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Applications of Conic Optimization and Quadratic Programming in the Investigation of Index Arbitrage in the Thai Derivatives and Equity Markets

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***Abstract*—**This research seeks to investigate the frequency and profitability of index arbitrage opportunities involving the SET50 futures, SET50 component stocks, and the ThaiDEX SET50 ETF (ticker symbol: TDEX). In particular, the frequency and profit of arbitrage are measured in the following three arbitrage tests: (1) SET50 futures vs. ThaiDEX SET50 ETF, (2) SET50 futures vs. SET50 component stocks, and (3) ThaiDEX SET50 ETF vs. SET50 component stocks are investigated. For tests (2) and (3), the problems involve conic optimization and quadratic programming as sub-problems. This research is first to apply conic optimization and quadratic programming techniques in the context of index arbitrage and is first to investigate such index arbitrage in the Thai equity and derivatives markets. Thus, the contribution of this study is twofold. First, its results would help understand the contribution of the derivatives securities to the efficiency of the Thai markets. Second, the methodology employed in this study can be applied to other geographical markets, with minor adjustments.

***Keywords*—**Conic optimization, Equity index arbitrage, Execution lags, Quadratic programming, SET50 index futures, ThaiDEX SET50 ETF, Transaction costs

I. Introduction

An index arbitrage is a financial transaction involving a purchase and/or sale of certain securities linked to a stock index whereby positive profits are earned with no risk. An index arbitrage is possible when there are temporary discrepancies in the prices of the securities. This study looks at three classes of securities, namely the SET50 component stocks, the SET50 index futures contracts, and the ThaiDEX SET50 ETF, henceforth denoted as TDEX for brevity.

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This study investigates index arbitrage opportunities occurring from the three major events: (1) the SET50 futures are overpriced (underpriced) relative to the price of the TDEX. In this case, an arbitrageur can buy (sell) the TDEX and simultaneously sell (buy) the futures, with zero initial investment, and take the reverse position when the prices are corrected for a guaranteed profit. Also, (2) when the price of SET50 futures is too low compared to the price of the SET50 index, an arbitrageur can sell the SET50 component stocks and buy the futures and take the opposite position when prices are corrected. Lastly, (3) when the price of the TDEX is too low compared to the prices of the SET50 index, an arbitrageur can sell the SET50 component stocks and buy the TDEX and take the reverse position when prices are corrected. To study arbitrage in the first event, arbitrage frequency and profit are measured from the mispricing of the SET50 futures against the TDEX, hereafter referred to as Test 1. To study arbitrage in the second event, frequency and profit are measured from the mispricing of the SET50 futures against the SET50 component stocks, hereafter referred to as Test 2. Lastly, in order to study arbitrage in the third event, frequency and profit are measured from the mispricing of the TDEX against the SET50 component stocks, henceforth referred to as Test 3.

While the investigation of index arbitrage in the first test involves only the TDEX and the SET50 futures and are numerically trivial to identify, arbitrage opportunities in the second and third tests involving the SET50 component stocks are not. Numerical optimization techniques are needed in the latter case. In particular, conic optimization and convex quadratic programming are used in formulating the research questions in the second and third tests.

The use of conic optimization and quadratic programming in an index arbitrage study differentiates this research from previous works on index arbitrage. Moreover, this research is first to investigate the opportunities and profit of index arbitrage in the Thai markets.

This research paper is organized as follows. Section II reviews literatures on the applications of optimization techniques in finance and the investigation of index arbitrage. Section III introduces the three classes of financial securities involved in this study. Section IV discusses about the data and methodology employed in this study. Section V presents the results of the study. Section VI concludes the paper.

II. Literature Review

The theory of portfolio optimization, also known as mean-variance optimization, was introduced by Harry Markowitz in 1950. The main idea of Markowitz's portfolio optimization problem is that a portfolio's return (mean) is a function of its risk (variance). The portfolio risk is measured from historical data while the portfolio return is the expected future return on the portfolio. The portfolio optimization problem can be mathematically set up in several ways. As summarized in [5], various formulations of portfolio optimization problem include 1) maximizing expected return for a given level of risk, 2) minimizing risk for a given value of expected return, 3) minimizing risk and maximizing expected return using a specified risk aversion factor, 4) minimizing risk regardless of the expected return, and 5) maximizing the expected return regardless of the risk.

In an investigation of index arbitrage, it is essential to know whether price discrepancy is present in each relevant security. Thus, a benchmark price is needed to compare with the actual, observed price. The benchmark price is usually derived from the theory of pricing corresponding to the security.

The price at time t of a stock index futures contract that will expire at time T, denoted as $F(t,T)$, can be expressed according to the cost-of-carry model as:

$$F(t,T) = S(t)e^{\frac{(r-\delta)(T-t)}{365}} \quad (1)$$

or

$$F(t,T) = S(t)e^{\frac{r(T-t)}{365}} - D(t,T) \quad (2)$$

Equation (1) is the cost-of-carry relationship for dividend expressed in continuously compounded annual yield δ. Equation (2) is the cost-of-carry relationship where the dividend paid between time t and T, denoted as $D(t,T)$, is discrete.

On another note, the price at time t of stock index futures that will expire at time T, $F(t,T)$, can be expressed relatively to the price of the exchange-traded fund, $ETF(t)$, as:

$$F(t,T) - ETF(t)e^{\frac{(r-\delta)(T-t)}{365}} \quad (3)$$

or

$$F(t,T) - ETF(t)e^{\frac{r(T-t)}{365}} - D(t,T) \quad (4)$$

Equation (3) is the futures-ETF relationship, adapted from the cost-of-carry model in equation (1), where the ETF pays continuously compounded annual yield δ. Equation (4) is the futures-ETF relationship in which the dividend paid between time t and T, denoted as $D(t,T)$, is discrete.

