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Dynamic Modeling of Underplateform Damper used in Turbomachinery

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***Abstract*—**The present work deals with the structural analysis of turbine blades and modeling of turbine blades. A common failure mode for turbine machines is high cycle of fatigue of compressor and turbine blades due to high dynamic stresses caused by blade vibration and resonance within the operation range of the machinery. In this work, proper damping system will be analyzed to reduce the vibrating blade. The main focus of the work is the modeling of under platform damper to evaluate the dynamic analysis of turbine-blade vibrations. The system is analyzed using Bond graph technique. Bond graph is one of the most convenient ways to represent a system from the physical aspect in foreground. It has advantage of putting together multi-energy domains of a system in a single representation in a unified manner. The bond graph model of dry friction damper is simulated on SYMBOLS-shakti® software. In this work, the blades are modeled as Timoshenko beam. Blade Vibrations under different working conditions are being analyzed numerically.

***Keywords*—**Turbine blade vibrations, Friction dampers, Timoshenko Beam, Bond graph modeling.

I. INTRODUCTION

THE common failure mode of turbine blade is high cycle fatigue of compressor and turbine blade due to high dynamic stresses caused by blades vibration and resonances within the operation range of the machinery. Since the advent of steam and gas turbines and compressors, and their application in various industrial sectors, blade failures (the majority of which are caused vibration related fatigue stresses) have proven to be a major cause of breakdown; often resulting in a catastrophic damage. The continuing industrial trend in recent years is to generate more power/thrust per unit mass per unit cost of equipment. In particular, the building of larger steam and gas turbines, which operate with close tolerances at higher speeds has become commonplace. The reliability and service life of such equipment is of a great economic importance, so that improvements in design methods can be minimized failures. Blade vibration is also an important guide to preventative maintenance, scheduling, which can be a major contributor to increased reliability and service life. Dynamic loads, which generate blade vibration, can arise from various sources. Rotor imbalances, non-concentric casings causing circumferential rotor clearance to vary and irregular intake geometry related pressure distribution within the air flow produce excitation forces at low integer engine orders.The stators located up and downstream of the rotor stage cause pressure fluctuations of higher integral orders. Vibrations asynchronous with rotor speed are excited by rotating stall, compressor surging and unfavorable interaction blades and flow, which may cause self exciting flutter vibrations. The blades are very structural members, particularly in the case of a compressor. Significant number of their natural frequencies can be in the frequency range of nozzle excitation forces. Though blades can be "de-tuned" to avoid resonance at the machine steady operating speed, they will often experience several resonances during start- up, shut down, and possibly overrun. A large number of failures and shut down can be explained by blade failure due to case by the vibration or flutter. In such cases, it may be possible to introduce damping either at the root, through slip at lower speeds, or through mechanical rubbing via shroud bands, base platform and/or lacing wires at higher speeds, where the root become interlocked. The several methods to reduce the dynamic stresses are namely (1) Decreasing the force of excitation. (2) Changing resonance frequency. (3) Increasing damping. Present paper investigates the proper damping system to reduce the blade vibration.

Increasing damping is one of the feasible way as the other methods effects the design conditions. This is a motivation for use of friction damper under the platform of blades. These dampers not only reduce vibration of the turbine greatly, moreover, do not extract gas flow filled and stream flow filled. The proposed research will explore the optimum use of friction damper and its modeling. A very limited research has been carried out for friction damper modeling through bond graphs.

Generally, blades are subjected to forced vibration by the variation of the flow of air. These stresses are produced by resonant vibrations, which significantly affect the life of turbo engine blades. A correct design of bladed disks required the capability to obtain the different resonant frequencies and the forced response of the turbine blade. The blade failures can be effectively reduced by employing dry friction/under-plateform damper. Dry friction damper dissipates energy in the form of heat due to the rubbing motion of the contacting surfaces resulting from the relative motion.

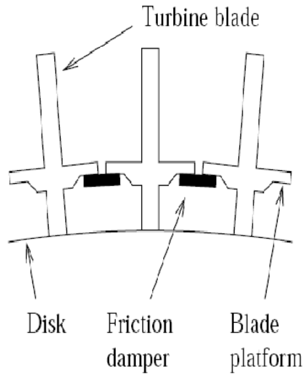

Fig. 1 Location of Friction damper

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The application of the dry fiction / under platform damper is loaded by centrifugal force against the underside of the platforms of two adjacent blades. The next subsection will focus the importance of friction damping used in turbomachinery.

A. Importance of Friction Damping Used in Turbo Machinery

When a body moves or tends to moves over another body, a force opposing the motion develops at the contact surfaces. This force which opposes the movement or tendency of movement is called frictional force. Friction has received considerable attention by researchers for several decades. While the most mechanical systems required to reduce friction and its effects. There are many applications where the presence of friction is desirable. The friction damper uses dry friction to dissipate energy and thus reduce the stresses. The advantage of using dry friction is that it provides a passive, inexpensive and environmentally robust way to improve turbine blade life. Friction damping takes place wherever two contact surfaces experience relative motion. The centrifugal force arising from the rotation of the disc forces the damper against the platform and energy is dissipated, when there is slip motion at the contact interfaces. Implementing the friction effectively to damp the vibrations becomes a challenging task for the modelers. The main cause for this is the non–linear relationship between the normal load and energy dissipation. Fig. 2 shows the typical nature of energy dissipated (E) versus the damper normal load (N) plot.

