specially modified tables. In this case the modified tables consist of random per-mutations of rank numbers that are mutually statistically independent. The method for generating modified tables is based on method used for inducing correlation among input variables [3].

Let $\boldsymbol{R}$ be an N x K matrix whose columns represent K permutations of integers 1, 2,...N. That is, matrix $\boldsymbol{R}$ is identical to the table of random permutations of rank numbers. Statistical correlation among columns of this matrix is described by the rank correlation matrix $\boldsymbol{T}$, where elements T_{ij}, i, j = 1, 2…K, are the Spearman rank correlation coefficients among columns i and j of matrix $\boldsymbol{R}$. The Spearman coefficient can be defined as

$$r_s = 1 - \frac{6\sum_i d_i^2}{N(N-1)(N+1)} \tag{2}$$

where d_i is the difference between the ranks of two samples, and N is the sample size.

It is obvious that matrix $\boldsymbol{T}$ is symmetrical and in the case of uncorrelated columns it is equal to unit matrix $\boldsymbol{I}$. Consider only realizations of matrix $\boldsymbol{R}$ for which matrix $\boldsymbol{T}$ is positive definite and let $\boldsymbol{S}$ be a lower triangular matrix such that

$$\boldsymbol{S} \, . \, \boldsymbol{T} \, . \, \boldsymbol{S}^T = \boldsymbol{I} \tag{3}$$

where

$$\boldsymbol{S} = \boldsymbol{Q}^{-1} \tag{4}$$

and

$$\boldsymbol{T} = \boldsymbol{Q} \, . \, \boldsymbol{Q}^T \tag{5}$$

Because matrix $\boldsymbol{T}$ is positive definite, the Cholesky factorization scheme may be used to find the lower triangular matrix $\boldsymbol{Q}$.

$$\boldsymbol{R_B} = \boldsymbol{R} \, . \, \boldsymbol{S}^T \tag{6}$$

Transformation results in an N x K matrix $\boldsymbol{R_B}$. Statistical correlation among columns of this matrix is described by the rank correlation matrix $\boldsymbol{T_B}$. As proved [3], matrix $\boldsymbol{T_B}$ will be close to unit matrix $\boldsymbol{I}$. That is, the difference between appropriate elements in matrix $\boldsymbol{T_B}$ and matrix $\boldsymbol{I}$ is lower than in the case of matrix $\boldsymbol{T}$ and matrix $\boldsymbol{I}$. We can now rearrange the values in each column of input matrix $\boldsymbol{R}$ so that they will have the same ordering as the corresponding column of matrix $\boldsymbol{R_B}$. As a result the rank correlation matrix $\boldsymbol{T}$ equals $\boldsymbol{T_B}$ and the statistical correlation among columns of matrix $\boldsymbol{R}$ and thus among columns of the table of random permutations of rank numbers is reduced.

The illustrative example of the table for N = 10 and K = 5 is shown in Tab. 1. There is shown (i) non-modified table, which was randomly generated (used in Latin Hypercube Sampling), and (ii) modified table with minimized statistical correlation among columns (used in Updated Latin Hypercube Sampling). In the first case the extreme value of the Spearman coefficient is equal to –0,47, in the second case it is equal to 0,07.

IV. Illustrative Results

To illustrate the possibility and basic features of the proposed strategy, the parametric study of the de-flections of concrete pavement is performed. The objective of the study is to identify those parameters theirs changes influence the pavement deflections the most. These parameters are grouped as follows:

- new arrangement of concrete slab – the new concrete mixture, the different arrangement of joints, or the different thickness,
- using the recycled materials in some (or all) layers of the pavement,
- improvement of the subgrade quality by adding some improving materials.