Reference [1] studies the profitability of index arbitrage involving Hong Kong's Hang Seng index futures, call and put options during their first year of trading, i.e., from October 1, 1993 to June 30, 1994. The study employs the futures-put-call parity relationship to determine mispricing of the futures with respect to the options. Unlike many studies that use transaction price data, this study uses (intraday) bid-ask prices in investigating index arbitrage profit. The author indicates that arbitrage frequency and profit are generally overstated when transaction price data are used as opposed to bid-ask prices. The relationship between the sizes of bid-ask spreads in the futures and options markets and the likelihood of arbitrage opportunity is observed.

Reference [3] uses transaction price data to investigate the profitability of index arbitrage involving the MMI index of the American Stock Exchange and the MMI futures traded on the Chicago's Board of Trade (CBT). The study period covers the first two years after the inception day of the MMI futures, that is, from July 24, 1984 to August 31, 1986.

Reference [4] employs the cost-of-carry model in the study of index arbitrage involving the SFE SPI 200^{TM} Index futures of the Australian Stock Exchange. The study also employs an autoregressive time series model to determine the relationship between price volatility and the size of mispricing. Results suggest that exogenous and endogenous price volatility have a positive correlation with the size of mispricing. The study uses intraday data during the four-year period from January 1, 2002 to December 15, 2005. The SFE SPI 200^{TM} Index futures contract was introduced in May 2000.

Reference [6] tests for index arbitrage between the S&P500 futures and the S&P500 index, using intraday transaction price data. While using the traditional cost-of-carry model to test for mispricing, the study goes beyond other arbitrage literatures by incorporating into its test the marking-to-market effect from trading futures. The sample period covers March 18, 1983 to December 17, 1987. The S&P500 futures started trading on April 21, 1982.

Reference [9] adapts the cost-of-carry model to test for mispricing in the S&P500 futures relative to the Standard and Poor's Depository Receipts (SPDRs), the exchange-traded fund that tracks the performance of the S&P500 index. The test is divided into periods of high and low volatilities, spanning approximately four years from March 1998 to October 2002. The SPDRs was introduced in 1993.

III. The three Classes of financial securities and Their Transaction costs

The three classes of financial securities involved in this study are the SET50 component stocks, the SET50 futures, and the TDEX.

The SET50 index is the stock index calculated from the top 50 stocks (in terms of market capitalization, liquidity, and compliance with requirements regarding the distribution of shares to minor shareholders) that are listed in the Stock Exchange of Thailand (SET). In calculating the SET50 index, each of the 50 stocks is given a weight proportional to its market capitalization, i.e., its price per share multiplied by total number of shares outstanding. The SET50 index is considered an indication of the Thai equity market performance. That is, the higher the SET50 index, the better is the market performance.

The SET50 futures are contracts in which the buyer agrees to purchase the SET50 index for a fixed price at a specified time (called expiration date) in the future. The SET50 futures are used to manage market risks as well as to generate profits from market movement predictions. A futures contract has the notional value equal to 1000 multiplied by the referenced index value (1000 is called the futures contract multiplier). For instance, if the referenced SET50 index value is 500, a SET50 futures contract has notional value of THB 500,000. To purchase a SET50 futures contract, investors are required to put in cash equal to 10 percent of the notional amount, that is, THB 50,000 in this case. Thus, futures contracts allow investors to obtain higher profit and incur higher loss (in percentage point) for the initial investment amount. However, investors incur substantially lower transaction costs in trading the SET50 futures than trading the SET50 component stocks. The SET50 futures were launched on April 28, 2006.

The ThaiDEX SET 50 ETF or TDEX is an exchange-traded fund, that is, open-ended fund traded via the Stock Exchange of Thailand, similar to ordinary equity shares. At least 65 percent of the fund's net asset value (NAV) is invested in equity, especially those comprising the SET50 index while the rest of the fund is invested in debt securities, money market, and/or money deposit. The net asset value (NAV) is the total assets of the fund minus its liabilities. The TDEX is designed so that its performance tracks the performance of the SET50 index with tracking error less than or equal to 1 percent annually. The TDEX is purchased and sold by unit investment, whose price is determined by the market and is approximately equal to one hundredth of the value of the SET50 index. That is, if the SET50 index is at 500, one investment unit would sell at approximately 5 Baht. The TDEX may trade at a price higher than the NAV per unit of the fund (trades at a premium) or lower than the NAV per unit of the fund (trades at a discount). The TDEX was launched on September 6, 2007, approximately 18 months after the SET50 futures was launched. Both the SET50 futures and the TDEX represent ways investors can make profit based on the market (SET50 index) performance.

In a stocks transaction, investors are charged with a commission fee between 0.15-0.25 percent of the matched value (price). To trade a SET50 futures contract, an investor incurs transaction costs that include exchange and clearing fees (THB 50 per contract or approximately 0.01 percent of contract notional value if the index level is at 500) and brokerage commission (THB 250-450 per contract or approximately 0.06-0.1 percent of notional value if the index level is at 500). In the case of TDEX, investors pay front-end fee (0.05 percent of NAV per unit), back-end fee (0.05 percent of NAV per unit), brokerage fee (0.1-0.2 percent of NAV per unit), management fee (0.4 percent of NAV per unit), trustee fee (0.015-0.025 percent of NAV per unit), and registrar fee (0.01 percent of NAV per unit).

