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Unsteady Water Boundary Layer Flow with Non-Uniform Mass Transfer

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***Abstract*—In the present analysis an unsteady laminar forced convection water boundary layer flow is considered. The fluid properties such as viscosity and Prandtl number are taken as variables such that those are inversely proportional to temperature. By using quasi-linearization technique the non-linear coupled partial differential equations are linearized and the numerical solutions are obtained by using implicit finite difference scheme with the appropriate selection of step sizes. Non-similar solutions have been obtained from the starting point of the stream-wise coordinate to the point where skin friction value vanishes. The effect non-uniform mass transfer along the surface of the cylinder through slot is studied on the skin friction and heat transfer coefficients.

***Keywords*—**Boundary layer, heat transfer, non-similar solution, non-uniform mass, unsteady flow.

I. Introduction

UNSTEADY boundary layer performs an important role in several engineering problems like start-up process and periodic fluid motion and in the area of convective heat and mass transfer. In unsteady flow problems, the boundary layer comprise different behavior owing to the inclusion of time dependent terms which influences in the pattern of fluid motion and boundary layer separation also creates complexity in the solution system discussed in [1], [2] and [3]. Many researchers discussed about two dimensional unsteady flow problems for different free stream velocity and flow configurations in [3], [4] and [5]. But their solutions are mainly restricted to similarity or self similar one because of the consequence of the mathematical difficulties involved in achieving the non-similarity solutions. Also the non-similar solutions have been obtained for the unsteady flows by some researchers in [6], [7] and [8]. Fluid properties often vary significantly with respect to the temperature in the circumstances where large or moderate temperature gradients exists across the fluid medium. More recently different studies have been reported for variable fluid properties such as viscosity and Prandtl number in [9], [10]. Mass transfer from a wall slot into the boundary layer is of interest for various prospective applications together with thermal protections, energizing the inner portion of boundary layer in adverse pressure gradient and skin friction reduction on control surfaces. The effect of uniform mass transfer applied in an unsteady forced convection boundary layer flow with variable viscosity and Prandtl number is studied by Eswara et.al in [11]. Despite of uniform mass transfer, finite discontinuities arise at the leading and trailing edges of the slot and those can be evaded by choosing a non-uniform mass transfer distribution along a stream-wise slot and it has been discussed by Minkowycz et al. in [12]. Recently Saikrishnan and Roy [13] have been reported about the influence of non-uniform mass transfer on steady boundary layer flows over a sphere and a cylinder with variable viscosity and Prandtl number. Many investigators discussed about the non-uniform mass transfer over different geometries in [14], [15], [16].

The outcome of the literature leads to study the simultaneous effects of non-uniform slot suction and temperature dependent fluid properties on unsteady water boundary layer flow over a cylinder. The non-similar solutions have been obtained starting from the origin of the stream-wise coordinate to the point where the skin friction value vanishes. These studies are useful for an engineer or a designer in determination of the surface heat requirements to stabilize the laminar water boundary layer flow over a cylinder. Procedure for Paper Submission

II. Analysis

Consider an unsteady, incompressible laminar non-similar water boundary-layer flow with temperature-dependent viscosity and Prandtl number over a two dimensional cylinder (see Fig. 1). U is the steady – state velocity at the edge of the boundary layer; u_∞ is free stream velocity and T_∞ is the free stream temperature. The x-axis runs along the free stream direction and the y-axis is perpendicular to it. Let the stream velocity and mass transfer (injection/suction) vary with the axial distance (x) along the surface and with the time (t). Assume that, the temperature difference between the wall and the free stream is small $(< 40^O\ C)$. In spite of the variation of the both density (ρ) and the specific heat (C_P) with temperature by less than 1% in the above mentioned temperature range, they are taken as constants [13].

The viscosity and the Prandtl number have been taken as given below [17], [18] and [19].

$$\mu = \frac{1}{(b_1+b_2T)} \quad \text{and} \quad Pr = \frac{1}{(c_1+c_2T)} \qquad (1)$$

$$b_1 = 53.41, b_2 = 2.43,\ c_1 = 0.068 \text{ and } c_2 = 0.004 \qquad (2)$$

The numerical data, utilized for these correlations, are taken from Ref. [19].

