Velocity Distribution in Open Channels: Combination of Log-law and Parabolic-law

Snehasis Kundu and Koeli Ghoshal

Abstract—In this paper, based on flume experimental data, the velocity distribution in open channel flows is re-investigated. From the analysis, it is proposed that the wake layer in outer region may be divided into two regions, the relatively weak outer region and the relatively strong outer region. Combining the log law for inner region and the parabolic law for relatively strong outer region, an explicit equation for mean velocity distribution of steady and uniform turbulent flow through straight open channels is proposed and verified with the experimental data. It is found that the sediment concentration has significant effect on velocity distribution in the relatively weak outer region.

Keywords—Inner and outer region, Log law, Parabolic law, Richardson number.

I. INTRODUCTION

Numerous investigations have been done to measure the turbulent mean velocity profile in open channel flows. For fully developed turbulent flows through straight rectangular open channels, the vertical distribution of mean velocity into the inner region ($y \leq 0.2h$, where y is the distance from bed and h is the flow depth) can be expressed well by the log law. But the log law deviates from the experimental results in the outer region. This deviation cannot be always modified either adjusting the von Karman constant κ or the additive constant present in the log law [1]- [3], as the log law measures the velocity which increases with increase of distance from bed. Vanoni [5] claimed that for wide open channels where aspect ratio(ratio of channel width b to water depth h) $Ar > 5$, there always exists a central region where there is no velocity dip and the log law can be extended up to the free surface. Later on [2] proposed that for wide open channels, in the central zone with $| d/h | < (Ar - 5)/2$ where $d = b/2 - z$ in which z is distance from the side wall, the side wall effect can be neglected and the flow is two-dimensional. The velocity distribution in this central section of open channels is characterized by the maximum velocity occurring at the free surface and the log law can be applied within the limits of experimental accuracy. In a vertical region near the side wall or in the corner zones, log law fail due to the presence of the dip-phenomenon which occurs from the side wall effect and reported a century ago [6]- [7]. In the outer region, the deviation of log law from experimental data is counted by adding the Coles' wake function to the log law [1], [8]. After the log law, the log-wake law is considered to be the most reasonable extension of velocity distribution [3].

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The log-wake law is applicable in the central section [15] but fails near the side wall regions and at the corner regions for wide open channels due to velocity dip-phenomenon. However flows through open channel with aspect ratio $Ar \leq 5$, due to the strong side wall shear stress, secondary velocity cannot be neglected and the maximum velocity always occurs below the free surface and in this case the log law and the log-wake law fail to predict the velocity near the free surface at the outer region.

Flows through open channels with aspect ratio $Ar \leq 5$, the velocity in the central section of the outer region($y > 0.2h$) can be described by a parabolic law. According to [9] it was proposed by Bazin from the experimental investigations. But the region of applicability of the parabolic law was not mentioned [4]. Vedula and Rao [4] proposed a general form of parabolic law which is as follows

$$\frac{u_{max} - u}{u_*} = C \left(1 - \frac{\eta}{\eta_{dip}} \right)^2 \qquad (1)$$

where u_{max} is the maximum velocity, u is the velocity at height η from bed, u_* is the shear velocity, C is a parameter, $\eta(= y/h)$ is the dimensionless distance from bed, y is the distance from channel bed, h is the flow depth, $\eta_{dip}(= y_{dip}/h)$ is the dimensionless distance of the maximum velocity from bed and y_{dip} is the location of maximum velocity from bed. According to [4] the junction point from which the parabolic law (1) is applicable up to the free surface ranges from 0.2 to 0.3 for sediment laden flows and they showed that the parameter C present in the parabolic law depends on this junction point. Later on [10] proposed the limitations and the region of validity of the parabolic law (1). They claimed from the experimental investigations that for the tangential parabola, the maximum value of the junction point is 0.5 when there is no dip-phenomenon and the value decreases with the presence of dip-phenomenon.

At the common boundary of inner and outer region both the laws i.e. the log law and the parabolic law are valid and velocity is single valued [10]. Combining the log law and the parabolic law, the binary law is proposed and verified by [4] for open channel flows with rough bed. Sarma *et al.* [10] studied the binary law for subcritical and supercritical flows in smooth and rough open channels with finite aspect ratios. From the previous paragraph it is clear that the region of applicability of the parabolic law depends on the choice of the junction point. Moreover the junction point cannot be expressed in terms of width and aspect ratio [10] and hence cannot be calculated from experimental data. Also to match the log law and the parabolic law at the junction point, extra conditions are needed. Therefore it is more reasonable to merge the

log law and the parabolic law into a single composite equation such that it can predict the velocity throughout the depth.

The main objectives of this paper are: (1) to propose a velocity distribution equation combining the logarithmic law in the inner region and the parabolic law in the outer region; (2) to show that the proposed log-parabolic law is valid in clear water, sediment-laden and quasi-neutral sediment-laden flows; (3) to analyze the effect of sediment concentration on velocity distribution in the outer region; and (4) to determine the effect of sediment suspension on the model parameters in sediment-laden flows.