In total, a one-side stock trade incurs 0.15-0.25 percent of traded value (round-trip fee = 0.3-0.5 percent). A one-side SET50 futures trade incurs THB 300-500 per contract or 0.07-0.11% of notional value if the index level is at 500 (round-trip fee = THB 600-1,000 per contract or 0.14-0.22 percent of the notional value). A one-side TDEX trade incurs 0.575-0.7 percent of net asset value per unit (NAV per unit). Since to exit the TDEX position incurs only the exit (back-end) fee, a round-trip transaction cost incurred to a TDEX investor is approximately 0.625-0.75 percent of NAV.

IV. Data and Methodology

Daily prices of the three classes of securities from the Stock Exchange of Thailand (SET) and Thailand Futures Exchange (TFEX) databases are collected. The data include (1) the daily closed price series of SET50 futures and the SET50 component stocks from April 2006 (inception month of SET50 futures) to most recent, (2) daily closed price series of the TDEX from September 2007 (inception month of the ETF) to most recent, and (4) daily closed price series of the SET50 index from April 2006 to most recent.

In total, 52 securities (50 stocks, SET50 futures, and the TDEX) are considered in this study. For the tests involving the TDEX, 363 daily prices are observed from September 6, 2007 to February 26, 2009.

A dividend of THB 0.14 per investment unit was paid to the TDEX holders on May 9, 2008 and dividends of THB 0.2 and 0.06 per unit were paid on June 19, 2009 and October 9, 2009, respectively.

The average of the 1-month Treasury bill yields is used to represent the risk-free rate, which is the rate at which an arbitrageur can borrow and lend. The average yields were 3.19 percent in 2008 and 1.2 percent in 2009.

The frequency of arbitrage opportunities and size of arbitrage profit are measured in the following three tests: (1) SET50 futures vs. ThaiDEX SET50 ETF, (2) SET50 futures vs. SET50 component stocks, and (3) ThaiDEX SET50 ETF vs. SET50 component stocks. The study is divided into two time periods, using the inception month of the TDEX as division point: (1) from April 2006 to August 2007 and (2) from September 2007 to February 2009. The results in the two time periods are compared in order to determine the effects of the TDEX on market efficiency. Market efficiency is inversely proportional to the frequency of arbitrage opportunities and the size of arbitrage profit. In other words, the smaller the frequency and size of index arbitrage, the more efficient the equity and derivatives markets are as a whole.

Table 1 helps visualize how the three arbitrage tests are implemented and categorized by security types and time periods.

TABLE I
CATEGORIZATION OF ARBITRAGE TESTS BY SECURITY TYPES AND TIME PERIODS

Arbitrage Test	Time Period	
	Apr 06- Aug 07 (17 months)	Sep 07- Feb 09 (16 months)
(1)	(1.1) SET50 futures vs. SET50 index (control test)	(1.2) SET50 futures vs. SET50 index (control test)
		(1.3) SET50 futures vs. TDEX[1]
(2)	(2.1) SET50 futures vs. SET50 component stocks (control test)	(2.2) SET50 futures vs. SET50 component stocks (control test)
(3)		(3.2) TDEX vs. SET50 component stocks

In order to gain insight into whether the introduction of the TDEX contributes to the efficiency of the Thai derivatives and equity markets, the frequency of arbitrage opportunities and the size of arbitrage profit are compared as follows:

(1) Compare (1.1) to (1.2) and (1.2) to (1.3)
(2) Compare (2.1) to (2.2) and (2.2) to (3.2)

It can be concluded that the introduction of the TDEX leads to more efficient markets if the following conditions hold true:

(1) the frequency of arbitrage opportunities and the size of arbitrage profit in (1.2) are smaller than in (1.1) *and* the frequency of arbitrage opportunities and the size of arbitrage profit in (1.3) are smaller than in (1.2)
(2) the frequency of arbitrage opportunities and the size of arbitrage profit in (2.2) are smaller than in (2.1) *and* the frequency of arbitrage opportunities and the size of arbitrage profit in (3.2) are smaller than in (2.2)

A. Test 1: SET50 futures vs. ThaiDEX SET50 ETF

For the first test, arbitrage between SET50 futures and the ThaiDEX SET50 ETF, the signal that arbitrage opportunity exists is adapted from [9] as follows:

$$\varepsilon = \left| F(t,T) - ETF(t)e^{\frac{(r-\delta_1)(T-t)}{365}} \right| - TC(t) \geq 0, \qquad (1)$$

where ε is the magnitute of mispricing or price discrepancy (that will likely lead to arbitrage if greater than zero), t is time at which arbitrage opportunity is observed, T is expiration time of SET50 futures contract, $F(t,T)$ is the price at time t of SET50 futures contract that will expire at time T, $ETF(t)$ is the price at time t of the TDEX, δ_1 is the expected annual dividend yield to be paid to the holder of the TDEX, and $TC(t)$ is the sum of transaction costs incurred in the arbitrage at time t.