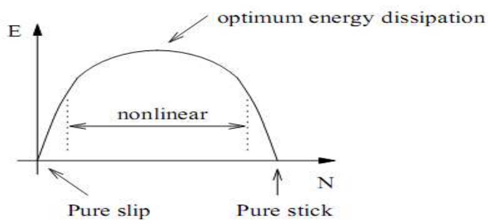

Fig. 2 Energy V/S Normal load

When a normal load is absent, the contact interface experiences pure slip only. Since no work is required to be done against friction, no energy is dissipated. On the other hand, very large normal loads cause the whole contact interface to stick. This results in no energy dissipation again since no relative motion is allowed at the interface. For normal loads that lie between these two extremities, energy is dissipated and the optimum value of energy dissipation lies within the range. However, the behavior of the contact interface is highly nonlinear in this region. This non linearity is due to the variation of friction parameters in space and time. The main difficulty lies in capturing the behavior of these parameters, which is the main difficulty of friction damper design.

The various methods employed to reduce the dynamic stresses are

- Decreasing the force of excitation,
- Changing resonance frequency, and
- Increasing damping.

One method is to reduce the force of excitation, but this implies changing the configuration of the blades and this is not feasible in most cases due to the other aero-design constraints. Another method is to change the forcing frequency, but this is feasible only for applications that require constant service speed. Thus, for applications such as jet engines, where service speed varies, this method cannot be employed efficiently. The third method is to use a device called a friction damper. This friction damper uses dry friction concept to dissipate energy and thus reduce the stresses. The advantage of using the dry friction is that it provides a passive, inexpensive and environmentally robust way to improve the turbine blade life.

The friction damping concept is frequently applied in turbo machinery applications mostly at hot locations to reduce resonance stresses. Friction has received considerable attention by researchers for several hundreds of years. While most mechanical systems need to reduce friction and it effects, there are many important applications, where friction is desired. Dry friction is used in such cases as a damping or isolation technique. A brief literature review on applications of dry friction dampers is presented here.

Friction has been considered as extremely active field of research, since its inception. Friction problems are nonlinear, and consequently, difficult to analyze exactly. According to Griffin [1], one important result of the recent research has been observed that the response of a mistuned bladed disk assembly is minimized by damper of with the same damper parameters that minimizes the resonant response of the system if it were tuned. The response of a tuned bladed disk can be characterized in terms of the response of a single blade and disk segment.

Zubieta et al. [2] presented a new testing method and analysis process to study the friction of a damper, which enables the characterization of both the static and dynamic behaviors in a single test. In this method, a new friction model with two degree of freedom is proposed, where one of them refers to the static behavior and the other to the dynamic behavior. Rao et al. [3] reported in his research that the effect of friction disappears with increasing speed and also the viscous damping ratio increased with strain in the blade. Lopez et al. [4] had shown that the maximum energy of dissipation and corresponding optimum friction force of friction dampers with stiff localized contacts and large relative displacements within the contact can be determined with sufficient accuracy using a dry friction model.

Firrone et al. [5] devised a novel method to compute the static balance of turbine blades with under platform dampers, focusing on the influence of the static loads on the system dynamics. Firrone [6] in their further study, presented a novel approach based on LDV measurements and their damping capability has been explored to study the cylindrical and wedge under platform damper, which was focused on to particular first bending modes of a two dummy blades system.

Different researchers have used various modeling technique to analyze the dynamics of dry friction dampers used in turbo machinery. Friction problems are nonlinear and, consequently difficult to analyze. In order to design a friction damper, its geometry and the geometry of the contact region, where it contacts the blade must be accurately. To do this, the design engineer must be able to calculate, how changing the damper / blade system will affect its resonant response and then choose a design which will further reduce the peak response as much as possible. The several modeling techniques as used to analyze the dynamics of dry friction dampers used in turbo machinery. Sanliturk et al. [7] presented a theoretical model for analyzing the dynamic characteristics of wedge –shaped under platform damper for turbine blade. He has shown that wedge shaped damper are prone to rolling motion, when subjected to radial platform motion and this can reduce the damping in certain mode of vibration usually described as the lower nodal diameter bladed disk modes. Ramaiah et al. [8] developed a mathematical model, which is used to count the micro-slip in the contact region. The important parameters, which affect energy dissipation between damper and the rigid surface are identified and studied by analyzing this model. This model can be used in the design of friction dampers for aircraft application to reduce resonant responses. Herrmann [9] included the vibration of the Euler- Bernoulli beam with allowance for damping. The presence of energy dissipation mechanisms is now greatly accepted in all models used for simulation of mechanical vibrations in elastic systems.

Narasimha et al. [10] described that dry friction can be efficiently used in such cases as a damping or isolation technique. Turbine blades, built up structures and transportation systems use friction to enhance their performance. They also investigated that the performance of cottage roof damper affecting the vibrations of turbine blade at different loading conditions. Phadke [11] had found that friction damping is an attractive technique because of its inexpensive, robust and environmentally tolerant nature. However, the inherent complexity involved with friction poses a challenge to develop friction models that can handle the rich interface behavior. Researchers working in this field have studied both low order and high order models. Then efforts indicate that there is a great need to develop a technique that correctly captures interface shear tractions which ultimately affect dissipation calculations. Keeping in view the scope of the research, the dry-friction damping can be broadly divided into two categories: (1) research efforts undertaken to model and analyze the nonlinear behavior of friction-damped systems and (2) studies that have principally focused on the modeling of the friction dampers.