TABLE I
TABLE OF PERMUTATIONS OF RANK NUMBERS

Run	Non-modified table					Modified table				
	Variable					Variable				
	1	2	3	4	5	1	2	3	4	5
1	1	3	4	1	5	1	3	4	1	3
2	8	6	10	2	4	8	6	10	2	2
3	5	5	9	3	7	5	5	9	6	5
4	9	4	1	10	3	9	4	1	9	4
5	6	10	7	8	1	6	10	5	8	1
6	10	2	2	6	6	10	2	3	3	8
7	2	1	5	9	10	2	1	7	10	6
8	4	7	6	4	8	4	7	6	5	9
9	7	8	8	7	9	7	8	8	7	10
10	3	9	3	5	2	3	9	2	4	7

TABLE II
DESCRIPTION AND THE RANGE OF PARAMETERS

No.	Parameter	Min	Max
X1	Thickness of concrete slab	180	250
X2	Young modulus of concrete	30000	45000
X3	Poisson's coefficient of concrete	0,19	0,21
X4	Thickness of top road base	150	250
X5	Young modulus of top road base	500	15000
X6	Poisson's coefficient of top road base	0,20	0,30
X7	Thickness of bottom road base	150	650
X8	Young modulus of bottom road base	80	200
X9	Poisson's coefficient of bottom road base	0,25	0,35
X10	Young modulus of subgrade	30	150
X11	Poisson's coefficient of subgrade	0,30	0,45
X12	Width of transversal joints	15	25
X13	Width of longitudinal joints	0	3
X14	Young modulus of joint material	50	1000
X15	Coefficient of friction in joints	0,1	0,9
X16	Temperature of upper surface	-10	40
X17	Temperature of lower surface	5	20

For the sake of simplicity only illustrative results are presented.

Finite element 3D model of the structure created by the finite element package ANSYS is used [4]. Its main features are:

- a parametric model, open and flexible,
- designed as a 3D model that models the four adjacent concrete slabs with subsequent layers and the surrounding soil,
- the interaction of adjacent slabs is realized with help of special contact elements which prevent the transmission of tensile stress,
- contact between a slab and a subsequent layer is also realized with the contact elements, which allow realistic modeling of lifting of corners and the center of the slab,
- the model can be loaded by thermal loading,
- displacements, rotations, strains, normal or tangential stresses etc. can be evaluated at any point in any layer of the pavement structure.

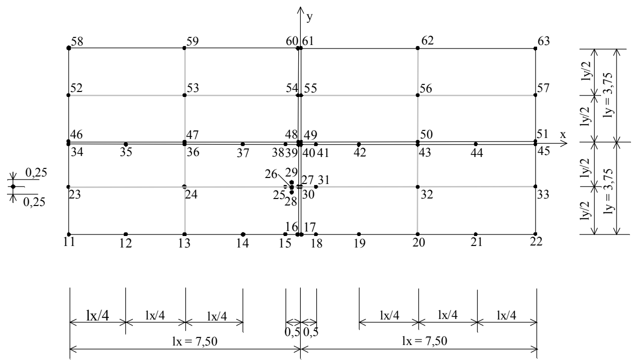

Fig. 1 Points for evaluation of stresses

Attention is paid to modeling of concrete slab contact with subsequent layer and interaction of adjacent slabs in joints. In these areas the so called contact problem occurs, in which tensile stresses cannot be transmitted. This is the case of structural non-linearity and the pavement modeling therefore becomes nonlinear. Thus the solution is divided into individual iteration steps and the Newton-Raphson method is used in each step. The actual contacts are modeled with help of special contact elements.

The behavior of the older type of rigid pavement is analyzed. This type of pavement is made from plain concrete, no dowels are used, and joints are made during laying of concrete. Dimensions of individual concrete slabs are 7,5 x 3,75 m, see Fig. 1. The structure is loaded by the self-weight of concrete slabs, by the thermal loading due to the temperature difference between the upper and lower surface of the slab, and by the load of intensity 50 kN at a distance of 0,25 m from the edge of slab - see point 26. Thus the total state of stress in the slab results from all three different sources of load acting together. The deflections are evaluated in 53 points on the upper surface of the concrete slabs, see Fig. 1.