II. Flow regions with Velocity Models

A. Logarithmic law in the inner region

Velocity distribution in the inner region where $0 < \eta \leq 0.2$ is described by the log law which is as follows

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \eta + B \qquad (2)$$

where u is the velocity at height η from bed, u_* is the shear velocity, κ is the von Karman constant, B is the additive constant and $\eta(= y/h)$ is the dimensionless distance from bed and h is the flow depth. The logarithmic law is valid throughout the inner region except on the channel bed where $\eta = 0$. Experimentally [11], [12], [13], and [14] showed that the log law is valid in the entire flow region and claimed that the value of the von Karman constant decreases with the sediment suspension. Later on [15] showed that the value of the von Karman constant in the region near to bed is not affected by the sediment suspension. Reference [16], and [17] supported Coleman's [15] argument. Therefore it can be concluded that the log law is applicable in the inner region with the value of the von Karman constant $\kappa = 0.4$ and when the log law is extended to the outer region, the value of κ decreases with the sediment suspension. This indicates that sediment suspension has significant effect on the velocity distribution in the outer region. In this study, keeping the value of von Karman constant κ as 0.4 throughout the depth, the effect of sediment suspension on velocity distribution is analyzed.

B. Parabolic law in the outer region

The velocity distribution in the outer region through an uniform open channel can be described by the parabolic law. The general forms of the parabolic law were proposed by [4] and [10]. But the junction point from where the parabolic law is applicable, can be extended into the inner region by adjusting the parameter present in the parabolic law. Many researchers pointed out that the parabolic law is applicable only in the outer region. Therefore in this study to fit the parabola in the outer region and to investigate the effect of suspended sediment on the velocity distribution in the outer region the parabolic law proposed by Bazin is taken. The parabolic law for the outer region $\eta > 0.2$ according to Bazin for a central section in open channels is as [9]

$$\frac{u_{max} - u}{u_*} = 6.3 \left(1 - \frac{\eta}{\eta_{dip}}\right)^2 \qquad (3)$$

where u_{max} is the maximum velocity, $\eta_{dip}(= y_{dip}/h)$ is the dimensionless distance of the maximum velocity from bed and y_{dip} is the location of maximum velocity from bed.

The log law and the parabolic law velocity profiles for inner and outer region respectively is plotted with the experimental data of Coleman [15] in Fig. 1. It should be noted from Fig. 1 that parabolic law (3) significantly deviates from the experimental measurements in the inner region and this deviation is denoted by $(u_{para} - u_{mea})/u_*$, where u_{para} is the value calculated from parabolic law (3) and u_{mea} is the measured value and u_* is the shear velocity. It can also be noted from Fig. 1 that beyond the inner region the velocity still deviates from parabolic as well as logarithmic profile. Coles' first pointed out that the deviation of velocity from log law in the outer region occurs due to the wake in the outer region. The log-wake law also deviates from experimental data near the free surface and it cannot satisfy the outer boundary condition [18]. To investigate the deviation of the velocity profile from the log law and the parabolic law more clearly beyond the inner region, the log law and the parabolic law are plotted in Fig. 2 with the data from [11], [15] and [19]. The point of deviation of parabolic law from the experimental data is denoted as η_p in Fig. 2. From Fig. 2 it can be seen that the values of η_p are 0.7, 0.6 and 0.46 and the aspect ratios for the velocity profiles plotted in Fig. 2 are 2.1, 3 and 11.9 for Coleman [15], Wang and Qian [19], and Vanoni [11] data respectively. One can observe that with the increase of aspect ratio, the value of η_p decreases gradually. This occurs because when the aspect ratio is small(usually less than 5), the side wall shear stress dominates over the free surface shear stress in the region $0.2 \leq \eta < \eta_p$ and as aspect ratio increases, the free surface shear stress dominates over the side wall shear stress in the central section. Also from Fig. 2 it is clear that the velocity in the region $0.2 \leq \eta < \eta_p$ neither predicted by the log law nor by the parabolic law. For sediment-laden flows, in the region where $\eta \geq \eta_p$, the effect of sediment concentration can be neglected as the sediment concentration becomes very low. Therefore to investigate the effect of sediment concentration in the outer region, it is more reasonable to divide the wake layer into two regions: relatively weak outer region where the side wall shear stress dominates over the free surface shear stress for narrow channels; and relatively strong outer region where the free surface shear stress is important than the side wall shear stress for narrow as well as wide open channels.

From the above discussion in the foregoing paragraphs it is evident that, the flow region may be divided into three regions: inner region which comprises 20% of the flow depth from the channel bed i.e., where $0 \leq \eta \leq 0.2$, the relatively weak outer region which roughly can be taken as $0.2 \leq \eta < \eta_p$ and the relatively strong outer region where $\eta_p \leq \eta \leq 1$ in which η_p is the point of divider. In the inner region logarithmic law is applicable and in the relatively strong outer region the parabolic law is applicable. Fig. 3 indicates the regions clearly.

C. Log-Parabolic law in the whole flow region

The log law and the parabolic law are two asymptotic expressions for velocity distribution in inner and relatively

strong outer region respectively. At the relatively weak outer region, the velocity cannot be predicted either by the log law or by the parabolic law. To merge these two expressions (2) and (3) into a single composite equation which can also predict the velocity in the relatively weak outer region, the following model is proposed

$$\frac{u}{u_*} = C - 6.3 \left[\Psi(\eta) \right]^2 - \alpha \ln \left[1 + \left(\frac{\lambda}{\eta} \right)^{\beta} \right] \Psi(\eta)(1 - \eta) \quad (4)$$

where $\Psi(\eta) = 1 - \eta/\eta_{dip}$ is a function of η, $\eta(= y/h)$ is the dimensionless distance from bed, C is a parameter determined from the maximum velocity condition $u = u_{max}$ at $\eta = \eta_{dip}$, parameters α and $\lambda(> 0)$ are determined with B, C and κ, and $\beta(> 0)$ is the shape parameter used to determine the shape between two asymptotes (2) and (3) of (4) in the relatively weak outer region. It is clear from Fig. 1 and Fig. 2 that the parabolic law (3) predicts velocity accurately in the relatively strong outer region and significantly overestimates the velocity in the inner region. So to diminish the value in the inner region the last term in the right hand side of (4) is subtracted from the parabolic law (3) and hence $\alpha > 0$.