Durali et al. [12] studied that the Timoshenko beam element may be used to model the turbine blade. Tsai [13] investigated that failure of blades may have different causes, such as corrosion from the environmental pollution, deteriorated material properties, incorrect operation, resonant vibration etc. In the dynamic analysis, the resonant vibration behavior can be analyzed and can be avoided based on the analytical results. Majkut [14] described the method to drive a single equation for free and forced vibrations of Timoshenko beam model, which may be used as a blade for the turbines. Allara [15] presented the hysteresis curves obtained with proposed model may be suitable to implement in the numerical solvers for the forced response calculation of turbine bladed discs with friction interfaces such as blade roots, shrouds and underplatform dampers. Cheng [16] presented a new analytical method, which may be applied as an impact damper reducing structure vibration. Stammers et al. [17] presented a semi active damper utilizing dry friction, with balance logic, for a class of sequential damping, which were used to minimize the force transmitted. Fischer et al. [18] investigated that the effect of nonlinearities on the amplitudes of the turbine blade, which may be to internal damping of the beam element. Whiteman et al. [19] presented analysis off the damping characteristics that indicated that the dry friction force is more effective in damping the fundamental mode. The other observation was that damping contribution by the displacement dependant dry friction damper is still linear structural like in nature, when multi modes are considered and relatively insensitive to changes in the amplitude of the response. This results indicates that in the case of turbine or compressor blades, this type of damper arrangement may be effective in the suppression or flutter. Kappagantu et al [20] studied the dynamics of an experimental beam excited by dry friction. The response of the beam included periodic and non periodic oscillations.

To analyze the turbine blade without the experiment is a tedious task and time consuming. More over the entire blade assembly required computation time. The bondgraph has an advantage to represent a system keeping physical as part in for ground. It has advantage of putting together multi energy domains of a system in a single representation in a unified manner. Adding complexity extent to straight forward and there is no burden of driving system equations. Many researchers model the friction dampers using finite element or other methods have certain limitations as dampers are not modeled in transferring the energies or dissipating the energies at the real contact. To investigate the optimum damper mass is another issue, which required to be handled properly by the researchers.

II. Friction Model

Underplatform dampers are physically very simple device, yet their nonlinear behavior is quite complicated and its analysis can be extremely difficult if all the details of dampers characteristics are to be included in the analysis. These difficulties arise due to many complicated factors like the temperature, frequency, surface roughness effects, the contact locations and their variation during vibration etc. In spite of the physical simplicity of these dampers, the effects on these dampers and other factors have not been fully explored by researchers. Based on engineering judgment, some simplifying assumptions, which are considered in the design of friction dampers, are as follows:

- Dampers flexibility are considered in the bondgraph model.
- Damper mass/inertia effects are considered in the model.
- Damper contact on left and right side can be represented as a point contact with three translational degrees of freedom.
- The turbine blade is identical from left and right side.
- The blade is moving with constant velocity.
- Damper and plateform surfaces remain in parallel and having point contact all the times.
- Centrifugal forces and gravitational forces are considered in the model.

The next section will present the relative motion across damper surfaces.

A. Relative Motion across Damper Surfaces

The theoretical formulation of the cottage roof dampers motion as presented in this section is based on a model [7], which is shown in Fig. 3. This model will quite helpful in making the damper model through bondgraphs. A local coordinate system may be attached to one of the platform and the instantaneous relative motion of the other platform can be obtained

$$r_{xyz} = i r_x + j r_y + k r_z \quad (1)$$

Where

$$r_x = |X| \cos(\omega t + \varnothing_x)$$
$$r_y = |Y| \cos(\omega t + \varnothing_y) \quad (2)$$
$$r_z = |Z| \cos(\omega t + \varnothing_z)$$

$$X = X_L - X_R$$
$$Y = Y_L - Y_R$$
$$Z = Z_L - Z_R \quad (3)$$

X_L, Y_L, Z_L, X_R, Y_R, Z_R in above Eqs.(1-3) are complex quantities representing both amplitude and phases at platform nodes. Likewise, X, Y and Z are the relative platform displacements with respect to some local coordinate system $\varnothing_x$, $\varnothing_y$, and $\varnothing_z$ the corresponding phase angles. In addition to that, ω is the angular velocity and t is time. It may also be noted that r_x, r_y, r_z and other parameters which are functions of them are not constant values but functions of the angular displacements, ωt. The damper formulation presented in this section requires calculations of these displacements for at least one vibrational cycle. However, equation does not consider ωt for simplicity and brevity.

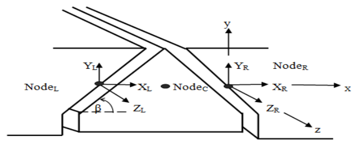

Fig. 3 Platform motion in three-dimensional (β is the platform angle, $Node_L$ and $Node_R$ are the platform left and right modes.)

The relative platform motion in Fig. 4 is three dimensional, having components in all three local directions depicting the relative displacement at two nodes. The relative displacements of the underplatform surfaces shown in Fig. 5 are calculated with assuming that the centrifugal force acting on the damper mass is larger than the gravitational force, therefore to keep the damper in contact with the platform surfaces at all times.

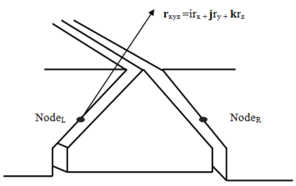

Fig. 4 Relative platform displacements

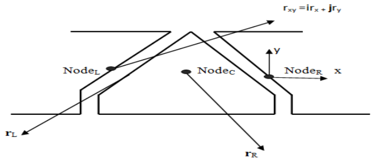

Fig. 5 Relative contact displacements on damper surfaces

A displacement triangle as shown in Figure 5 can be drawn relating the relative displacements of the platform in x-y plane (r_{xy}) and the relative contact displacements r_L and r_R since the directions of all the vectors in Fig. 5 and if the magnitude of the (r_{xy}) are known, the magnitudes of the r_L and r_R may be calculated as

$$|\mathbf{r}_L| = \frac{|\mathbf{r}_{xy}|(\sin(\alpha)+\cos(\alpha)\tan(\beta))}{2 sin\beta} \quad (4)$$

$$|\mathbf{r}_R| = \frac{|\mathbf{r}_{xy}|\cos(\alpha)}{\cos(\beta)} - |\mathbf{r}_L| \quad (5)$$

Similarly, damping forces may also be computed as given in reference [7]. This displacement vector and damping forces will be used in creating bondgraph model of the friction damper. The geometrical considerations and dynamic load factor are also considered for modeling through bondgraphs.