Because both the SET50 futures and the TDEX track the SET50 index, the fair price of the futures should be very close, if not equal to the price of the ETF, less dividends paid to the ETF holder. If the difference at time t between the prices of the SET50 futures and the ETF is sufficiently large, arbitrageurs can sell the overpriced security and simultaneously buy the underpriced security, then do the reverse trade as the overpriced security becomes underpriced and the underpriced becomes overpriced. In reality, transaction costs and execution lags must be taken into account in an arbitrage test. The price difference between the securities must more than cover the total transaction costs for an arbitrageur to make profit. As execution lags may cause the arbitrage observed at the time an order is placed to not prevail at the time it is executed, they must be incorporated in an arbitrage test. This research will take into account such execution lags by measuring profit at the execution time, denoted t^+, of the arbitrage opportunity observed at time t. The trading rules once price discrepancy is observed are:

- If $F(t,T) - ETF(t)e^{\frac{(r-\delta_1)(T-t)}{365}}$ is positive, an arbitrageur can make profit by selling the SET50 futures and buying the TDEX. The arbitrageur holds his position until the reverse signal is observed at time s, that is, until $F(s,T) - ETF(s)e^{\frac{(r-\delta_1)(T-t)}{365}}$ becomes negative. He then takes the opposite position by buying the SET50 futures and selling the ETF. Profit is calculated as follows:

$$\pi(t^+) = \left(F(t^+,T) - ETF(t^+) - TC(t^+)\right) + \left(-F(s^+,T) + ETF(s^+) - TC(s^+)\right), \qquad (2)$$

- If $F(t,T) - ETF(t)e^{\frac{(r-\delta_1)(T-t)}{365}}$ is negative, an arbitrageur makes profit by buying the SET50 futures and selling the ETF, holding the position until the reverse signal is observed, then closing the position by selling the SET50 futures and buying the ETF. Profit is calculated as follows:

$$\pi(t^+) = \left(-F(t^+,T) + ETF(t^+) - TC(t^+)\right) + \left(F(s^+,T) - ETF(s^+) - TC(s^+)\right), \qquad (3)$$

[1] TDEX is the ticker symbol of the ThaiDEX SET50 ETF.

where t^+ is the time (after t) at which a trade is executed to open arbitrage position, s is the time (after t^+) at which the first reversed arbitrage signal is observed, s^+ is the time (after s) at which a trade is executed to close arbitrage position, $\pi(t^+)$ is profit at time t^+, $F(t^+,T)$ is the price at time t^+ of the SET50 futures, and $ETF(t^+)$ is the price at time t^+ of the TDEX.

B. Test 2: SET50 futures vs. SET50 component stocks

For the second test, arbitrage between the SET50 futures and the SET50 component stocks, the arbitrage signal is:

$$\varepsilon = |F(t,T) - S(t)e^{\frac{(r-\delta_2)(T-t)}{365}}| - TC(t) \geq 0, \quad (4)$$

where $S(t)$ is the value at time t of the SET50 index, r is the 1-month Treasury yield and δ_2 is the estimated annual dividend yield paid on the SET50 index.

Once price discrepancy is observed, the trading rules are:

- If $F(t,T) - S(t)e^{\frac{(r-\delta_2)(T-t)}{365}}$ is positive, an arbitrageur can make profit by selling the SET50 futures and buying the SET50 component stocks. Unlike previous arbitrage studies which suggest buying the quantity of stock that is proportional to its weight in the SET50 index, this study determines the quantity of a stock to be purchased or sold by optimization, using daily futures returns as the benchmark. The arbitrageur will hold this position until the reverse signal is observed. Profit is calculated as follows:

$$\begin{aligned}\pi(t^+) &= \left(F(t^+,T) - w_1 S_1(t^+) - \ldots - w_{50} S_{50}(t^+)\right) \\ &+ \left(-F(s^+,T) + w_1 S_1(t^+) + \ldots + w_{50} S_{50}(t^+)\right) \\ &- \left(TC(t^+) + TC(s^+)\right),\end{aligned} \quad (5)$$

- If $F(t,T) - S(t)e^{(r-\delta_2)(T-t)}$ is negative, an arbitrageur makes profit by buying the SET50 futures and selling the SET50 component stocks and holding this position until the reverse signal is observed. Likewise, the quantity of a stock to be purchased is determined by optimization, using daily futures returns as the benchmark. Profit is as calculated as follows:

$$\begin{aligned}\pi(t^+) &= \left(-F(t^+,T) + w_1 S_1(t^+) + \ldots + w_{50} S_{50}(t^+)\right) \\ &+ \left(F(s^+,T) - w_1 S_1(t^+) - \ldots - w_{50} S_{50}(t^+)\right) \\ &- \left(TC(t^+) + TC(s^+)\right).\end{aligned} \quad (6)$$

C. Test 3: ThaiDEX SET50 ETF vs. SET50 component stocks

Because the TDEX tracks the SET50 index performance and invests over 95percent of its fund in the SET50 component stocks, arbitrage test between the TDEX and the SET50 component stocks can be implemented as follows. Let ε be the arbitrage signal.

$$\varepsilon = | ETF(t)e^{-\delta_1\frac{(T-t)}{365}} - S(t)e^{-\delta_2\frac{(T-t)}{365}} | - TC(t) \geq 0, \quad (7)$$

where $S(t)$ is the value at time t of the SET50 index, δ_1 is the expected annual dividend yield to be paid to the holder of the TDEX, δ_2 is the estimated annual dividend yield paid on the SET50 index. Once price discrepancy is observed, the trading rules are:

- If $ETF(t,T) - S(t)$ is positive, an arbitrageur can make profit by selling the TDEX and buying the SET50 component stocks. The quantity of each of the SET50 stocks is determined by optimization. The arbitrageur then take the reverse position when prices are corrected. Profit is calculated as follows:

$$\begin{aligned}\pi(t^+) &= \left(ETF(t^+) - w_1 S_1(t^+) - \ldots - w_{50} S_{50}(t^+)\right) \\ &+ \left(-ETF(s^+) + w_1 S_1(t^+) + \ldots + w_{50} S_{50}(t^+)\right) \\ &- \left(TC(t^+) + TC(s^+)\right),\end{aligned} \quad (8)$$