In the inner region for small value of η, the function $\Psi(\eta)$ is approximated as $\Psi(\eta) \approx 1$. From the proposed model it is clear that in the inner region near to the bed, η is very small and for a fixed value of λ, $\left(\frac{\lambda}{\eta} \right)^{\beta}$ is very large and one can approximate the third term on the right hand side of (4) as

$$\ln \left[1 + \left(\frac{\lambda}{\eta} \right)^{\beta} \right] \Psi(\eta)(1 - \eta) \rightarrow \beta [\ln \lambda - \ln \eta] \quad (5)$$

and (4) tends to the form for the inner region

$$\frac{u}{u_*} = \alpha\beta \ln \eta + C - 6.3 - \alpha\beta \ln \lambda \quad (6)$$

Imposing the boundary condition $u = u_{max}$ at $\eta = \eta_{dip}$ into (4) and comparing (2) with (6) one gets the value of the parameters C, α and λ respectively as

$$C = \frac{u_{max}}{u_*} \quad (7)$$

$$\alpha = \frac{1}{\kappa\beta} \quad (8)$$

$$\lambda = \exp \left[\kappa \left(\frac{u_{max}}{u_*} - 6.3 - B \right) \right] \quad (9)$$

From (8) and (9) it is clear that $\alpha > 0$ and $\lambda > 0$ which we have mentioned earlier.

Similarly, for the outer region when $\eta \rightarrow 1$

$$\ln \left[1 + \left(\frac{\lambda}{\eta} \right)^{\beta} \right] \Psi(\eta)(1 - \eta) \rightarrow 0 \quad (10)$$

i.e., (4) matches asymptotically to the parabolic law (3) in the relatively strong outer region.

Substituting the values of C and α from (7) and (8) into (4) one can write the velocity equation in the defect form as

$$\frac{u_{max} - u}{u_*} = 6.3 \left[\Psi(\eta) \right]^2 + \frac{1}{\kappa\beta} \ln \left[1 + \left(\frac{\lambda}{\eta} \right)^{\beta} \right] \Psi(\eta)(1 - \eta) \quad (11)$$

In this study (11) is referred as the log-parabolic law which is the main contribution of this study. Equation (11) matches asymptotically with the log law when η is very small nearer to bed and with the parabolic law when η is nearer to 1 at the free surface. Like the log law, (11) is invalid at the solid boundary where $\eta = 0$. The first term on the right hand side represents the parabolic law and the second term represents the log law. The velocity profile is calculated from (11) for open channel flows throughout the depth if the value of the parameters are given. To complete the investigation, it now remains to evaluate the parameters B, u_*, η_{dip}, λ and β.

III. DETERMINATION OF PARAMETERS AND FITTING

The main parameters in (11) are u_{max}, u_*, η_{dip}, β and λ. The shear velocity u_* is calculated using the formula given by [18]. For the shape parameter the method of least-squares is used using the help of MATLAB. The value of λ is calculated from (9). To compute the value of λ the value of B is needed and it is calculated using the least-squares method. The value of the parameter η_{dip} is calculated from the formula given by [20].

A. Least-squares method for parameter B

To accurately estimate the value of the additive parameter in the universal log law, the least-squares method is used. The least-squares approximation can be represented as

$$S = \sum_{i=1}^{n} \left[\frac{u_i}{u_*} - \frac{1}{\kappa} \ln(\eta_i) - B \right]^2 \quad (12)$$

in which S is the sum of the squares of the residuals, n is the number of sample points (η_i, u_i) which satisfies $\eta_i \leq 0.2$ and κ is the given von Karman constant. The model parameter B can be found by minimizing S i.e. by solving the following equation

$$\frac{\partial S}{\partial B} = 0 \quad (13)$$

This can be written as

$$\sum_{i=1}^{n} \left[\frac{u_i}{u_*} - \frac{1}{\kappa} \ln(\eta_i) - B \right] = 0 \quad (14)$$

Further (14) can be written as

$$B = \frac{1}{n} \left[\sum_{i=1}^{n} \frac{u_i}{u_*} - \frac{1}{\kappa} \sum_{i=1}^{n} \ln(\eta_i) \right] \quad (15)$$

The model parameter B can be calculated explicitly using (15) from the data. A MATLAB programme has been written to find the value of the parameter B.

