

Weak measurement theory for discrete scales

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Abstract—With the increasing spread of computers and the internet among culturally, linguistically and geographically diverse communities, issues of internationalization and localization and becoming increasingly important. For some of the issues such as different scales for length and temperature, there is a well-developed measurement theory. For others such as date formats no such theory will be possible. This paper fills a gap by developing a measurement theory for a class of scales previously overlooked, based on discrete and interval-valued scales such as spanner and shoe sizes. The paper gives a theoretical foundation for a class of data representation problems.

Keywords—Data representation, internationalisation, localisation, measurement theory

I. INTRODUCTION

For many years the problems of internationalization (i18n) and localization (l10n) have been important to those who have to prepare software or any kind of documentation for world markets. With the rise of the World Wide Web this has become an even more pressing issue, as the volume and scope of information grows [1]. The “traditional” components of i18n concern character sets, currency and number formats, etc. The rules for different cultures can be complex and quite adhoc. For example, while a currency amount in US dollars might be written as “\$1,234.56”, in France an amount would be written as “1 234,56Fr” [2].

Many programming and delivery systems now have mechanisms for dealing with much of this by using locales [3] to specify cultural, linguistic and geographic regions. The World Wide Web Consortium has also addressed some aspects of i18n with the ability to specify locale preferences and information in HTML documents [4] and also with a Scenario document discussing topics that web service designers should be aware of [5]. Web backend systems such as JSP support localization of Web pages through the JSP Standard Tag Library [6]

On the opposite side, there has been centuries of work done to standardise and provide a solid mathematical foundation for “scientific” measures such as length, pressure, time, and so on. This has been so successful that nowadays we scarcely think of the issues involved.

There is a class of problems that falls between these two extremes. It is typified by the examination by Tex Texin [7], [8], [9] of the different scales used for shoe sizes in different countries. Cultures have not co-operated to produce scales which have any mathematical foundation. There is structure to the problem, but no theory. The problem is conversion between different scales which measure the same property in quite different ways.

This paper provides the mathematical foundation to this type of problem. We introduce the concept of *weak* measurement in the context of example problems. We give representational theorems to show that common practice has a theoretical basis that extends beyond the examples.

The contribution of this paper is that it resolves some issues in data conversion problems that arise primarily through internationalization and extends the theory of measurement to cover new cases. The theory is also applicable to other non-cooperative systems, such as pay scales for different classes of employee, or for approximations such as rounding currency values. We show that there is a rigorous mathematical theory which can cover these cases. The theory is not exhaustive and we close with some further areas of investigation.

II. MEASUREMENT THEORY

There are many occasions in which we wish to quantify some property. For example, the size of a shirt, the amount of money in a wallet, your height. All of these measure the value of a property using some units system: For example, height is measured in inches, centimetres and so on. The scientific, industrial and medical worlds abound in different types of measurements, many of which are shown in the Units of Measure Dictionary[10] lists and in the Unified Code for Units of Measure [11].

A measure takes some property and evaluates it in some scale system. Representational measure theory [12], [13], [14], [15] is concerned with different axiomatisations and developing a representation theorem for each axiomatisation. For example, the simplest type of measure is the *ordinal* measure which simply concerns a transitive order \preceq on a set X . A function $f : X \rightarrow \mathfrak{R}$ is an (ordinal) measure if it satisfies the axioms

$$\forall a, b \in X, a \preceq b \rightarrow f(a) \leq f(b) \quad (1)$$

and

$$\forall a, b \in X, f(a) \leq f(b) \rightarrow a \preceq b \quad (2)$$

For example, if one block of wood is shorter than another block of wood, then the length of the first block measured in centimetres will be shorter than the length measured in centimetres of the second block.

All standard scientific measures satisfy these axioms: length, mass, temperature, pressure and so on and usually satisfy additional axioms. Scales which are only ordinal include hardness scales, intelligence quotients and so on, where the values don't really matter, just the ordering relationship between them.