- If $ETF(t)e^{-\delta_1\frac{(T-t)}{365}} - S(t)e^{-\delta_2\frac{(T-t)}{365}}$ is negative, an arbitrageur makes profit by buying the TDEX and selling the SET50 component stocks. The arbitrageur takes the reverse position once prices are corrected. Profit is as follows:

$$\begin{aligned}\pi(t^+) &= \left(-ETF(t^+) + w_1 S_1(t^+) + \ldots + w_{50} S_{50}(t^+)\right) \\ &+ \left(ETF(s^+) - w_1 S_1(t^+) - \ldots - w_{50} S_{50}(t^+)\right) \\ &- \left(TC(t^+) + TC(s^+)\right).\end{aligned} \quad (9)$$

In the standard Markowitz portfolio investment theory, the expected return of the portfolio is a function of the risk of the portfolio. Given that the portfolio is well-diversified, the higher the risk of the portfolio, the higher is the return. The expected portfolio return is the weighted average of the return of each component securities in the portfolio, and is mathematically represented as follows:

$$E(\overline{R}) = w_1 r_1 + w_2 r_2 + \ldots + w_n r_n = \sum_{i=1}^{n} w_i r_i, \quad (10)$$

where $\overline{R}$ is portfolio return, w_i is the weight of the i^{th} security, i.e. the proportion of Baht invested in each security to the total Baht investment, r_{it}, or denoted above as r_i for brevity, is the (continuously compounded) return of the i^{th} security at time t. $r_{i,t} = \ln(P_{i,t} / P_{i,t-1})$, where $P_{i,t}$ is the price at time t of the i^{th} security, and $P_{i,t-1}$ is the price at time t-1 of the i^{th} security.

The risk of the portfolio is represented by the variance of the portfolio return, or mathematically:

$$\sigma^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} w_i w_j \quad (11)$$

where σ^2 is portfolio variance (risk) and σ_{ij} is covariance between return of security i and return of security j.

The covariance matrix is estimated from market data and is mathematically represented as:

$$\hat{\Sigma} = \begin{pmatrix} \hat{\sigma}_{11} & \cdots & \hat{\sigma}_{1n} \\ & \ddots & \\ \vdots & \hat{\sigma}_{ij} & \vdots \\ & \ddots & \\ \hat{\sigma}_{n1} & \cdots & \hat{\sigma}_{nn} \end{pmatrix} \quad (12)$$

where $\hat{\sigma}_{ij}$ represents the covariance between security i and security j. A conic optimization technique is then applied onto the estimated covariance matrix, $\hat{\Sigma}$, to ensure that it is positive semidefinite. The formulation of the problem is as follows:

$$\min_{\Sigma} d_F(\Sigma, \hat{\Sigma}) \quad \Sigma \in \left\{ X \in IR^{n\times n} : X \geq 0 \right\}, \quad (13)$$

where $\hat{\Sigma}$ is the estimated covariance matrix and

$$d_F(\Sigma, \hat{\Sigma}) := \sqrt{\sum_{i,j} \left(\sigma_{ij} - \hat{\sigma}_{ij}\right)^2}. \quad (14)$$

The now positive semidefinite covariance matrix, Σ, is then used in the following quadratic programming problem in order to calculate the number of each securities to be purchased or sold in Test 2: SET50 futures vs. SET50 component stocks, and Test 3: ThaiDEX SET50 ETF vs. SET50 component stocks. The quadratic programming problem is formulated as follows:

$$\begin{aligned} \min_{w} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} w_i w_j \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i r_i \geq r^* \\ & \sum_{i=1}^{n} w_i = 1, \end{aligned} \quad (14)$$

where the benchmark return r^* equals the average return of the SET50 futures in Test 2 and equals the average return of the TDEX in Test 3.

V. RESULTS

Execution lags of 0, 1, and 2 days are incorporated in measuring arbitrage profits. Generally speaking, the number of profitable arbitrage trades decreases as the length of lags increases, with institutional investors having higher chances of making arbitrage profits than individual investors.

Because institutional investors normally trade in larger amount than individual investors, the commission incurred to institutional investors is lower than that incurred to individual investors. In the following result tables, the “low commission” refers to the lower commission bracket and/or the institutional investors while “high commission” refers to the higher commission bracket and/or individual investors.

Table II reports the mispricing frequency (taken into account transaction costs) corresponding to Test 1.1: SET50 futures vs. SET50 index during the April 2006-August 2007 period, Test 1.2: SET50 futures vs. SET50 index, and 1.3: SET50 futures vs. TDEX during the September 2007-February 2009 period. In general, the mispricing frequency is higher in the test involving the TDEX (71.3 and 64.9 percent for low and high transaction brackets) than in that involving the SET50 index (61.9 and 53.9 percent for low and high transaction brackets). In other words, using the TDEX and the SET50 index as benchmarks, the SET50 futures is mispriced relatively to the TDEX more often than to the SET50 index. In arbitrage involving the SET50 futures and the TDEX, the majority of the mispricings are buy signals while in arbitrage involving the SET50 futures and SET50 index, the majority of the mispricings are sell signals. That is, the SET50 futures is often underpriced relatively to the TDEX but is generally overpriced relatively to the SET50 index.