Totally 17 input parameters influencing deflections are taken into account - the thickness of individual layers, physical and mechanical properties of materials, characteristics of joints, and the temperature of the upper and lower surface of slabs, see Tab. 2 (units in MPa, mm, °C). The range of all parameters is divided into 30 intervals and the same number of simulation runs is performed.

To measure the relative influence of each parameter on the output, the sensitivity coefficient based on Spearman rank correlation coefficient is proposed. It is not limited to the linear relationship among parameters and output in the computational model like the Pearson correlation coefficient. The sensitivity coefficient is defined as

$$r_{sk} = 1 - \frac{6\sum_{n}(m_{kn} - m_n)^2}{N(N-1)(N+1)} \tag{7}$$

where r_{sk} is the sensitivity coefficient among the k-th parameter X_k and the output, m_{kn} are the rank numbers of the parameter X_k, m_n are the rank numbers of the output, and N is the number of intervals (simulation runs). The higher is the sensitivity coefficient, the higher is the sensitivity of the output to the appropriate parameter. Sensitivity lower than 0,30 (in absolute value) can be explained as practically no influence, higher than 0,30 as a low influence, higher than 0,50 as a moderate influence, higher than 0,70 as a high influence, and sensitivity higher than 0,90 as a dominant influence. The sensitivity coefficients (and thus the influence of all parameters on output) are analyzed within one procedure and the key parameters with the largest influence are immediately identified.

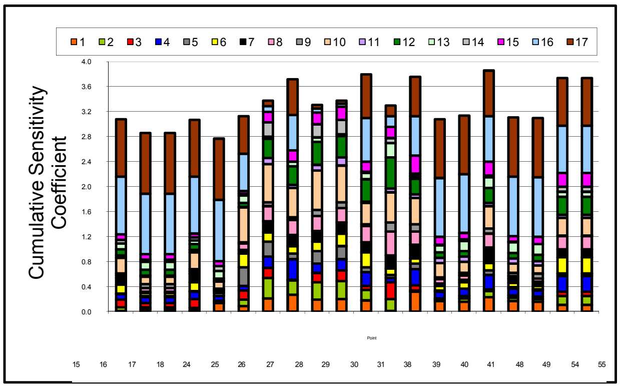

Fig. 2 Sensitivity coefficients for some points of concrete slabs

The illustrative results in some important points on the upper surface of the concrete slabs are shown in Fig. 2. In the figure sensitivity coefficients of individual parameters are shown – that is, there is shown influence of individual parameters on variability of deflections in these points. The larger is the sensitivity coefficient, the larger is the influence.

At some points only the minimum number of parameters show influence. These are corners and centers of plates that are influenced by only two parameters - the dominant influence show the temperature of the upper and lower surface of the concrete slab, i.e. the temperature gradient. Influence of these parameters is dominant also in all other points with the exception of points near the external load application, where the influence of temperature is minimal. On the other way, in these points the influence of Young modulus of subgrade is high or very high. Also influence of width of transversal joint is significant in some points. All other input parameters show low or zero. It is obvious, that influence of individual parameters is generally different in different points of concrete slabs.

V. Conclusion

The method is proposed to improve the efficiency of parametric studies and tests planning. All input parameters are taken into account, but only a few simulation runs are performed to assess the relative importance of each parameter. The Updated Latin Hypercube Sampling is used to choose the desired combinations of input values for the individual simulation runs or tests. The most important parameters with the largest influence are immediately identified within one procedure.

Acknowledgment

The research was supported by the project TA01020326 "Optimization of design and realization of low capacity road pavements " of the Technology Agency of Czech Republic and FR-TI3/727 “Advanced Cellular Concrete Technology based on Industrial Wastes for Energy-Saving Construction” of the Ministry of Industry and Trade of Czech Republic.

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