For an arbitrary relational structure $\langle X, \preceq \rangle$ the fundamental questions are

- Does a measure function exist?
- What is the relation between any two measure functions (i.e. what are the allowable transformations that preserve the property of being a measure function)?

The simplest representation theorem for this structure is given by [12] as their Theorem 1:

Theorem 1: Let X be a finite set and \preceq a total order on X . Then there exists a measure f satisfying axioms (1) and (2). Further, if g is any other function also satisfying these axioms, then there is a function $h : \mathfrak{R} \rightarrow \mathfrak{R}$ such that $g = h \circ f$ and h is strictly monotonic.

The proof of the first part is constructive: assign the smallest element the value 1, the next smallest the value 2 and so on. The second part follows from the axioms and ensures that we can convert between any two measures of a property such as length (e.g. from centimetres to lightyears) without loss of information by applying a suitable function. *Note:* Many texts frame the theorem in terms of a *weak* order, and then use the equivalence class of equal elements to transform this to a total order in order to prove the theorem. We simplify the exposition throughout by using a total order.

The theorem can easily be extended to weak orders on countable sets.

The meaning of the theorem is that properties such as length can have a variety of measures such as inches, centimetres and so on. Firstly, such a measure must exist, and given any two measures we can transform values from one to the other by a monotonic function such as multiplication by 2.54.

Many of the measures of physical properties such as length also have a binary operation (corresponding to addition of lengths) and there are representation theorems for these ratio scales. For example, for ratio scales such as length, the additive properties of length are preserved between different measures (so that adding one inch is the same as adding 2.54 cms). These representation theorems form the theoretical basis for the algorithms used by Web sites such as “Unit Conversion and measurement made easy” at <http://01conversion.com/> which lists the categories Length, Mass, Area, Volume, Speed, Temperature, Pressure and Power, and within the category of Length will “Convert between angstrom, cable length, centimeter, chain, fathom, foot, ...”. The representation theorems also form the basis for the XML “conversion of units” documents[16] which define scale conversions such as

```
<UnitOfMeasure uid="m"> Metre
</UnitOfMeasure>

<UnitOfMeasure uid="ft"> US Survey foot
  <ConversionToBaseUnit baseUnit="#m">
    <numerator>12.</numerator>
    <denominator>39.37</denominator>
  </ConversionToBaseUnit>
</UnitOfMeasure>
```

TABLE I
SPANNER SIZES

| measure (inch) | mm | UNF inch |
|----------------|----|----------|
| .354 | 9 | . |
| .375 | . | 3/8 |
| .394 | 10 | . |
| .433 | 11 | . |
| .438 | . | 7/16 |
| .472 | 12 | . |
| .500 | . | 1/2 |
| .512 | 13 | . |
| .551 | 14 | . |
| .563 | . | 9/16 |

III. WEAK MEASURE - MOTIVATION

Scientific scales have good mathematical properties and theoretical background. Things such as date formats have no properties. This paper addresses some cases where there are apparent measures but which do not yet have a proper theoretical background. The first of these cases concerns discrete measures.

Nuts and bolts come in pairs: a metric $8mm$ bolt is best turned with a metric $8mm$ spanner. The standard ISO bolt sizes form a finite set $\{7mm, 8mm, 10mm, 11mm, 13mm, 14mm, 17mm, 19mm, 22mm, 24mm, 27mm\}$. There is an obvious measure function into the millimetre scale, which obeys axioms (1) and (2). The ISO bolt sizes form an ordinal scale but not a ratio scale: you can't add a $7mm$ and an $8mm$ spanner to get a $15mm$ spanner!

There are other sets of spanners and bolts, with another main group being the A/F spanners. These also form a finite set, $\{1/4\ inch, 5/16\ inch, \dots\}$. Again there is an obvious measure function into the inch scale, and from there into the millimetre scale (multiply by 25.4).

A partial table of spanner sizes is given in Table I. A full table is given at <http://www.vars.freewire.co.uk/tech/charts/spanners.htm>.