B. Calculation of shear velocity u_*

The shear velocity is an important parameter in open channel hydraulics which is used to dimensionless mean flow velocity, turbulence intensities etc.. It is defined $u_* = \sqrt{\tau_b/\rho_0}$ where τ_b is the average bed shear stress, ρ_0 is the fluid density. There are several methods to determine the shear velocity [18]. Using the conformal mapping over the rectangular cross-section of the channel into a half plane, [18] proposed two

different explicit formulas for determining the shear velocity in a central vertical depending on the value of the aspect ratio b/h. The shear velocities are expressed as

$$
\begin{aligned}
u_* &= \sqrt{ghS\Phi_1} \quad \text{when} \quad b/h \le 2.5 \\
&= \sqrt{gbS\Phi_2} \quad \text{when} \quad b/h > 2.5
\end{aligned} \tag{16}
$$

where g is the gravitational acceleration, h is the flow depth, b is the width of the channel, S is the energy slope and the functions Φ_1 and Φ_2 are defined as

$$
\Phi_1 = \frac{4}{\pi} \tan^{-1} e^{\left(-\frac{\pi h}{b}\right)} + \frac{4b}{\pi^2 h} \left[1 - \left(1 + \frac{\lambda \pi h}{b} \right) e^{-\lambda \frac{\pi h}{b}} \right] \tag{17}
$$

in which $\lambda = 0.6691$ and

$$
\Phi_2 = \frac{1}{\pi} \sin^{-1} \left(\tanh \frac{\pi h}{b} \right) \tag{18}
$$

The shear velocity for at the centerline is calculated from (16). A comparison between the calculated values with the measures values is plotted in Fig. 4.

C. Calculation of parameter η_{dip}

It is known from experimental investigations that the location of maximum stream wise velocity y_{dip} is mainly depends on the lateral position z from side wall of the measured velocity profiles in rectangular open channels. Several investigators proposed empirical relation of dependence of the relative distance of the location of velocity dip to the water depth, $\eta_{dip} = y_{dip}/h$ with the relative lateral distance to the water depth z/h. Wang *et al.* [21] obtained the following empirical relation of linear trend of sine wave function based on measured data from literature as

$$
\eta_{dip} = 0.44 + 0.212 \left(\frac{z}{h} \right) + 0.05 \sin \left(\frac{2\pi}{2.6} \frac{z}{h} \right) \tag{19}
$$

where $z/h < 2.6$. He added that if the aspect ratio $Ar = b/h < 5.2$, the maximum velocity occurs below the free surface and if $Ar > 5.2$, maximum velocity may occur at the water surface. He remained silent about the occurrence of velocity dip at the vicinity of side wall regions for wide open channels. Later Yang *et al.* [20] proposed another empirical relation between dimensionless velocity dip position η_{dip} and relative lateral distance to the water depth, z/h as follows

$$
\eta_{dip} = \frac{1}{1 + 1.3 \exp\left(-\frac{z}{h}\right)} \tag{20}
$$

A comparison between these two empirical relations is plotted in Fig. 5. Is is clear from the figure that they give almost same result when $z/h < 2.5$. For larger value of z/h the relation proposed by [21] deviates more from experimental results whereas the equation proposed by [20] matches more accurately with the experimental data of Coleman [15], Vanoni [11], and Wang and Qian [19]. For calculating the velocity in narrow and wide open channels from (11), the value of the parameter η_{dip} is therefore calculated from the relation (20).

D. Procedure for fitting the profile

1) The value of the parameter B in the log law is computed from (15) by fitting the best straight line by least-squares method into the inner region $\eta \le 0.2$.
2) With this value of B, the value of λ is calculated from (9).
3) The shear velocity u_* is calculated from (16).
4) Then the value of the parameter η_{dip} is calculated from (20).
5) Taking $\beta = 1$ as an initial approximation the value is improved from (11) by using least-squares technique in MATLAB to get the best fitting equation.
6) Then the velocity is calculated from (11) using the values of the parameter u_*, β, λ and η_{dip} obtained from previous steps.

IV. TEST OF THE LOG-PARABOLIC LAW

The velocity measurements from experiments of Coleman [15], Vanoni [11] and Wang and Qian [19] is used to verify the applicability of log-parabolic law (11) in open channel flows.

Fig. 3 indicates that there is velocity defect in the inner region defined as $(u_{para} - u_{mea})/u_*$, where u_{para} is the value calculated from parabolic law (3) and u_{mea} is the measured value taken from data and u_* is the shear velocity. This velocity defect is defined from (3) and (11) as

$$
\frac{u_{para} - u_{mea}}{u_*} = \frac{1}{\kappa \beta} \ln \left[1 + \left(\frac{\lambda}{\eta} \right)^\beta \right] \Psi(\eta)(1 - \eta) \tag{21}
$$

To assure whether this velocity defect defined by (21) is proportional to $\ln \left[1 + (\lambda/\eta)^\beta \right] (1 - \eta/\eta_{dip}) (1 - \eta)$, as predicted by (11) it is appropriate to investigate the relationship between the quantities $(u_{para} - u_{mea})/u_*$ and $\ln \left[1 + (\lambda/\eta)^\beta \right] (1 - \eta/\eta_{dip}) (1 - \eta)$. Fig. 3 shows a velocity profile taken from RUN 2 data of Coleman's [15] experiment where aspect ratio $Ar = 2.08$. The data have been replotted in Fig. 6 in the form $(u_{para} - u_{mea})/u_*$ against $\ln \left[1 + (\lambda/\eta)^\beta \right] (1 - \eta/\eta_{dip})(1 - \eta)$. Fig. 6 shows that the points can be represented by a straight line with a correlation coefficient 0.85 and slope 2.6655. Also it can be seen from (21) that the slope of the straight line in Fig. 6 is $1/(\kappa\beta) = 2.6685$ where $\kappa = 0.4$ and $\beta = 0.9369$. The value of the parameter β is taken from the best fitting equation by least-squares method which is clearly indicated in Fig. 3. Thus the plot in Fig. 6 indicates that the formula (11) given in defect form for the vertical distribution of velocity profile for the entire flow regime is actually reasonable and Fig. 3 establishes the validity of (11) clearly.