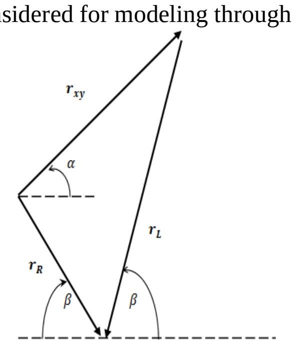

Fig. 6 Displacement triangle relating r_{xy} to r_L and r_R

B. Equation of Lateral Vibration of Timoshenko Beams

Timoshenko beam is considered in the model as turbine blade elements. Consider the free body diagram of an element of a Timoshenko beam [22, 24] as shown in Fig. 6, where M(x, t) the bending moment is, V(x, t) is the shear force, and f(x, t) is the external force per unit length of the beam. The inertia force acting on the element of the beam is the force equation of motion in the z direction, which provided

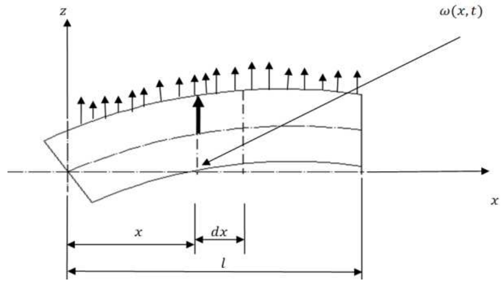

Fig. 7 A beam in bending

$$-(V + dV) + f(x,t)dx + V = \rho A(x)dx \frac{\partial^2 w}{\partial t^2}(x,t) \quad (6)$$

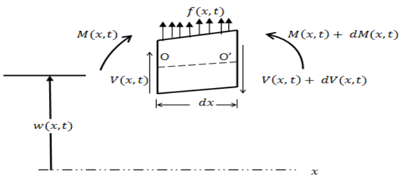

Fig. 8 Shear force and bending force during bending

Where ρ is the mass density and A (x) is the cross- sectional area of the beam. The moment equation of motion about the y axis passing through point O in Fig. 7 may be may be presented as

$$(M + dM) - (V + dV)dx + f(x,t)dx\frac{dx}{2} - M = 0 \tag{7}$$

By writing

$dV = \frac{\partial V}{\partial x}dx$ and $dM = \frac{\partial M}{\partial x}dx$

and disregarding the other terms involving second powers in dx, Eqs. (6) and (7) can be re written as

$$-\frac{\partial V}{\partial x}(x,t) + f(x,t) = \rho A(x)dx\frac{\partial^2 \mathrm{w}}{\partial t^2}(x,t) \tag{8}$$

$$-\frac{\partial M}{\partial x}(x,t) - V(x,t) = 0 \tag{9}$$

By using the relation V $= \frac{\partial M}{\partial x}$ into Eq. (9)

$$-\frac{\partial M}{\partial x}(x,t) + f(x,t) = \rho A(x)dx\frac{\partial^2 \mathrm{w}}{\partial t^2}(x,t) \tag{10}$$

From the elementary theory of bending of beams (also known as the Euler-Bernoulli or thin beam theory), one may easily write

$$\mathrm{M}(x,t) = EIx\frac{\partial^2 \mathrm{w}}{\partial t^2}(x,t) \tag{11}$$

Where E is Young's modulus and $I(x)$ is the moment of inertia of the beam cross section about the y axis. Insert Eq. (11) into Eq. (12), one may obtain the equation of motion for the forced lateral vibration of a non uniform beam, which will reduce to

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2}(x,t) = f(x,t) \tag{12}$$

This equation is quite useful in determination of forces during vibration of Timoshenko beams.

C. Effects of Shear Deformation on Beams

The determination of shear deformation is very important for modeling. When the cross- sectional dimensions are not very small, compared to the length of the beam, one may require considering the effects of rotary inertia and shearing deformation in the element of the beam as shown in Fig.9. The angle γ between the tangent to the deformed centre line (O´T) and normal to the face (O´N) denotes the shear deformation of the element [ref. 22]. Since positive shear on the right face Q´R´ acts downward, one may obtain from Fig. 9

$$\gamma = \emptyset - \frac{\partial w}{\partial x} \tag{14}$$

Where, $\emptyset$ denotes the slope of the deflection curve due to the bending deformation alone. It may be noted that because of shear alone, the element undergoes distortion but no rotation. The bending moment M and the shear force V are related to $\emptyset$ and ω as per the following relation.

$$\mathrm{M} = EI\frac{\partial \emptyset}{\partial x} \tag{15}$$

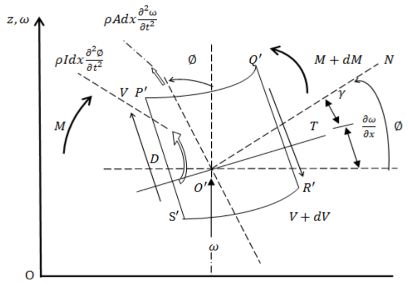

Fig. 9 An element of Timoshenko beam

$$\mathrm{V} = kAG\gamma = kAG\left(\emptyset - \frac{\partial w}{\partial x}\right) \tag{16}$$

Where G is the modulus of rigidity of the material of the beam and k is a constant, also known as Timoshenko's shear coefficient [22, 24], which mainly depends on the shape of the cross – section. Generally, for a rectangular section the value of k is 5/6. For rotation about a line passing through point D and parallel to y-axis, one may write the following equations:

$[M(x,t) + dM(x,t)] + [V(x,t) + dV(x,t)]dx +$
$f(x,t)dx\frac{dx}{2} - M(x,t) = \rho I(x)dx\frac{\partial^2\emptyset}{\partial t^2}$ = Rotary inertia of the element

(17)

Using the relations once again $dV = \frac{\partial V}{\partial x}dx$ and $dM = \frac{\partial M}{\partial x}dx$ and adopting the procedure given in ref. [22, 24], one may obtain the desired equation of motion for the forced vibration of a uniform beam:

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} - \rho I\left(1 + \frac{E}{kG}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\rho^2 \mathrm{I}}{kG}\frac{\partial^4 w}{\partial t^4} + \frac{EI}{kAG}\frac{\partial^2 f}{\partial x^2} - \frac{\rho I}{kAG}\frac{\partial^2 f}{\partial x^2} - f = 0 \tag{18}$$

For free vibration, f =0, and equation (18) reduces to

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} - \rho I\left(1 + \frac{E}{kG}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\rho^2 I}{kG}\frac{\partial^4 w}{\partial t^4} = 0 \tag{19}$$

This Timoshenko beam theory is used to model turbine blade, which incorporates effects of shear in the model.

III. Bondgraph Modeling

A. Modeling of Timoshenko Beam Element

The effect of shear is considered in this model as stated above. However, incorporation of shear into the bondgraph model is also very significant. Initially it may be treated as a Rayleigh beam as given in ref. [23] considering additional deformation of the element due to the shear force. From variational principles, the Timoshenko model may be written as

$$\frac{\partial}{\partial x}\left[\ \left\{\frac{\partial}{\partial x}y(x,t) - \psi\right\}\right] = \rho A\frac{\partial}{\partial t^2}$$

$$\frac{\partial}{\partial x}\left[EI\frac{\partial}{\partial x}\psi\right] + kGA\left\{\frac{\partial}{\partial x}y(x,t) - \psi\right\} = \rho I\ \frac{\partial}{\partial t^2}\psi \tag{20}$$

Where ψ is the angular deformation due to bending , G is the shear modulus , k is a factor depending on the beam cross- sectional shape and is used for averaging the relation between the shear force and shear angle for the cross- section. Timoshenko model of a beam element with bending deformation ψ and shear deformation β may be easily shown in Fig. 10 (a) and Fig. 10 (b).The total angular deflection at a section may be written as

$$\frac{\partial}{\partial x} y\,(x,t) = \psi + \beta \tag{21}$$

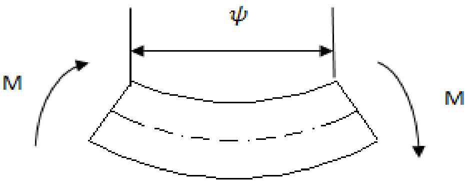

Figure 10(a): Bending effects in Timoshenko beam

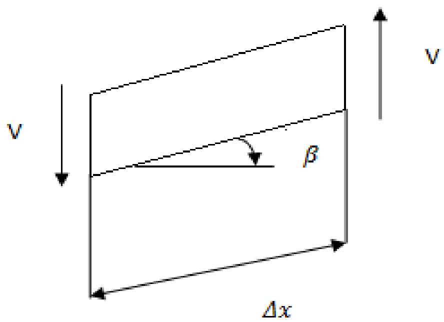

Figure 10(b): Shear effects in Timoshenko beam

The shear force and bending moment equations can be written as

$$\mathrm{V}\,(x) = kGA\beta \tag{22}$$

$$\mathrm{M}\,(x) = EI\frac{\partial}{\partial x}\psi \tag{23}$$

The unloaded static equations may be written from Eq. (23) as

$$\frac{\partial}{\partial x}[kGA\,\{\frac{\partial}{\partial x}y\,(x,t) - \psi\}] = 0 \tag{24}$$

$$\frac{\partial}{\partial x}[EI\frac{\partial}{\partial x}\psi] + kGA\{\frac{\partial}{\partial x}y\,(x,t) - \psi\} = 0 \tag{25}$$

In order to find a C- field relationship, One may introduce an intermediate deformation variable z in such a way that

$$z = y\,(x,t) - \mathrm{B}\,x, \tag{26}$$

and $\frac{\partial}{\partial x} z = \frac{\partial}{\partial x} y(x,t) - \beta = \psi$ (27)

It is also important that an unloaded beam element has the same value of shear force and thus the same β all along.

$$\mathrm{V}\,(x) = kGA\,\{\frac{\partial}{\partial x}y\,(x,t) - \psi\} = -EI\frac{\partial^3}{\partial x^3}\,Z \tag{28}$$

From Eqs. (23) and (27), one may have

$$\mathrm{M}\,(x) = EI\frac{\partial}{\partial x}\psi = EI\frac{\partial^2}{\partial x^2}\psi \tag{29}$$

The expressions for shear force V(x) and M (x) in the variable z (x) are similar to those of Euler-Bernoulli beam displacement y (x). From Eqs. (28) and (29), the elements of a C- field can be expressed as the C- field provides the relationship between the forces and moments to the corresponding deformation as

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} = [K]\begin{Bmatrix} z_1 \\ \psi_1 \\ z_2 \\ \psi_2 \end{Bmatrix} \tag{30}$$