Table III reports the number of profitable arbitrage trades corresponding to Test 1.1: SET50 futures vs. SET50 index during the April 2006-August 2007 period, Test 1.2: SET50 futures vs. SET50 index, and 1.3: SET50 futures vs. TDEX during the September 2007-February 2009 period. An arbitrage trade comprises of two transactions: the opening trade and the closing trade which takes the opposite side to the opening trade. And intuitively, the closing trade occurs after the opening trade. The two transaction cost brackets (high for individual and low for institutional traders) and the three execution lags (0, 1, and 2 days) are taken into account. In general, the number of profitable arbitrage trades is higher in the low transaction cost bracket, that is, institutional investors are able to make more profitable arbitrage trades than individual investors because they incur lower transaction costs. Moreover, the number of profitable arbitrage trades declines as the length of execution lags increases. The SET50 index-SET50 futures combination allows slightly higher number of arbitrage trades than the TDEX-SET50 futures combination.

Table IV shows average profit corresponding to the trades in Table III, that is, Test 1.1: SET50 futures vs. SET50 index during the April 2006-August 2007 period, Test 1.2: SET50 futures vs. SET50 index, and 1.3: SET50 futures vs. TDEX during the September 2007-February 2009 period. Table V reports in percentage point the average profit per futures contract. In general, institutional investors (low transaction cost bracket) obtain higher arbitrage profit than individual investors (high transaction cost bracket). This is mainly due to the difference in transaction costs paid by the two types of investors.

Table VI reports the mispricing frequency from Tests 2.2: SET50 futures vs. SET50 component stocks and 3.2: TDEX vs. SET50 component stocks for the period from September 2007 to February 2009. There are generally more mispricing signals in trades involving the SET50 futures than those involving the TDEX. Specifically, mispricing frequencies are, respectively of low and high transaction brackets, 87.3 and 78.5 percent for trades involving the SET50 futures and 71.3 and 58.5 percent for trades involving the TDEX.

Table VII indicates that there is significantly higher number of profitable arbitrage trades involving the SET50 futures than those involving the TDEX. There are over 140 profitable SET50 futures arbitrage trades in all scenarios (combinations of transaction costs and execution lags). However, the number of profitable arbitrage trades involving the TDEX is less than five in all scenarios.

Table VIII reports the profit per arbitrage trade for Test 2.2 and 2.3 while Table IV reports the profit per futures contract for Test 2.2 and profit per 100,000 TDEX units for Test 2.3. Generally, the SET50 futures trades are more profitable than the TDEX trades.

The results from Tests 2.2 and 2.3 indicate that the TDEX is a (slightly) more efficient derivatives security than the SET50 futures. That is, the TDEX is less frequently mispriced with respect to the SET50 index. In other words, the SET50 futures is more frequently mispriced with respect to the SET50 index. This is intuitive because the TDEX is designed to track the performance of the SET50 index. The number of profitable arbitrage trades involving the TDEX is much less than that involving the SET50 futures. The significant difference may be caused by the much higher transaction costs involving trading the TDEX (than involving the SET50 futures). Not only TDEX are investors required to pay commission fees, they are also required to pay front-end and back-end fees as well as management fees. The stocks investors only have to pay commission fees.

Profitable arbitrage trades involving the TDEX vs. the SET50 component stocks occur less frequently than those involving the SET50 futures vs. the SET50 component stocks. This is due to the fact that the SET50 index is more aligned with the TDEX price than with the SET50 futures. Moreover, the average profit from the TDEX trades is usually lower than that obtained from the SET50 futures trades. This is due to the fact that the TDEX incurs much higher transaction costs than the SET50 futures.

Furthermore, the SET50 futures price is more aligned with the SET50 index than with the TDEX. As evident in Fig.2, the SET50 index usually lies in between the price of the TDEX and the price of the SET50 futures. This phenomenon explains why there are more mispricing signals and profitable arbitrage trades involving the SET50 futures vs. TDEX than the SET50 futures vs. SET50 index, as shown in Tables II and III.

VI. Conclusion

This study employs optimization techniques including the conic and quadratic optimizations in investigating the frequency and profitability of index arbitrage involving the Stock Exchange of Thailand's SET50 futures, SET50 component stocks, and the ThaiDEX SET50 ETF (TDEX). This sample period spans the inception of the TDEX to most recent, that is, from April 28, 2006 to February 26, 2009.

The uniqueness of this research lies in the application of optimization techniques in the study of index arbitrage. Moreover, different levels of transaction costs and execution lags are also incorporated before measuring arbitrage profits from the trades. Even with the presence of transaction costs and execution lags, arbitrage opportunities exist though the number of profitable arbitrage trades reduces as the number of lags increases. However, the (conditional) average profit from an arbitrage trade is still high for all lags.

Compared with the results from other studies, the frequency of arbitrage in this study is lower. However, this does not mean that the Thai derivatives markets are more efficient than other markets as comparing the results from this study to those in literature review is like comparing apples to oranges. As most index arbitrage studies are conducted on intraday data, further studies on the Thai markets must be done using intraday data of the Thai securities as well. However, the SET50 futures intraday price data are undisclosed to the public as a part of market manipulation prevention policy by the Thailand's Futures Exchange

Once the SET50 futures intraday price data become available, this study can be properly extended.