Fig. 7 compares velocity profiles obtained from log-parabolic law with experimental data of Coleman [15]. Two clear water profiles (RUN 1 and 21) and one water-sediment mixture profile (RUN 13) is plotted in this figure in a rectangular scale and in a logarithmic scale in Fig. 7(a) and Fig. 7(b) respectively. The aspect ratios for the runs 1, 13 and 21 are 2.07, 2.08 and 2.1 respectively and in all cases maximum velocity occurs below the free surface. The value of the parameters B, β, η_{dip} and λ is given in I. One can find

from the table that the shape parameter decreases with the Richardson number whereas the parameter λ increases with Richardson number. It clearly shows that the density gradient has significant effect on the velocity profile. From Fig. 7 it can be concluded that, the log-parabolic law predicts the velocity well throughout the depth with velocity dip correctly. This indicates that log-parabolic law is applicable for clear water and sediment-laden flows through narrow open channels.

Fig. 8 compares log-parabolic law with the experimental data of Vanoni [11]. From all data, one profile (RUN 3) in clear water and two profiles (RUN 18 and 21) in water-sediment mixture is plotted in the figure in both rectangular scale and logarithmic scale in Fig. 8(a) and Fig. 8(b) respectively. The aspect ratios for three runs are 5.75, 5.99 and 11.9. The values of the parameter η_{dip} from (16) and the values of the other parameters β and λ are indicated in II. From the figure it is clear that log-parabolic law measures the velocity profiles well throughout the depth of the channel. It indicates that log-parabolic law is applicable for predicting the velocity profiles in clear water and sediment-laden flows through wide open channels.

Experimental data of Wang and Qian [19] is taken to check the validity of log-parabolic in quasi-neutral sediment-laden flows. Wang and Qian [19] performed experiment for three types of particles: SF(clear water + fine plastic particle), SM(clear water + medium plastic particle) and SC(clear water + coarse plastic particle). Due to uniform distribution of plastic particles in clear water, the experiment can be treated as quasi-neutral sediment-laden flow [22]. Fig. 9 compares log-parabolic law with some experimental data from experiment of Wang and Qian [19]. In all cases the aspect ratio is 3 and hence velocity dip occurs. In this case also, log-parabolic law predicts velocity profile well throughout the depth for narrow open channels.

A. Density gradient effect on the parameters β and λ

To test the effect of density gradient on the model parameters β and λ the experimental data of Coleman [15] and Vanoni [11] is taken. The density gradient is quantified by the Richardson number R_i which is defined as the ratio of the sediment suspension energy to the turbulent production [22]. The Richardson number is defined as [22]

$$R_i = \frac{gh\eta_{dip}}{u_*^2} \frac{\rho_s - \rho_0}{\rho_0} \frac{C_{0.05} - C_1}{1 + \frac{\rho_s - \rho_0}{\rho_0} C_m} \quad (22)$$

where g is gravitational acceleration, h is flow depth, η_{dip} is the dimensionless distance of the velocity dip position from bed, u_* is shear velocity, ρ_0 is the mass density of water, ρ_s is the mass density of sediment, $C_{0.05}$ is the volumetric sediment concentration at $y = 0.05h$, C_1 is the volumetric sediment concentration at the top of the boundary layer and C_m is the mean sediment concentration.

The calculated values of Richardson number for all Coleman's [15] and Vanoni's [11] experimental profiles are shown in I and II respectively. The variation of the shape parameter β with the Richardson number R_i is plotted in Fig. 10 with the experimental data of Coleman [15] and Vanoni [11]. It is clear
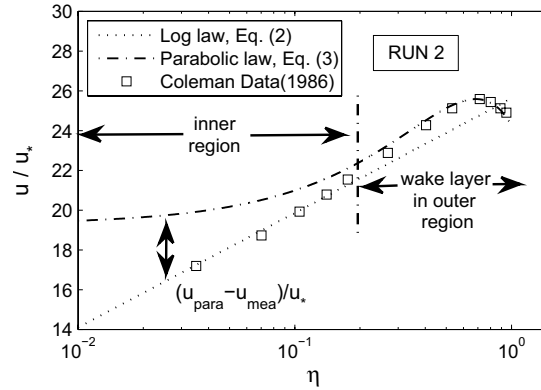

Fig. 1. Typical velocity profile with velocity dip [data from Coleman(1986)]

from figure that Richardson number has significant effect on the shape parameter β. From the figure one can see that when $R_i = 0$, $\beta = 1$ which is consistent with respect to theoretical aspect. A curve fitting line fitted these data well with R^2 value 0.7. The variation of shape parameter β with the Richardson number R_i can be expressed as

$$\beta = \frac{1}{1 + 15.4 \exp\left(-\frac{4.5}{R_i^{0.2}}\right)} \quad (23)$$

The values of β is irregular when $R_i < 5$ which is due to the difficulty of measuring very low concentration in the fluid.