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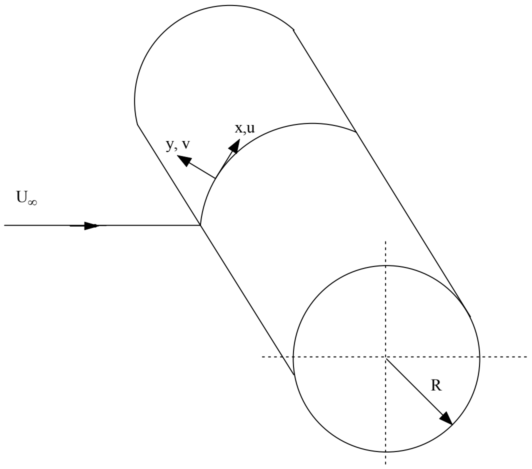

Fig1: Flow model and coordinate system

The governing equations of momentum and thermal boundary layers of this problem are in [20]

$$u_x + v_y = 0 \tag{3}$$

$$u_t + uu_x + vu_y = (u_e)_t + u_e(u_e)_x + \rho^{-1}\left(\mu u_y\right)_y \tag{4}$$

$$T_t + uT_x + vT_y = \rho^{-1}\left[(\mu/Pr)T_y\right]_y + \left(\mu/\rho c_p\right)\left(u_y\right)^2 \tag{5}$$

The initial and boundary conditions are

$$u(x,y,0) = u(x,y), \quad v(x,y,0) = v(x,y), \quad T(x,y,0) = T(x,y) \tag{6}$$

$$u(x,0,t) = 0, v(x,0,t) = v_w(x,t), T(x,0,t) = T_w(x,t)$$

$$u(x,\infty,t) = u_e(x,t), T(x,\infty,t) = T_\infty. \tag{7}$$

where u and v are velocity components in the x and y directions respectively, R is radius of the cylinder, μ is variable viscosity, Pr is variable Prandtl number, u_e is the potential flow velocity at the edge of the boundary layer, $v_w(x,t)$ denotes the surface mass transfer distribution.

Choose a stream function $\psi(x,y,t) = u_\infty R\left(\frac{2\xi}{Re}\right)^{1/2}\phi(t^*)f(\xi,\eta,t^*)$ with the non-similar variables $\xi = \int_0^x \left(\frac{U}{u_\infty}\right) d\left(\frac{x}{R}\right)$, $\eta = \left(\frac{Re}{2\xi}\right)^{1/2}\left(\frac{U}{u_\infty}\right)\left(\frac{y}{R}\right)$ and define the dimensionless temperature and time variables $G = \frac{T-T_\infty}{T_w-T_\infty}$, $t^* = (3/2)Re(\mu_e/\rho R^2)t$ respectively. $\phi(t^*)$ is a time function with a continuous first derivative for $t^* \geq 0$, representing the unsteadiness in the free stream. The continuity equation (3) is identically satisfied. The remaining governing eqns. (4)-(5) can be made dimensionless and become,

$$\left(NF_\eta\right)_\eta + \phi\left[fF_\eta + \beta(1-F^2)\right] - P\left[F_{t^*} - \phi^{-1}\phi_{t^*}(1-F)\right] = 2\xi\phi\left(FF_\xi - f_\xi F_\eta\right) \tag{8}$$

$$\left(NPr^{-1}G_\eta\right)_\eta + \phi fG_\eta + NEc(u_e/u_\infty)^2F_\eta^2 - PG_{t^*} = 2\xi\phi\left(FG_\xi - f_\xi G_\eta\right) \tag{9}$$

with boundary conditions

$$F(\xi,0,t) = 0, \quad G(\xi,0,t) = 1,$$
$$F(\xi,\infty,t) = 1, \quad G(\xi,\infty,t) = 0. \tag{10}$$

where

$$N = \mu/\mu_\infty = \frac{(b_1+b_2T_\infty)}{(b_1+b_2T)} = \frac{1}{(1+a_1G)}$$

$$Pr = \frac{1}{(c_1+c_2T)} = \frac{1}{(a_2+a_3G)}$$

$$a_1 = \frac{b_2\Delta T_w}{(b_1+b_2T_\infty)}, \ a_2 = c_1 + c_2T_\infty, \ a_3 = c_2\Delta T_w$$