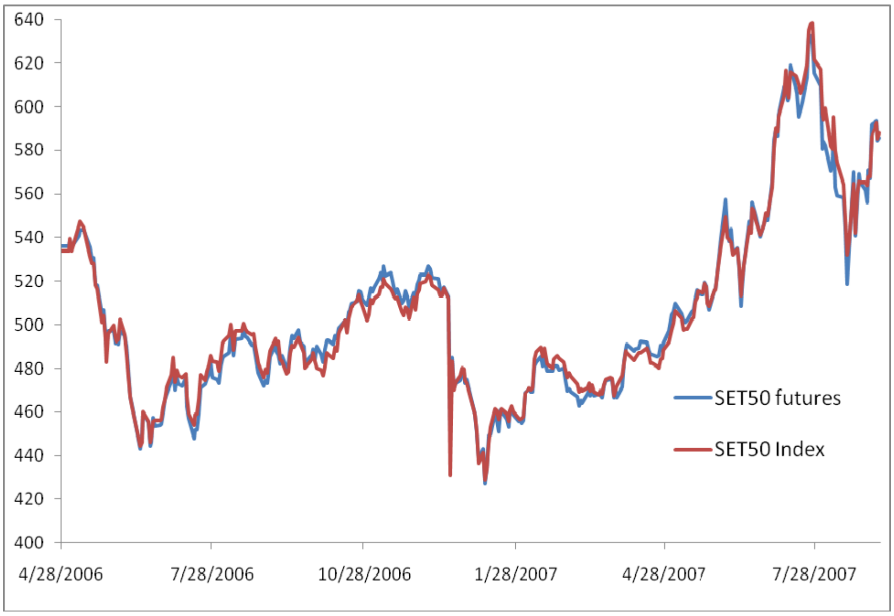

Fig. 1 Time series of the SET50 futures and SET50 index from April 2006 to August 2007

Fig. 2 Time series of the SET50 index, SET50 futures, and TDEX from September 2007 to February 2009

TABLE II
MISPRICING FREQUENCY FROM SUB-TESTS 1.1, 1.2, AND 1.3[2]

Subsample	Total Obs	Types of Mispricing Signal	Mispricing Frequency			
			SET50 Futures vs. TDEX		SET50 Futures vs. SET50 Index	
			Transaction Costs[5]		Transaction Costs	
			THB 300 + 0.575%	THB 500 + 0.7%	THB 300 + 0.575%	THB 500 + 0.7%
Apr 06 - Aug 07	332	Buy[3]	NA	NA	36 (10.8)	27 (8.1)
		Sell[4]	NA	NA	148 (44.6)	129 (38.9)
		Total	NA	NA	184 (55.4)	156 (47.0)
Sep 07 - Feb 09	362	Buy	218 (60.2)	204 (56.4)	87 (24.0)	80 (22.1)
		Sell	40 (11.1)	31 (8.6)	137 (37.8)	115 (31.8)
		Total	258 (71.3)	235 (64.9)	224 (61.9)	195 (53.9)

[2] Subtests 1.1 and 1.2 involve arbitrage between the SET50 futures and the SET50 index and subtest 1.3 involves arbitrage between SET50 futures and the TDEX. The numbers in parentheses are frequencies in percentage points.

[3] Buy signal refers to the scenario in which the SET50 futures is underpriced relatively to the other security (arbitrageurs must buy the futures and sell the other security in order to make profit).

[4] Sell signal refers to the scenario in which the SET50 futures is overpriced relatively to the other security (arbitrageurs must sell the futures and buy the other security in order to make profit).

[5] Transaction costs for one-way trade. To close (cash out) the position, investors incur twice the amount of the costs. The one-way transaction costs for a SET50 futures trade are 300 and 500 Baht per contract for a large-volume trade and small-volume trade, respectively. The one-way transaction costs for a TDEX trade are .575% and .7% for a large-volume trade and small-volume trade, respectively. The one-way transaction costs for a SET50 index trade are assumed to be equal to those for a TDEX trade.

TABLE III
NUMBER OF PROFITABLE ARBITRAGE TRADES FROM SUB-TESTS 1.1, 1.2, AND 1.3

Subsample	Execution Lags	Total Obs	Types of Mispricing Signal	Frequency of Profitable Arbitrage Trades[5]			
				SET50 Futures vs. TDEX		SET50 Futures vs. SET50 Index	
				Transaction Costs		Transaction Costs	
				THB 300 + 0.575%	THB 500 + 0.7%	THB 300 + 0.575%	THB 500 + 0.7%
Apr 06 - Aug 07	0	332	Buy	NA	NA	5	2
			Sell	NA	NA	7	3
			Total	**NA**	**NA**	**12**	**5**
	1	332	Buy	NA	NA	2	0
			Sell	NA	NA	8	4
			Total	**NA**	**NA**	**10**	**4**
	2	332	Buy	NA	NA	3	2
			Sell	NA	NA	4	3

			Total	**NA**	**NA**	**7**	**5**
Sep 07 - Feb 09	0	362	Buy	7	3	10	7
			Sell	20	10	19	14
			Total	**27**	**13**	**29**	**21**
	1	362	Buy	5	2	7	3
			Sell	17	8	16	12
			Total	**22**	**10**	**23**	**15**
	2	362	Buy	2	1	9	3
			Sell	25	12	16	12
			Total	**27**	**13**	**25**	**15**

[5] An arbitrage trade involves two transactions, one to open the position and the other is the reverse trade to close the position. In this case, each transaction occurs on two different days, with the reverse trade occurring after the opening trade

TABLE IV
CONDITIONAL ARBITRAGE PROFIT IN BAHT FROM SUB-TESTS 1.1, 1.2, AND 1.3

Subsample	Execution Lags	Total Obs	Types of Mispricing Signal	Arbitrage Profit			
				SET50 Futures vs. TDEX		SET50 Futures vs. SET50 Index	
				Transaction Costs		Transaction Costs	
				THB 300 + 0.575%	THB 500 + 0.7%	THB 300 + 0.575%	THB 500 + 0.7%
Apr 06 - Aug 07	0	332	Buy	NA	NA	3802.00	1987.20
			Sell	NA	NA	2067.00	316.36
	1	332	Buy	NA	NA	1171.60	0.00
			Sell	NA	NA	1618.70	1146.80
	2	332	Buy	NA	NA	6346.50	4508.70
			Sell	NA	NA	2469.40	625.77
Sep 07 - Feb 09	0	362	Buy	142.00	3012.00	2070.80	875.27
			Sell	1466.00	276.00	2549.50	1382.00
	1	362	Buy	649.75	39.00	2517.90	1328.30
			Sell	2271.80	1083.00	1537.40	367.30
	2	362	Buy	6836.80	4923.00	666.41	4982.10
			Sell	660.25	697.00	2754.20	1594.20