However, if you have a car with metric bolts and you only have A/F spanners then you make do by choosing the smallest spanner that will fit the bolt. For example, for a $9mm$ bolt you would choose the $3/8\ inch$ spanner. It isn't perfect but will often do. In the measure-theoretic terms we are proposing here, the question arises: is the set of A/F spanner values a possible measure for metric bolts? This question will arise whenever we have discrete values of a similar property which have devised different physical structures with different values. *Note:* This is not a desirable situation, but arises when different groups independently define different discrete systems.

The short answer is “no”. Between $1/2\ inch$ and $9/16\ inch$ are two metric sizes, $13mm$ and $14mm$. So both of the $13mm$ and $14mm$ bolts will require a $9/16\ inch$ spanner. (Similarly between $10mm$ and $11mm$.) That is, the function f from metric bolts to A/F spanner sizes measured in inches has special values

$$f(13mm) = 9/16$$

and

$$f(14mm) = 9/16$$

which break axiom (2) of an ordinal measure ($f(14mm) \leq f(13mm)$ but $14mm \not\leq 13mm$). While the physical requirement is for such a mapping, it is not a measure.

IV. WEAK MEASURE - DEFINITION

We drop the “only if” part of the measure definition to define a *weak measure*.

Definition 1: A function $f : X \rightarrow \mathfrak{R}$ is a weak (ordinal) measure on a relational structure $\langle X, \preceq \rangle$ if it satisfies the axiom

$$\forall a, b \in X, a \preceq b \rightarrow f(a) \leq f(b) \tag{3}$$

From the identity $P \rightarrow Q \equiv \neg Q \rightarrow \neg P$ we can obtain

$$\forall a, b \in X, f(a) < f(b) \rightarrow a \prec b \tag{4}$$

so what we lose in a weak measure are the features associated with equality. Essentially, a weak measure is a homomorphism rather than an isomorphism so that properties such as

$$\forall a, b \in X, f(a) = f(b) \rightarrow a = b \tag{5}$$

no longer hold.

To avoid possible confusion we will often refer to a measure as a *strong* measure, to distinguish it from the weak measure defined above. In order theoretic terms, a weak measure is a monotonic function into \mathfrak{R} while a strong measure is a strictly monotonic function into \mathfrak{R} .

V. REPRESENTATION THEOREM FOR WEAK MEASURES

The case of spanners is just one example of a weak measure, and we used its “natural” weak measure to justify the definition. Another example might be the differing pay scales for TAFE level teachers and University lecturers. Many higher education institutions in Australia are “dual-sector” - what should a TAFE teacher be paid when lecturing a University course? These are examples of a weak measure between two finite sets. Other common weak measures arise through approximations to a larger set of values, such as giving a person’s age in years (people with different birthdates can have the same age), or rounding off sums of money to the nearest dollar, where many different values are mapped onto the same value.

In general it is necessary to prove the existence of a weak measure and to detail its properties. This we do now. We restrict our attention to finite sets for simplicity although the results should be extensible to countable systems.

Theorem 2: Let X be a finite set and \preceq a total order on X . Then there exists a weak measure f on $\langle X, \preceq \rangle$, which is also a (strong) measure. Further, if g is any weak measure satisfying axiom (3), then there is a function $h : \mathfrak{R} \rightarrow \mathfrak{R}$ from strong to weak measures such that $g = h \circ f$ and h is monotonic.

This differs from Theorem 1 in dropping the strictness requirement from monotonicity.

Proof: Firstly, any measure f is also a weak measure, and there is always a measure for such a relational structure, from Theorem 1. Since f is an isomorphism, set $h = g \circ f^{-1}$. It is straightforward to show that h is monotonic, while the example of spanners shows that it need not be strictly monotonic. ■

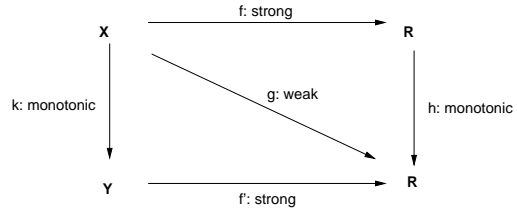


Fig. 1. Weak and strong measures

VI. DIFFERING MEASURE SETS

The size of a spanner measures the width across the spanner flats. The A/F spanners form one set, the metric spanners form another. Each set is ordered in size, and there are strong measures from each set into length scales.