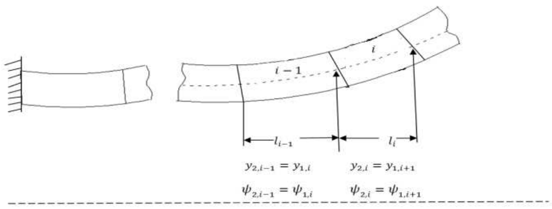

Fig. 11a C_ Element of a Timoshenko beam model

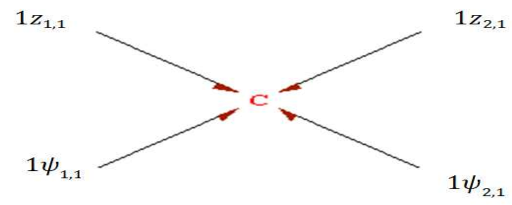

Fig. 11b C— Element of a Timoshenko beam model for 1 section

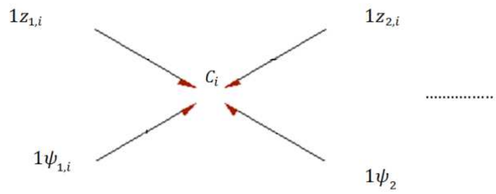

Fig. 12 C— Element of a Timoshenko beam model for i[th] section

This C- field can be used to model individual elements of a reticulated beam as shown in Fig. 11(b). It may please be noted that this model must have representation for actual deformation$y\,(x,t)$, whereas the C- field is defined by Eq. (30). The local coordinate S^i for each element may be given as.

$x - x_i = s^{(i)}$ for $x_i \le x \le x_{i+1}$

Where x_i is the distance of the i^{th} element from the global origin. It may be observed from Figure 12 that the shear compliance with stiffness kGA is in differential causality. This causality may be removed by considering inertia associated with shear deformation.

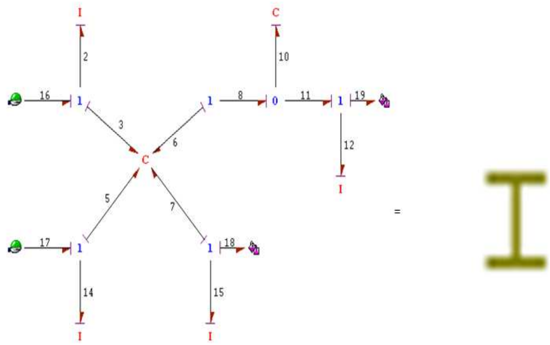

Fig. 13 Bond graph model of Timoshenko beam

The elements of the C- field provided in Eq. (30) may also be computed from energy consideration. The potential energy of the element may be written as

$$V=\frac{1}{2}\{y_1\ \psi_1\ z_2\ \psi_2\}[K]\{y_1\ \psi_1\ z_2\ \psi_2\}^T+\frac{1}{2}\,kGA\beta^2 \quad (31)$$

The C- field equation in terms of $\{y_1\ \psi_1\ y_2\ \psi_2\}^T$ may be shown in Fig.13 and written as

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} = [K]\begin{Bmatrix} z_1 \\ \psi_1 \\ z_2 \\ \psi_2 \end{Bmatrix} \quad (32)$$

The potential energy can be easily obtained after following the steps as given in ref. [23], which denotes

$$V=\frac{1}{2}\{y_1\ \psi_1\ z_2\ \psi_2\}\ [T]^T\ [K]\ [T]\ \{y_1\ \psi_1\ z_2\ \psi_2\}^T + \frac{1}{2}\,kGA\{\,b_1\ y_1+\ b_2\,\psi_1+\ b_3\,y_2+\ b_4\,\psi_2\}^2$$

$$=\frac{1}{2}\{y_1\ \psi_1\ z_2\ \psi_2\}\ [K]\ \{y_1\ \psi_1\ z_2\ \psi_2\}^T \quad (33)$$

Where $[T]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The Eq. (33) will be used to calculate the components of stiffness matrix, which will be used to model the turbine blade element.

B. Physical System of Turbine Blade with Dry Friction Damper (Considering Timoshenko Beam Model)

The physical system of the turbine blade with cottage shaped friction damper has been shown in Figure 14. The turbine blades have been considered as the cantilever beams and friction dampers are placed in between the two blades. The cantilever beam considered in this case is Timoshenko beam. In this work, multi-body approach is applied to investigate the dynamic modeling of friction dampers used in turbo machinery using bond graphs technique and simulation has been performed on SYMBOLS-shakti® software [21], which is a modeling, simulation, and control systems software for a variety of scientific and engineering applications.

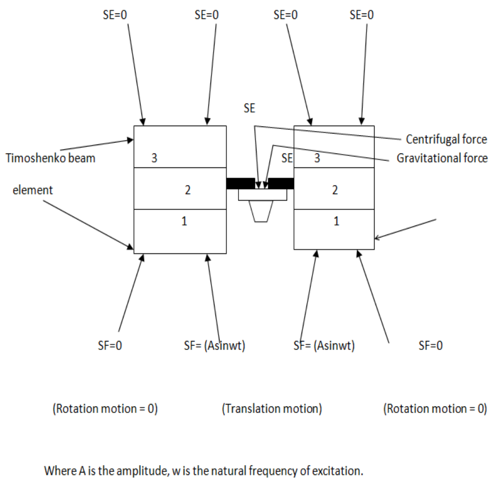

Fig. 14 Physical system of turbine blade with friction damper considering Timoshenko beam model

C. Modeling of Friction Damper

The friction damper is modeled in between two adjacent blades. In modeling the blade, three reticules of Timoshenko beam have been used and the friction damper is used after 1st reticules of the beam. The friction damper is fitted beneath the platform, which experiences centrifugal forces and gravitational forces. The beam also expresses the shear deformation and rotary inertia. The detailed bond graph model is shown in Figure 15. In this bondgraph model, I62 is modeling the mass of the damper. SE61 element is modeling the centrifugal force exerted on the damper. Element SE49 and SE50 are showing the modeling of gravitational force acting on damper mass. Stiffness element C60 and C57 modeling the stiffness of the damper. The geometric considerations are also taken care in the modeling of friction dampers.