Similarly, the variation of the parameter λ with the Richardson number is plotted with the experimental data of Coleman [15] and Vanoni [11] in Fig. 11. It can be seen that the value of λ increases with the increase of the Richardson number R_i. A linear relation exist between λ and R_i. A curve fitted line is fitted with the data with $R^2 = 0.74$. The variation of parameter λ with the Richardson number R_i can be expressed as

$$\lambda = 0.1 + 0.006R_i \quad (24)$$

From this relation we have $\lambda = 0.1$ when $R_i = 0$, which is consistent with the experimental results. The value of λ scatters when $R_i < 5$ due to very low concentration in the fluid.

V. CONCLUSION

The velocity distribution in steady and uniform turbulent flow through straight open channels is reanalyzed theoretically from the previous flume experimental investigations. The theoretical analysis shows that:

1) The wake layer in the outer region of flow field may be divided into two regions, relatively weak outer region and relatively strong outer region and the parabolic law (3) agrees well with the experimental data in the relatively strong outer region.

2) The proposed log-parabolic law (11) is valid in clear water, sediment-laden and quasi-neutral sediment-laden flows throughout the depth of the channel.

3) The density gradient i.e., the Richardson number R_i has significant effect on the shape parameter β and the

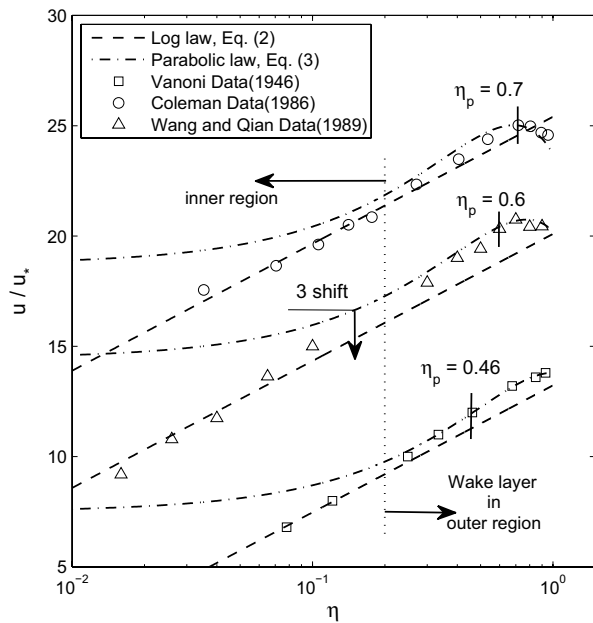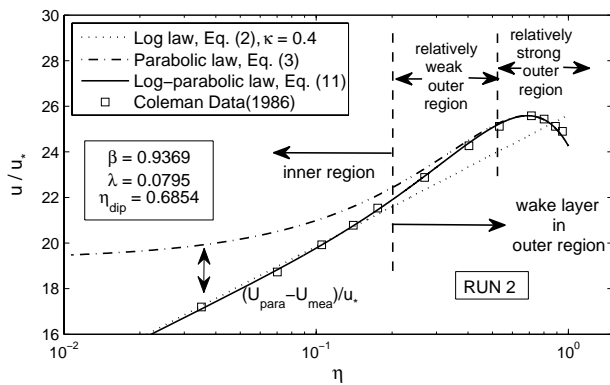Fig. 2. Velocity profiles for narrow and wide open channels[data from Vanoni(1946), Coleman(1986), and Wang and Qian(1989)]

Fig. 3. Comparison of log law, parabolic law and log-parabolic law for Coleman(1986) RUN 2.

parameter λ for sediment-laden flows. From computed results, empirical relations for the shape parameter β and the parameter λ with the Richardson number R_i are proposed. For clear water flows, $\beta = 1$ and the value decreases exponentially with the increase of the Richardson number in sediment-laden flows. The parameter λ increases linearly with the Richardson number.

4) The sediment suspension has significant effect on the velocity distribution in the relatively weak outer region.
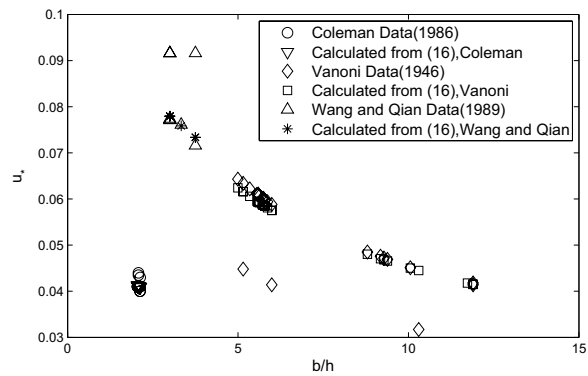
Fig. 4. Measured and calculated values of u_* from (15) for Coleman(1986), Vanoni(1946) and Wang and Qian(1989) data.
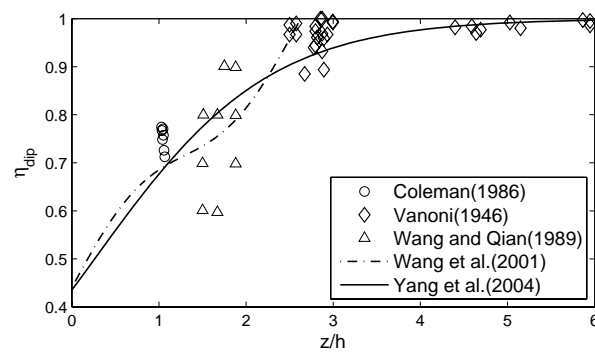