$$\Delta T_w = (T_w - T_\infty), \ u = u_eF, \ u_e = U\phi(t^*)$$

$$v = (2\xi Re)^{-1/2}U\phi\left[f + 2\xi f_\xi + (\beta-1)\eta F\right]$$

$$\beta(\xi) = \left(2\xi/U\right)\left(dU/d\xi\right), \ P = 3\xi\left(u_\infty/U\right)^2,$$

$$Ec = u_\infty^2/\left[c_p(\Delta T_w)\right], \ f = \int_0^\eta F d\eta + f_w$$

$$f_w = -\xi^{-1/2}(Re/2)^{1/2}\phi^{-1}\int_0^x (v_w/u_\infty)d(x/R) \tag{11}$$

Here f_w is the stream function value at wall. For a circular cylinder, the unsteadiness as well as non-similarity both is due to the external velocity at the edge of the boundary layer and the normal component of the velocity at the surface. The free-stream velocity distribution at the edge of the boundary layer for the case of two-dimensional flow over a circular cylinder can be expressed as,

$$u_e/u_\infty = 2\sin\bar{x}\,\phi(t^*), \ U/u_\infty = 2sin\bar{x}, \ \bar{x} = x/R \tag{12}$$

The expressions for $\xi, \beta, f_w, P, C_f, N$ are respectively given by,

$$\xi = 2(1-\cos\bar{x}), \quad \beta = 2\cos\bar{x}/(1+\cos\bar{x}),$$
$$P = 3/[2(1+\cos\bar{x})] \tag{13}$$

$$f_w = \begin{cases} 0, & \bar{x} \leq \bar{x}_0 \\ A\phi^{-1}(2Q_1)^{-1/2}C(\bar{x},\bar{x}_0), & \bar{x}_0 \leq \bar{x} \leq \bar{x}_0^*, \\ A\phi^{-1}(2Q_1)^{-1/2}C(\bar{x}_0^*,\bar{x}_0), & \bar{x} \geq \bar{x}_0^*, \end{cases} \tag{14}$$

where the function

$$C(\bar{x},\bar{x}_0) = 1 - \cos\{\omega^*(\bar{x}-\bar{x}_0)\},$$
$$Q_1 = 1-\cos\bar{x}, Q_2 = 1+\cos\bar{x}.$$

here v_w is taken as,

$$v_w = \begin{cases} -u_\infty\left(\frac{Re_L}{2}\right)^{-1/2}A\,\omega^*\sin\{\omega^*(\bar{x}-\bar{x}_0)\}, & \bar{x}_0 \leq \bar{x} \leq \bar{x}_0^* \\ 0, & \bar{x} \leq \bar{x}_0 \text{ and } \bar{x} \geq \bar{x}_0^* \end{cases} \tag{15}$$

where ω^* and $\bar{x}_0$ are the free parameters which determine the slot length and slot locations respectively. The function $v_w(\bar{x})$ is continuous for all values of $\bar{x}$ and it has non-zero values only in the intervals $[\bar{x}_0, \bar{x}_0^*]$. The reason for taking such a function is that it allows the mass transfer to change slowly in the neighborhood of the leading and the trailing edges of the slots. The parameter $A > 0$ refer that there is suction. It is convenient to express eqns. (8) and (9) in terms of $\bar{x}$ instead of ξ. Equation (12) gives the relation between ξ and $\bar{x}$ as

$$\xi\frac{\partial}{\partial\xi} = B(\bar{x})\frac{\partial}{\partial\bar{x}} \tag{16}$$

where, $B(\bar{x}) = \tan\frac{\bar{x}}{2}$. Substituting (16) into equations (8) and (9), we obtain

$$\left(NF_\eta\right)_\eta + \phi\left[fF_\eta + \beta(1-F^2)\right] - P\left[F_{t^*} - \phi^{-1}\phi_{t^*}(1-F)\right] = 2B(\bar{x})\phi\left(FF_{\bar{x}} - f_{\bar{x}}F_\eta\right) \tag{17}$$

$$\left(NPr^{-1}G_\eta\right)_\eta + \phi fG_\eta + NEc(u_e/u_\infty)^2F_\eta^2 - PG_{t^*} = 2B(\bar{x})\phi\left(FG_{\bar{x}} - f_{\bar{x}}G_\eta\right) \tag{18}$$