D. Integrated Model of Turbine Blade with Friction Damper (Considering Timoshenko Beam)

The integrated model of Timoshenko beam as turbine blade with friction damper is a vectorized bond graph model interfacing sub-models with, Timoshenko beam by its respective icon. The blade is modeled with three reticules (Beam elements). The significance of using more reticules improves the modeling accuracy, mainly in the higher modes. However, it also requires high capacity computational facility. The friction damper is modeled by the resistive element, *R* and disc is rotated by a constant speed, ω as shown in Figure 15.

The next section will present the simulation study of the bond graph model shown in the Figure 15.

IV. Simulation Results

The bond graph model of the turbine blade with taken the boundary conditions of cantilever beam conditions was simulated using a Runga -Kutta fourth order method on the software SYMBOLS Shakti® [21]. The bond graph model with object-oriented sub models of the turbine blade was shown in Fig. 15.

A. Description of Simulation Rig

The bond graph model of the turbine blade with friction damper as shown in Fig. 15 was simulated on the software SYMBOLS Shakti®. The shaft is rotated by a constant speed source. The simulation rig consists of solid rotating bladed discs with blades, which is discretized into six reticules with one end attached to the discs, whereas another one is free like a cantilever beam. The dimensional data are as follows: L_{beam} = 1.2 m, material and blade properties are as follows: E =1.93 e11 N/mm^2, ρ =7850kg/m^3

All the parameters presented in the Table.1 are used for simulation. The results of simulation are presented in the next sub-section.

B. Frictional Forces

The frictional forces between the left beam and right beam are present due to the friction damper shown in Fig.16 and Fig. 17 at 0.1 kg of damper mass. The friction force at right beam is more than the left beam.

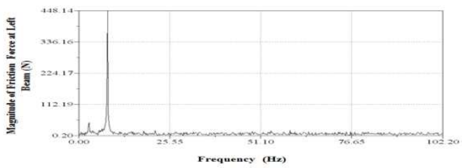

Fig. 16 Magnitude of Friction force at left beam at 0.1 kg of mass

TABLE I
PARAMETERS AND VALUES USED FOR SIMULATION

Parameter	Value
Length of each reticules	0.2 m
Modulus of elasticity for the beam material	2.11e11 N/mm^2
Density of the material of beam	7850 kg/m^3
Radius of the turbine blade	0.4 m
Moment of inertia of beam	6.69 e-7 m^4
Internal damping co-efficient of the beam material	2.50×10^{-4} Ns/m
Frequency of excitation	50
Amplitude of excitation	0.2 m
Mass of friction damper (Variable)	0.1 to 0.5 Kg
Surface area of the damper	.003m^2
Damping coefficient of the friction	0.7 N.m.s.
Reynolds Number	10,000
Inertia distance of contact	.01 m
Inertia of the damper	4.17 e-4 m^4
Frequency of turbine blade	200
Elasticity of the material of the damper	1.93 e11N/mm^2
Second moment of area of damper	1.25 e-8 m^4
Poisson ratio	0.3
Modulus of rigidity	8.1e10
Shape factor	0.95
Damper mass	0.1-0.5 Kg
Thick ness of the damper	0.08m

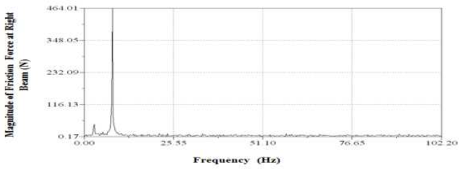

Fig. 17 Magnitude of Friction force at right beam at 0.1 kg of mass

C. Energy dissipation

The energy dissipated between the left beam and right beam is shown in Fig. 18 and Fig. 19 at 0.1 kg of damper mass. This energy is dissipated in terms of heat or the other due to the contact of blade surface and the friction dampers. The energy dissipation at right beam is more than the left beam.

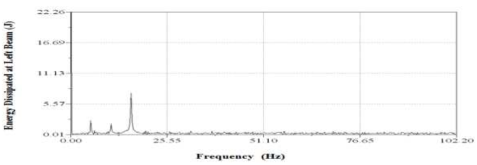

Fig. 18 Energy Dissipated at left beam at 0.1 kg of mass

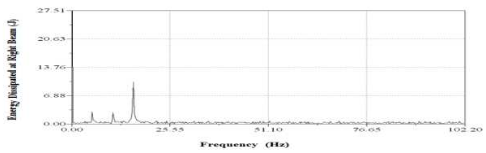

Fig. 19 Energy Dissipated at right beam at 0.1 kg of mass

The Fig. 20 shows the energy dissipation of right and left beam with mass of friction damper. It shows that when mass increases by 0.1 kg, but the energy dissipation at left beam is less than the right beam, this energy is dissipated in terms of heat or the other due to the contact of blade surface and the friction dampers at variable masses from 0.1 kg, to 0.5 kg. So it is clear from this comparison graph between left beam and right beam, the energy dissipation is always more on right beam. So there is always possibility of failures of right beam. To avoid such kind of failure, one may use the appropriate damper mass between left and right beam. In this work, it may be seen from the plot 0.2 kg damper mass is appropriate to reduce the energy dissipation as energy dissipation is maximum for small damper mass.

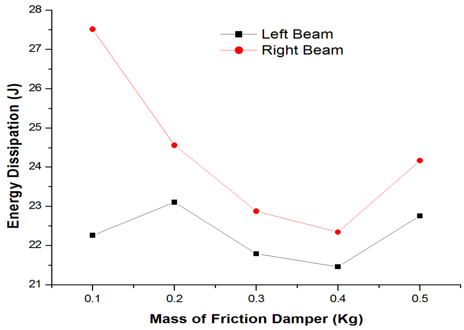

Fig. 20 Energy Dissipation between left and right beam (0.1 to 0.5 kg.)