Fig. 5. Relation between velocity dip position η_{dip} and relative lateral distance to water depth z/h for open channel flows(after Yang $et\ al.$(2004)).
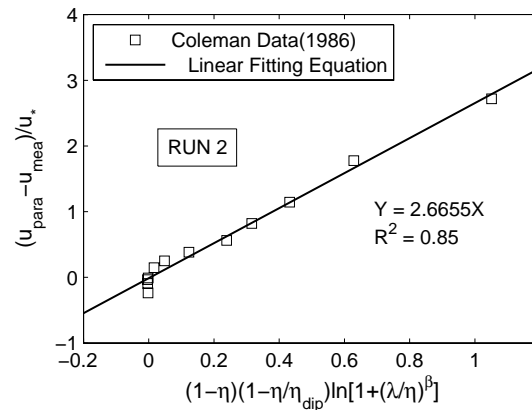

Fig. 6. Relation between velocity difference $(u_{para} - u_{mea})/u_*$ and $\ln\left[1 + (\lambda/\eta)^\beta\right]\left(1 - \eta/\eta_{dip}\right)(1 - \eta)$.

REFERENCES

[1] D. Coles, "The law of the wake in turbulent boundary layer," *Journal of Fluid Mechanics*, vol. 1, pp. 191-226, 1956.
[2] I. Nezu, and W. Rodi, "Experimental study on secondary currents in open channel flow," *Proceedings of the 21st IAHR Congress*, pp. 115-119, Melbourne: IAHR, 1985.
[3] I. Nezu, and H. Nakagawa, "*Turbulence in open-channel flows*," Balkema, Rotterdam NL., 1993.
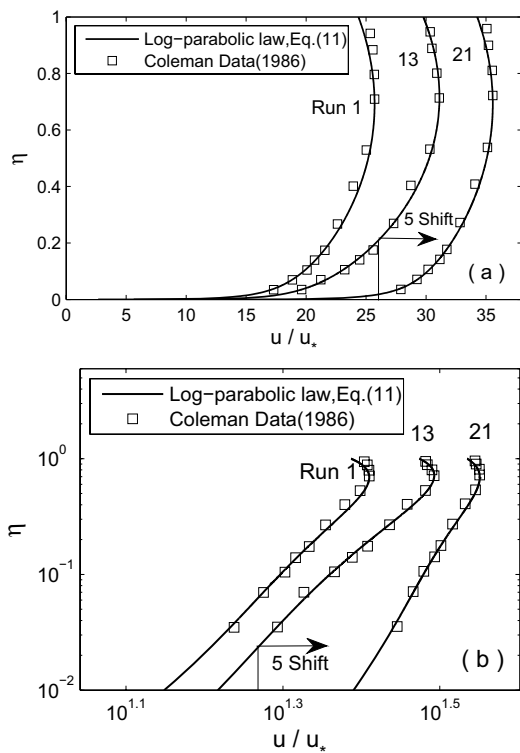[4] S. Vedula, and A. R. Rao, "Bed Shear from Velocity Profiles: A New

Fig. 7. Comparison of measured and computed velocity profile for Coleman(1986) RUN 1(clear water), 13(water-sediment mixture) and 21(clear water).
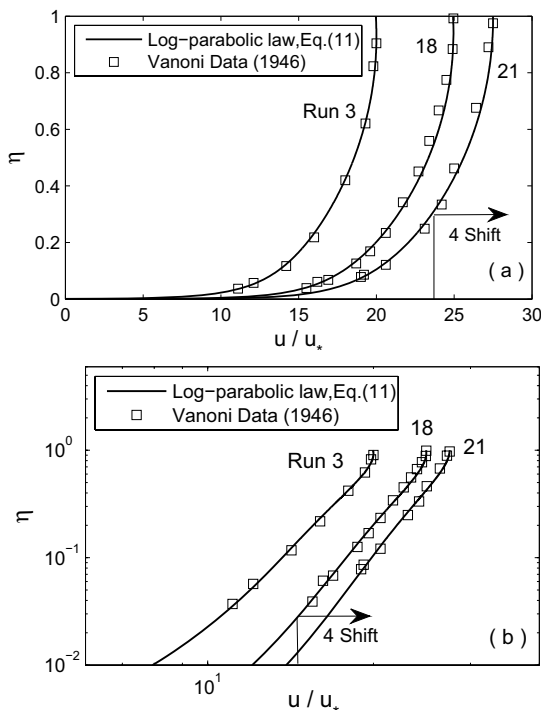

Fig. 8. Comparison of measured and computed velocity profile for Vanoni(1946) RUN 3(clear water), 18 and 20(water-sediment mixture).
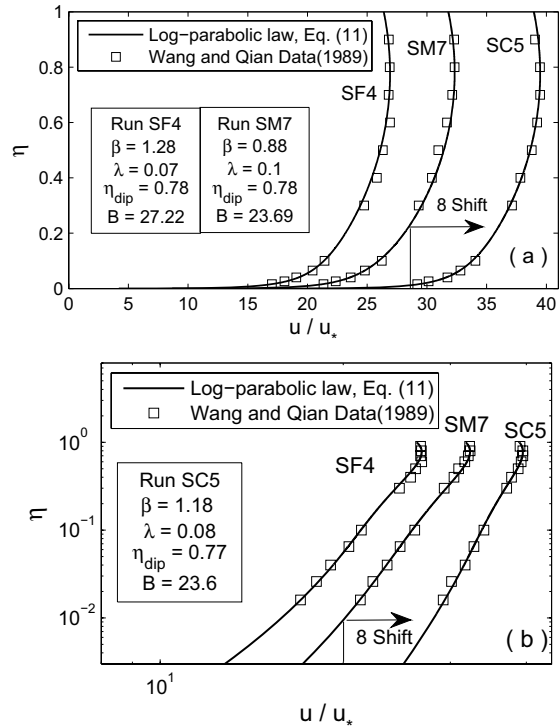