The boundary conditions become

$$F(\bar{x},0,t) = 0, \quad G(\bar{x},0,t) = 1$$
$$F(\bar{x},\infty,t) = 1, \quad G(\bar{x},\infty,t) = 0. \tag{19}$$

The skin-friction coefficient at the wall can be expressed in the form

$$C_f(Re)^{1/2} = 4Q_2(Q_1)^{1/2}\phi(t^*)\left(F_\eta\right)_w \tag{20}$$

where, $C_f = \frac{2\left[\mu\left(\frac{\partial u}{\partial y}\right)\right]_w}{\rho u_\infty^2}$ and $N_w = \frac{1}{1+a_1 G_w} = \frac{1}{1+a_1}$ = constant.
Similarly, the heat transfer coefficient in terms of Nusselt number can be written as

$$Nu(Re)^{-1/2} = 2^{1/2}\cos(\bar{x}/2)\left(G_\eta\right)_w \tag{21}$$

where, $Nu = \frac{R\left(\frac{\partial T}{\partial y}\right)_w}{(T_\infty - T_w)}$.

III. Results and Discussion

The set of non-linear coupled partial differential equations (17) and (18) with boundary conditions (19) have been solved numerically using an implicit finite difference scheme in combination with the Quasilinearization technique. It is a generalization of the Newton-Raphson approximation technique in functional space. The detailed explanation about Quasilinearization technique is given in [21]. The grid sizes $\Delta\eta, \Delta\bar{x}, \Delta t^*$ have been optimized and taken as, $\Delta\eta = 0.01$, $\Delta t^* = 0.01$ throughout the computation. In the $\bar{x}$ direction, $\Delta\bar{x} = 0.01$ for $\bar{x} \le 1.5$ and $\Delta\bar{x} = 0.0001$ for $\bar{x} > 1.5$ in order to ensure the convergence of the numerical solution to the exact solution. The value of η_∞ (i.e. the edge of the boundary layer) is taken as 6.0. A convergence criteria based on the relative difference between the current and previous iteration values is employed. When the difference reaches less than 10^{-4} the solution is assumed to have converged and the iterative process is terminated.

Computations have been carried out for various values of time $t^*(0 \le t^* \le 2)$ and mass transfer parameter $A(0 \le A \le 1)$. In all numerical computations the variable viscosity and Prandtl number is considered. Skin friction and heat transfer parameters $\left((F_\eta)_w, (G_\eta)_w\right)$ have been calculated for both constant and variable fluid properties of the fluid flow at different times $t^* = 0$ and $t^* = 2$. With the purpose of getting validity of the present method, at time $t^* = 0$, the results are compared with the results of Surma Devi and Nath [8] and Eswara and Nath [11]. It shows that the results are in very good agreement.

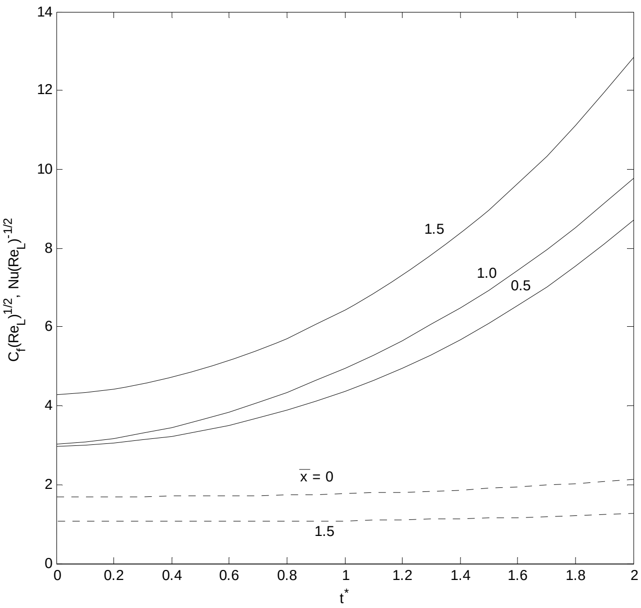

Fig.2 Effect of variable fluid properties (_______) and constant fluid properties (..........) on skin friction coefficient at times $t^* = 0$ and 1, with $\phi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25, Ec = 0, Pr = 0.7$.