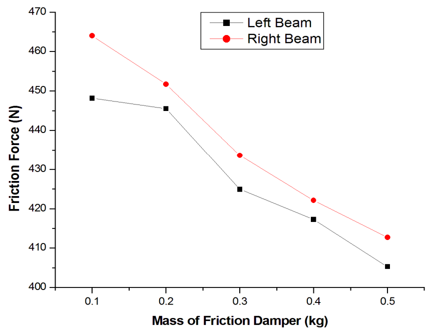

Fig. 21 Frictional Force between left and right beam (0.1 to 0.5 kg.)

The Fig. 21 shows the friction force at left and right beam with friction damper mass. It is clear from the plot that the friction forces less on the left beam and more on the right beam, when mass of damper increases from 0.1 kg to 0.5 kg. More over friction force decrease with increase in damper mass. So, right beam is more prone to failures. To avoid such kind of failure, one may use the appropriate damper mass, which can balance the effect of friction forces on left and right beam. Simulation study suggests that one may use 0.2 kg damper mass to reduce the vibration and moreover to increase the effectiveness of the turbine blade.

V. CONCLUSION

The structural analysis of the turbine blade with friction dampers are shown in this work. The different ways of friction damping along with their mathematical analysis are also elaborated. The bondgraph modeling of the turbine blade is presented considering the turbine blade as cantilever Timoshenko beam. It is also very important that the whole bladed disk is not required for the dynamic analysis; two blades with friction damper can show the dynamic behaviour of the blades. The unique model of dry friction damper as proposed may be easily applied in power plants to increase the productivity and to reduce failures of blades. The effect of friction damper on turbine blade has been investigated using the variable fiction damper mass. It concluded that the optimum mass of the damper is important for reducing the resonant vibration of the turbine blade. It is very significant to note that dry friction damping is most effective method to reduce vibration, with increase in excitation force the damper optimized to withstand must also be the increase in load which is also validated in the reference [7]. It is also very important that the frictional force and energy damped are quite reduced in increase damper mass. However, an optimum mass of damper is required to reduce the vibration of the turbine blades.

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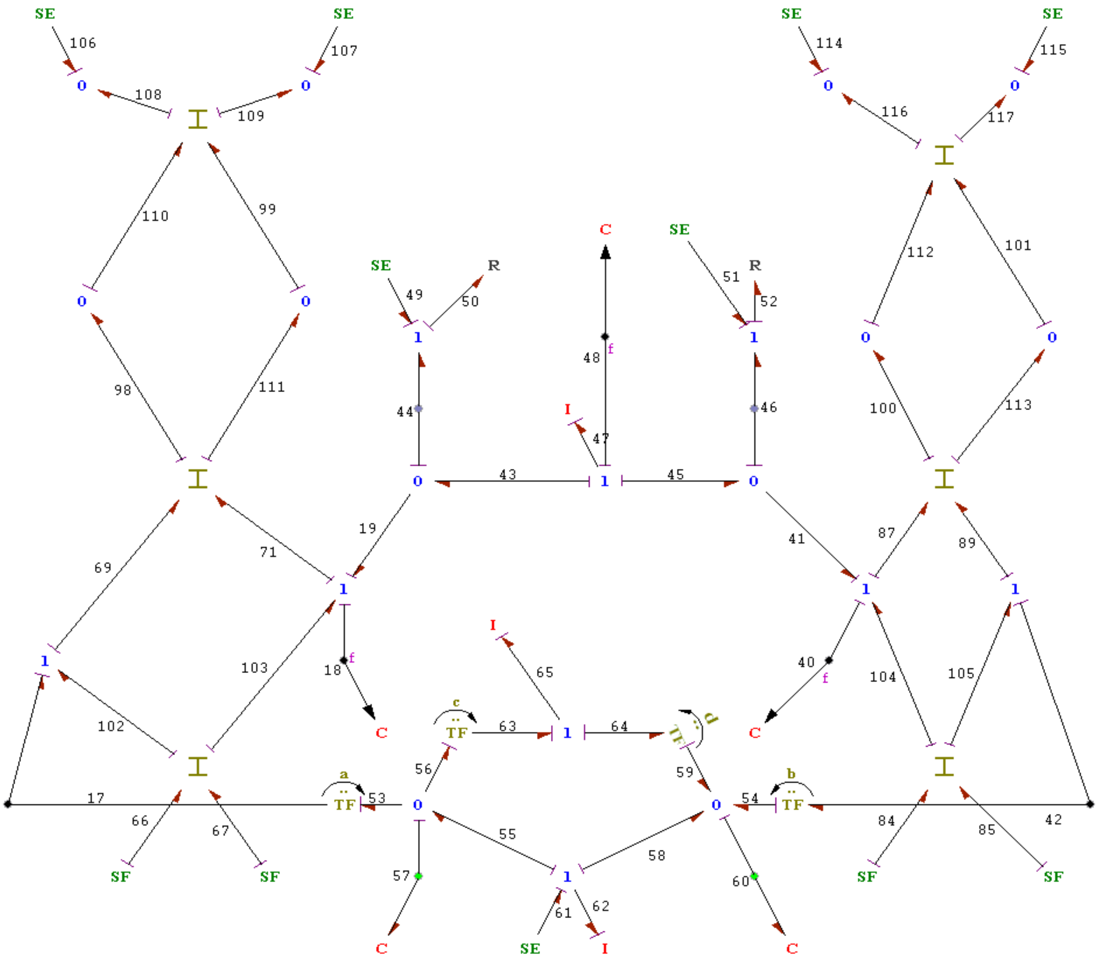

Fig. 15 Integrated bondgraph model of turbine blade (considering Timoshenko beam) with friction damper