Using a spanner from one set for a bolt from another means that we are performing a mapping from one spanner set to the other. This mapping would be “sensible” in some sense, so that we would choose the “next” size up rather than any other. Alternatively, one might say that the bolt should be chosen to be the next size down! Either way, the mapping should preserve the order of the original set. A mapping $k : X \rightarrow Y$ should satisfy the monotonic condition:

$$\forall x, y \in X, x \preceq_X y \rightarrow k(x) \preceq_Y k(y)$$

A monotonic function induces a weak measure in the following way

Theorem 3: Let X and Y be two sets each with a total order and let $k : X \rightarrow Y$ be monotonic. Then for any total measure f' on Y , $g = f' \circ k$ is a weak measure on X

We can summarise the last two theorems in the commuting diagram [17] of Figure 1. This figure can be read in many ways, since the strong measures f and f' are invertible. If we are *measuring* spanner sizes to find the right one, then we are using the upper triangle $g = h \circ f$. If we are *defining* a relation between spanner sets, then we use the lower triangle $g = f' \circ k$. The commuting diagram links these two views, so that for example, given k then for any strong measures $h = f' \circ k \circ f^{-1}$

VII. EXPLOITING STRUCTURE

The representation theorem for weak ordinal measures shows that there is always a monotonic function from any measure to any weak measure. Being monotonic is not a great deal of help in practice, as there are many, many possibilities. For example, a function that takes all metric bolts to $1/4$ inch is monotonic, but would be useless for turning bolts since a spanner of that size is too small to fit em any metric bolts.

The size of a spanner is measured as a length property. In general lengths are *ratio measures*, so that millimetres can be converted to inches by simple multiplication. However, neither set of spanners form a scale and this property does not appear to be directly useful. However, we can use the fact that they are both measured by lengths to embed them into a larger set of *all* spanner sizes and then use the common length measure to order them.

TABLE II
SPANNER SIZES

| bolt (mm) | spanner (inch) |
|-----------|----------------|
| 9 | 3/8 |
| 10 | 7/16 |
| 11 | 7/16 |
| 12 | 1/2 |
| 13 | 9/16 |
| 14 | 9/16 |

TABLE III
SHOE SIZES

| English | Paris Points | mm |
|---------|--------------|-----|
| 2 | 34 | 227 |
| | | 229 |
| 3 | 35 | 233 |
| | | 237 |
| 4 | 36 | 240 |
| | | 245 |

Let $\langle X, \preceq_X \rangle$ and $\langle Y, \preceq_Y \rangle$ be two relational structures and consider the set $Z = X \cup Y$, along with a precedence relation \preceq on Z which coincides with the precedence relations on X and Y . That is

$$\forall a, b \in X, a \preceq b \equiv a \preceq_X b \tag{6}$$

and

$$\forall a, b \in Y, a \preceq b \equiv a \preceq_Y b \tag{7}$$

$\langle Z, \preceq \rangle$ preserves order on the subsets X and Y while also introducing order between the elements of the two sets.

For example, we can consider X to be the set of metric spanners while Y is the set of A/F spanners and Z is the set of both types of spanner, ordered by size. We now consider the situation where we have no metric spanners, that is, all spanners are mapped to A/F spanners. This implies a function $g : Z \rightarrow Y$ which preserves order as much as possible but also keeps the A/F spanners unchanged:

$$\forall a, b \in Z, a \preceq b \rightarrow g(a) \preceq g(b) \tag{8}$$

$$\forall a \in Y, g(a) = a \tag{9}$$

It is then straightforward to show that for any measure f on $\langle Y, \preceq_Y \rangle$ that $f \circ g$ is a weak measure on $\langle X, \preceq_X \rangle$. Further, if we have an element x of X that is immediately bounded by two elements a and b of Y then either $x = a$ or $x = b$.