The effect of variable fluid properties and constant fluid properties on skin-friction coefficient $C_f(Re_L)^{1/2}$ is studied at different times $t^* = 0$ and $t^* = 1$. It is observed that the effect of variable fluid properties is to decrease the skin friction and the point of vanishing skin friction moves downstream. At time $t^* = 0$ the present results are compared with the results of Saikrishnan and Roy [13] and Eswara and Nath [11] those are matching well.

The effects of skin friction and the heat transfer coefficients $\left(C_f(Re_L)^{1/2}, Nu(Re_L)^{-1/2}\right)$ on time at different stream-wise locations have been showed in Fig. 2. It is observed that as time increases, both $C_f(Re_L)^{1/2}$ and $Nu(Re_L)^{-1/2}$ increased. The response is the same at all stream-wise locations. The effect of time is more on skin friction coefficient comparing to heat transfer coefficient because of the velocity gradient is a function of time (t^*) which is directly related to the momentum equation.

The velocity and temperature profiles have been shown in Fig. 3 and it has been found that as time increases and $A \ge 0$, the thickness of both momentum and thermal boundary layer decrease. Thus, it is found that, unsteadiness decreases the thickness of both momentum and thermal boundary layer when the free stream is accelerating. The reason is that suction decreases the viscosity causes increase in fluid velocity and hence decrease the thickness of both velocity and temperature boundary layers.

The effect of suction $(A \ge 0)$ on $C_f(Re_L)^{1/2}$ and $Nu(Re_L)^{-1/2}$ at different times $t^* = 0$ and $t^* = 2$ have been shown in Fig. 4. It is found that as time increases, both the skin friction $C_f(Re_L)^{1/2}$and the heat transfer $Nu(Re_L)^{-1/2}$ increase with suction ($A \ge 0$). At time $t^* = 0$, when there is no mass transfer the skin friction value attains maximum value $\left(C_f(Re_L)^{1/2} = 4.253\right)$ at $\bar{x} = 1.0$ whereas at time $t^* = 2$, it attains maximum value $\left(C_f(Re_L)^{1/2} = 12.8\right)$ at $\bar{x} = 1.0556$. When $A \ge 0$, the skin friction and the heat transfer coefficients increase as the slot starts and attain their maximum values before the trailing edge of the slot. Finally, the values of $C_f(Re_L)^{1/2}$and $Nu(Re_L)^{-1/2}$ decrease from their maximum values and $C_f(Re_L)^{1/2}$ reaches zero but $Nu(Re_L)^{-1/2}$ remain finite at time $t^* = 0$ whereas both $\left(C_f(Re_L)^{1/2} \text{ and } Nu(Re_L)^{-1/2}\right)$ are remain finite at time $t^* = 2$. Suction causes the point of vanishing skin friction to move downstream.