TABLE V
CONDITIONAL ARBITRAGE RETURN FROM SUB-TESTS 1.1, 1.2, AND 1.3

Subsample	Execution Lags	Total Obs	Types of Mispricing Signal	Arbitrage Return (Holding Period Return)			
				SET50 Futures vs. TDEX		SET50 Futures vs. SET50 Index	
				Transaction Costs		Transaction Costs	
				THB 300 + 0.575%	THB 500 + 0.7%	THB 300 + 0.575%	THB 500 + 0.7%
Apr 06 - Aug 07	0	332	Buy	NA	NA	0.65%	0.34%
			Sell	NA	NA	0.35%	0.05%
	1	332	Buy	NA	NA	0.20%	0%

			Sell	NA	NA	0.28%	0.20%
	2	332	Buy	NA	NA	1.09%	0.77%
			Sell	NA	NA	0.42%	0.11%
Sep 07 - Feb 09	0	362	Buy	0.05%	0.99%	0.70%	0.29%
			Sell	0.50%	0.09%	0.88%	0.48%
	1	362	Buy	0.21%	0.01%	0.85%	0.45%
			Sell	0.78%	0.37%	0.53%	0.13%
	2	362	Buy	2.24%	1.61%	0.22%	1.68%
			Sell	0.23%	0.24%	0.95%	0.55%

TABLE VI
MISPRICING FREQUENCY FROM SUB-TESTS 2.2 AND 3.2[6]

Subsample	Total Obs	Types of Mispricing Signal	Frequency of Mispricings (lags = 0)			
			SET50 Futures vs. SET50 Stocks		TDEX vs. SET50 Stocks	
			Transaction Costs		Transaction Costs	
			THB 300 + 0.15%	THB 500 + 0.25%	0.575% + 0.15%	0.7% + 0.25%
Sep 07 - Feb 09	362	Buy	130 (35.9)	116 (32.0)	258 (71.3)	211 (58.3)
	362	Sell	186 (51.4)	168 (46.4)	0 (0)	0 (0)
	362	Total	316 (87.3)	284 (78.5)	258 (71.3)	211 (58.3)

[6] Subtest 2.2 involves arbitrage between the SET50 futures and the SET50 component stocks in the period from September 2007 to February 2009 while Subtest 2.3 involves arbitrage between the TDEX and SET50 component stocks in the same period.

TABLE VII
NUMBER OF PROFITABLE ARBITRAGE TRADES FROM SUB-TESTS 2.2 AND 3.2

Subsample	Execution Lags	Total Obs	Types of Mispricing Signal	Frequency of Profitable Arbitrage Trades			
				SET50 Futures vs. SET50 Stocks		TDEX vs. SET50 Stocks	
				Transaction Costs		Transaction Costs	
				THB 300 + 0.15%	THB 500 + 0.25%	0.575% + 0.15%	0.7% + 0.25%
Sep 07 - Feb 09	0	362	Buy	79	50	0	0
			Sell	81	72	4	1
			Total	**160**	**122**	**4**	**1**
	1	362	Buy	65	49	0	0
			Sell	80	75	0	0
			Total	**145**	**124**	**0**	**0**
	2	362	Buy	62	50	0	0
			Sell	83	77	3	3
			Total	**145**	**127**	**3**	**3**

TABLE VIII
CONDITIONAL ARBITRAGE PROFIT IN BAHT FROM SUB-TESTS 2.2 AND 3.2

				Arbitrage Profit			
				SET50 Futures vs. SET50 Stocks		TDEX vs. SET50 Stocks	
				Transaction Costs		Transaction Costs	
Subsample	Execution Lags	Total Obs	Types of Mispricing Signal	THB 300 + 0.15%	THB 500 + 0.25%	0.575% + 0.15%	0.7% + 0.25%
Sep 07 - Feb 09	0	362	Buy	19817.00	18774.00	0	0
			Sell	8589.20	7593.00	12070.00	493.51
	1	362	Buy	21583.00	20562.00	0	0
			Sell	7611.30	6614.10	0	0
	2	362	Buy	5899.20	4853.40	0	0
			Sell	6123.50	5129.50	3110.60	417.14

TABLE IX
CONDITIONAL ARBITRAGE RETURN FROM SUB-TESTS 2.2 AND 3.2

				Arbitrage Return (Holding Period Return)			
				SET50 Futures vs. SET50 Stocks		TDEX vs. SET50 Stocks	
				Transaction Costs		Transaction Costs	
Subsample	Execution Lags	Total Obs	Types of Mispricing Signal	THB 300 + 0.15%	THB 500 + 0.25%	0.575% + 0.15%	0.7% + 0.25%
Sep 07 - Feb 09	0	362	Buy	6.79%	6.43%	0%	0%
			Sell	2.95%	2.61%	1.97%	0.16%
	1	362	Buy	7.40%	7.04%	0%	0%
			Sell	2.61%	2.27%	0%	0%
	2	362	Buy	2.02%	1.66%	0%	0%
			Sell	2.10%	1.76%	1.02%	0.14%

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