This substantially restricts the possible weak measures on X . It includes the measure that maps metric sizes to the next A/F up, the measure that maps metric sizes to the next A/F down, or to any mix of up or down. This is quite satisfactory for a general characterisation of weak measures on two finite subsets of a third set.

If we want to go one stage further to a uniqueness type of theorem, then we can do so by placing further restrictions on the function $g : X \cup Y \rightarrow Y$. For example, to gain uniqueness of sizing spanners upwards:

Theorem 4: Let $g : X \cup Y \rightarrow Y$ satisfy equations (8) and (9). In addition, let g satisfy

$$\forall x \in X, y \in Y, y \prec x \rightarrow g(y) \prec g(x)$$

Then g is unique

This uniqueness theorem allows us to construct tables such as the Table II, which maps metric bolts upwards to A/F spanners, corresponding to the function k of Figure 1:

Similar tables could be constructed for payment scales for TAFE teachers if the policy was to pay upwards for University courses, although management would be more likely to use a similar uniqueness theorem for payments downwards.

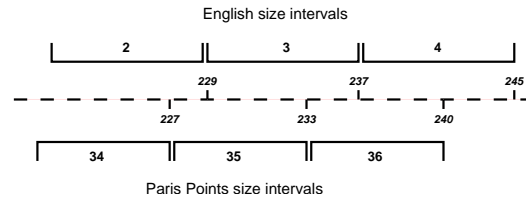


Fig. 2. Shoe size intervals

VIII. RELATION BETWEEN WEAK MEASURES - MOTIVATION

In the previous sections we considered the relation between a strong measure and a weak measure. The case of shoe sizes shows that we need to consider the relation between pairs of weak measures. While Texin [8] gives a table of conversions from one shoe measure to another, he also gives details on how shoe sizes are calculated, which allows a more rigorous analysis.

A small portion of the table of sizes as related to their actual length measure in millimetres is shown in Table III.

Shoe sizes are based on the length of a person's foot. For example, the Paris Points size of 35 corresponds to a foot length of between 228mm and 233mm. Thus we have a function from lengths into the Paris Point sizes $\{34, 35, 36, \dots\}$. This is basically a step function. Using the "obvious" ordering of $34 < 35 < 36 \dots$ it is a monotonic function

The function from foot length to English shoe sizes $\{2, 3, 4, \dots\}$ is also monotonic. For example, any length between 230mm and 237mm maps onto 3.

The size measures the available space in a shoe. The correct size has been chosen if a foot fits into the size, but not into the next smallest one. So a "size three" foot will be bigger than 229mm but smaller than or equal to 237mm. This is shown in interval format in Figure 2.

There is a strong measure from Paris Points sizes into any length scale. For example, in the millimetre scale, a Paris Points size of 35 has a value of 233. Similarly there is a strong measure from English shoe sizes into the millimetre scale, with an English shoe size of 2 having a value of 229. Consequently, from Theorem 2 there is a weak measure from foot lengths to Paris Points lengths, and another from foot lengths to English lengths.

A Continental buyer who is using the internet to buy shoes from England may be faced with the following question: "I normally buy shoes with Paris Points size of 35. What is the corresponding English size?" Conversion of shoe sizes from one scale to another, such as Paris Points to English sizes is then a mapping between two weak measures. However,

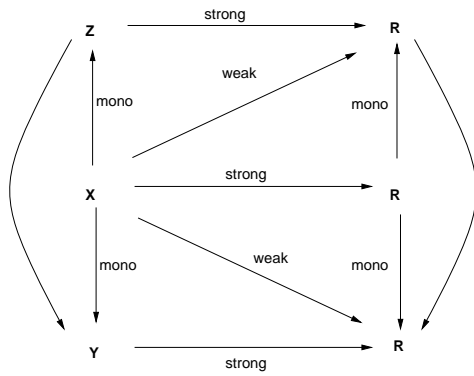


Fig. 3. Mapping between weak measures

this mapping is *not* functional: a Paris Points size of 35 corresponds to a foot length of between 228mm and 233mm and this could map to an English size of 2 or 3.