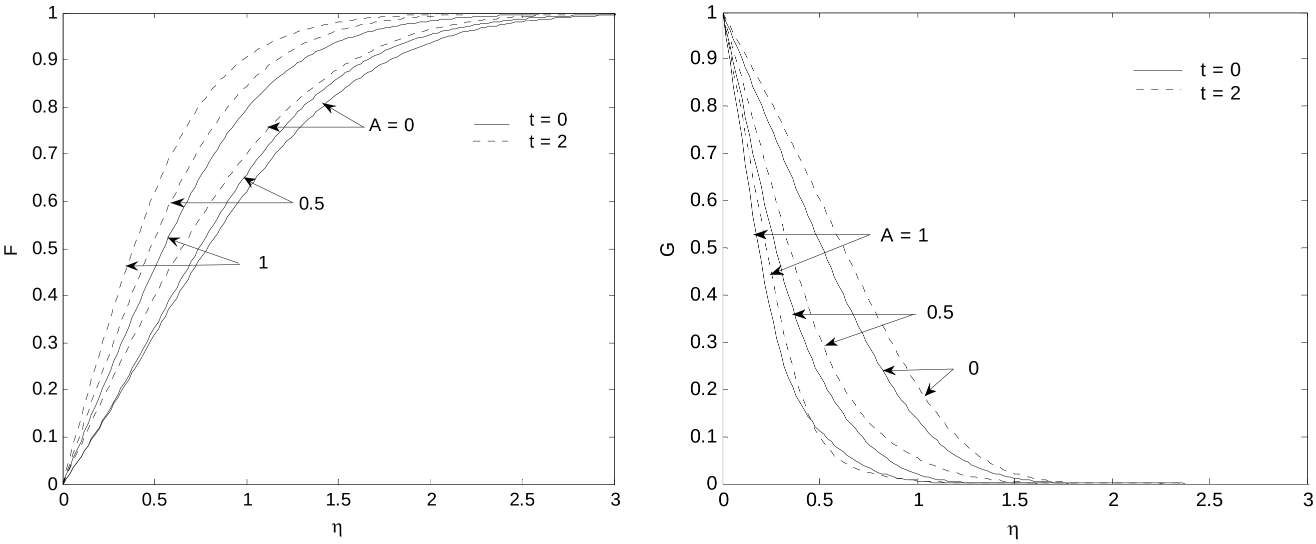

Fig.3 Effect of mass transfer parameter ($A \geq 0$) on (a) Velocity profile and (b) Temperature profile for $t^* = 0$ (..........) and $t^* = 2$ (_______) with $\phi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25$, $Ec = 0$, $\bar{x} = 1.75, T_\infty = 18.7^0C, \Delta T_w = 10.0^0C, w^* = 2\pi$, variable viscosity and Prandtl number. Slot position [1.2 – 1.7].

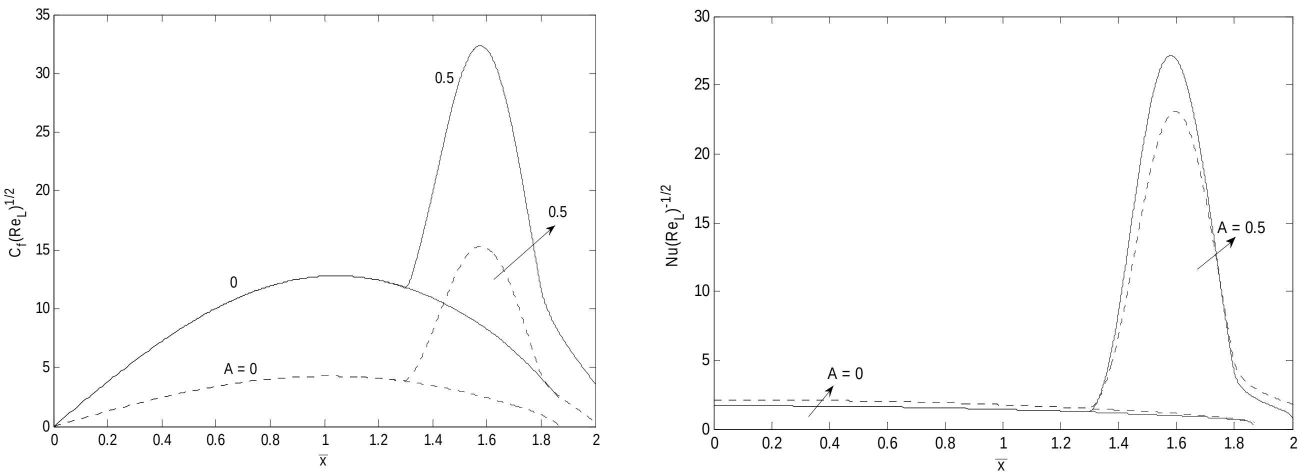

Fig.4. Effect of mass transfer parameter ($A > 0$) on (a)Skin friction coefficient and (b) Heat transfer coefficient at times $t^* = 0$ (------------) and $t^* = 2$ (________) with $\phi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25$, $Ec = 0$, $T_\infty = 18.7^0C$, $\Delta Tw = 10.0^0C, w^* = 2\pi$, variable viscosity and Prandtl number. Slot position [1.3-1.8].

IV. Conclusion

Non-similar solutions of an unsteady water boundary layer flow over cylinder with non-uniform slot suction (injection) have been obtained starting from the origin of the stream wise co-ordinate to the point of vanishing skin friction. Non uniform slot suction and moving the slot downstream delay the occurrence of reverse flow. At time $t^* = 0$, the percentage of the downstream movement of the separation point is 7.5% whereas at time $t^* = 2$, the percentage of the downstream movement of the reverse flow regime is 7.5%, when the mass transfer ($A = 0.5$) is applied into the flow. It is found that suction decreases the thickness of both momentum and thermal boundary layer as time increases. As time increases, there is a significant increase in the skin friction and heat transfer parameters. The effect of unsteadiness is more significant on the skin friction as compared to the heat transfer.

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