The question of mapping Paris Points sizes into English sizes can be answered with reference to Figure 3. The top rectangle of this figure is just Figure 1, as is the bottom rectangle. The curved arrow on the left is the mapping between the Paris Points set {34, 35, 36, ...} and the English set {2, 3, 4, ...}, while the curved arrow on the right is the mapping between the two different weak measures.

IX. RELATIONS BETWEEN WEAK MEASURES

Theorem 5: Let g_1 and g_2 be two weak measures on $\langle X, \preceq \rangle$. Then

$$\forall a, b \in X, g_1(a) < g_1(b) \rightarrow g_2(a) \leq g_2(b)$$

Proof: If $g_1(a) < g_1(b)$ then $a \prec b$ and so $g_2(a) \leq g_2(b)$ ■

Thus any two weak measures keep the same order, but possibly with some overlap. The inequalities cannot be tightened. For example, lengths of 229mm and 233mm are both Paris Points of 35 but are of English sizes 2 and 3 respectively.

We first deal with the curved arrow on the right-hand side of Figure 3. We seek to find a version of the monotonic Theorem 2 between two weak measures. Suppose g_1 and g_2 are two weak measures. Then from theorem 2 there is a strong measure f and monotonic functions h_1 and h_2 such that $g_1 = h_1 \circ f$ and $g_2 = h_2 \circ f$. As h_1 is not strictly monotonic, we cannot take its inverse as a *real-valued function*. But we can define an inverse *set-valued function*

$$h_1^{-1}(x) = \{y : h_1(y) = x\}$$

We can then form $h = h_2 \circ h_1^{-1}$ as a set-valued function and then for any $x \in X$

$$g_2(x) \in h \circ g_1(x)$$

We make this clearer by example. A size 36 Paris Points shoe has a length in millimetres of 240, and any foot with a millimetre length between 234 and 240 will have a weak measure of 240. The inverse function h_1^{-1} will be

$$h_1^{-1}(240) = \{x : x \in [234, 240]\}$$

TABLE IV
SHOE SIZE CONVERSION

| English | Direction | Paris Points |
|---------|-----------|--------------|
| 3 | → | 35, 36 |
| 3, 4 | ← | 36 |

h_2 will map these values to the weak English millimetre lengths of {237, 245}. That is, lengths of between 234 and 237 will map to the length of the English shoe size 3 with length 237, while lengths of between 238 and 240 will map to the length of the English shoe size 4, which is 245. That is

$$h_2 \circ h_1^{-1}(240) = \{237, 245\}$$

Having established a set-mapping from one weak measure to another, we now look at its properties. We need a generalisation of pointwise monotonic functions to set-valued functions, which we call *set-monotonic*.

Given a relational structure $\langle X, \preceq \rangle$ with total order \preceq we can create a new relational structure $\langle 2^X, \preceq \rangle$ on sets of elements of X with an induced partial order. For example, we want to be able to say that $\{1, 2, 3, 4\} \leq \{3, 4, 5\}$ but $\{1, 2, 4\} \not\leq \{3, 4, 5\}$ since in the second case 3 is not in the first set.

Definition 2: Given two subsets A and B of X , define $A \preceq B$ if

$$\forall a \in A, \forall b \in B, \quad \text{either } a \preceq b \\ \text{or } a \in B \text{ and } b \in A$$

We can then define a property of *set-monotonic* by

Definition 1: A set-valued function h from X into 2^Y is set-monotonic if

$$\forall x, y \in X, x \preceq y \rightarrow h(x) \preceq h(y)$$

using the induced set relation above. It is strictly set monotonic if the inequality is replaced by a strict inequality.

Then we have

Theorem 6: Let h_1 and h_2 be two monotonic functions and let h be the set-valued mapping $h = h_2 \circ h_1^{-1}$. Then h is set-monotonic

Proof: If $a < b$, then $h_1^{-1}(a) \prec h_1^{-1}(b)$ and then $h_2 \circ h_1^{-1}(a) \preceq h_2 \circ h_1^{-1}(b)$ ■

Now we can turn to the curved arrow on the left-hand side of Figure 3. Once we have defined a mapping between two weak measures then we can use the strong measures f_1 and f_2 to “pull back” the mapping between weak measures to the mapping between the sets Y and Z :

Theorem 7: Given h as above, define $h' : Z \rightarrow Y, h' = f_2^{-1} \circ h \circ f_1^{-1}$. Then h' is a set-monotonic function.

This theorem then allows to write the table of shoe size conversions as in Table IV. Any other mappings between weak measures would have a similar representation.

X. IMPLEMENTATION ISSUES

The focus in representational measurement theory is the existence of, and mappings between, strong measures. The site <http://01conversion.com> allows one to translate values from one scale to another. In a similar vein, the site <http://www.xe.com> allows one to translate money from one scale

(e.g. US dollars) to another (e.g. euros). Bobbit[16] makes such scale transformations more explicit by defining functions of the form

$$Y = (A + B \times X)/(C + D \times X)$$

This will handle many scale transformations but not even all scientific ones: it fails to accommodate the logarithmic scale transformations such as bel to sound intensity.

In this paper we have considered transformations involving weak measures. If we follow a similar pattern to the above, then for strong to weak transformations we would look for a way of specifying the monotonic function h on the right of Figure 1. This is straightforward, but would miss the motivational point. What we would like to specify is the monotonic function on the *left* of Figure 1. The theory requires the right-hand side to justify the left-hand side, but once given the left-hand side we can deduce the monotonic function to the right.

That is, for a weak measure we would like to specify the mapping from one source set to another, as in metric spanners to A/F spanners. This could be simply a lookup table of metric values to A/F values.

In the case of mappings between weak mappings, again we are less interested in the right-hand curved arrow of Figure 3 and more interested in the curved arrow on the left-hand side of the figure. Given the left-hand side function, the right-hand side function follows. Again, this is a simple lookup table but the values are set-values rather than simple values.

Such lookup functions can be easily written in any programming language, and could for example be functional Javascript [18] embedded in HTML pages.

XI. FURTHER WORK

In practice, shoe and feet sizes are not rigid. A person can squash into a shoe that is too small, or pad a shoe that is too large. This suggests that it may be worth extending the theory of measurement to fuzzy sets, in particular to fuzzy intervals. The enticingly titled "Fuzzy Measure Theory" by Wang and Klir[19] is actually about extending the Borel measure theory to fuzzy sets and has nothing to do with representational measurement theory. The paper by Dubois et al[20] states that a thesis by X Wang identified forty different methods of comparing fuzzy intervals. Clearly there will be considerable complexity in extending measurement theory to fuzzy intervals.

Currency transactions appear to be related to scale transformations, in that money in one currency can be converted to another currency. However, such transactions are functional but not invertible: if you transfer from one currency to another and then back again you end up with less than you started with. This is not covered by the theory given here.

XII. CONCLUSION

This paper has discussed a class of "measures" that require a weakened definition of representational measure. We have defined the concept of weak measure and given representation

theorems for weak measures. We have also discussed transformations between strong and weak measures, and between weak and weak measures. Weak measures often arise due to transformations between "incompatible" sets, and we have shown how specification of such transformations can lead to weak measures. Finally, in discussing implementation issues, we have shown that simple mechanisms will suffice.

This work is of particular interest to the internationalisation community, since weak measures may easily arise due to independently derived measurement systems. However, whenever approximations to values are done, weak measures may also occur, and this area may also be important.

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