Fig. 9. Comparison of measured and computed velocity profile for Wang and Qian(1989) RUN SF3, SM3 and SC5(water-plastic particle mixture).
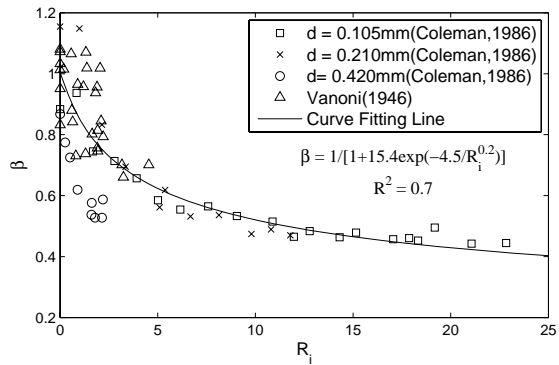
Fig. 10. Effect of density Gradient R_i on the shape parameter β.

Approach," *Journal Hydraulic Engineering*, vol. 111, no. 1, pp. 131-143, 1985.

[5] V. A. Vanoni, "Velocity Distribution in Open Channels," *Civil Engineering, ASCE*, vol. 11, no. 6, pp. 356-357, 1941.

[6] J. B. Francis, "On the cause of the maximum velocity of water flowing in open channels being below the surface," *Trans.* ASCE, vol. 7(1), no. 1, pp. 109-113, 1878.

[7] F. P. Stearns, "On the current meter, together with a reason why the maximum velocity of water flowing in open channel is below the surface," *Trans.* ASCE, vol. 7, pp. 331-338, 1883.

[8] J. O. Hinze, "*Turbulence*," McGraw-Hill, New York, 1975, pp. 698-699.

[9] G. H. Keulegan, "Laws of turbulent flow in open channels," *J. Res., Nat. Bureau of Standards*, vol. 21, pp. 707-741, 1938.

[10] K. V. N. Sarma, B. V. R. Prasad, and A. K. Sarma, "Detailed Study of Binary Law for Open Channels," *Journal Hydraulic Engineering*, vol. 126, no. 3, pp. 210-214, 2000.

[11] V. A. Vanoni, "Transportation of suspended sediment by water,"
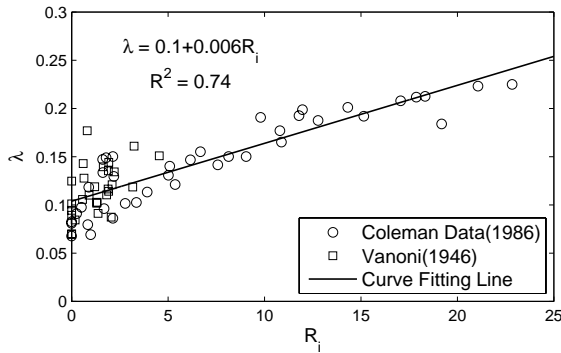
Fig. 11.   Effect of density Gradient R_i on the parameter λ.

TABLE I
CALCULATION OF RICHARDSON NUMBER FROM COLEMAN'S (1986)
EXPERIMENTAL DATA

Run	β	η_{dip} (m)	λ	B	$C_{0.05}$ (10^{-3})	C_1 (10^{-3})	C_m (10^{-3})	R_i
1	0.88	0.68	0.08	25.69	0.000	0.00	0.00	0.000
2	0.94	0.69	0.08	25.62	0.840	0.09	0.25	0.846
3	0.75	0.68	0.09	25.11	1.650	0.16	0.45	1.687
4	0.71	0.69	0.10	25.00	2.700	0.23	0.67	2.784
5	0.66	0.69	0.11	24.68	3.820	0.34	0.92	3.921
6	0.59	0.69	0.13	24.49	4.890	0.42	1.12	5.015
7	0.55	0.69	0.15	24.38	5.950	0.48	1.40	6.159
8	0.56	0.68	0.14	24.10	7.210	0.53	1.58	7.578
9	0.53	0.68	0.15	24.05	8.530	0.52	1.79	9.048
10	0.51	0.69	0.17	24.10	10.31	0.64	2.06	10.89
11	0.46	0.69	0.20	24.20	11.35	0.62	2.25	11.97
12	0.48	0.68	0.19	23.69	11.86	0.58	2.24	12.78
13	0.46	0.69	0.20	23.81	13.53	0.79	2.71	14.31
14	0.48	0.69	0.19	23.85	14.26	0.77	2.84	15.15
15	0.46	0.69	0.21	23.82	16.18	0.99	3.04	17.06
16	0.45	0.68	0.21	23.77	17.27	0.94	3.18	18.33
17	0.46	0.69	0.21	23.70	16.68	0.77	2.77	17.87
18	0.50	0.68	0.18	23.64	17.95	0.93	3.12	19.18
19	0.44	0.69	0.22	23.55	19.79	0.95	3.27	21.06
20	0.44	0.69	0.22	23.53	21.48	1.05	3.37	22.84
21	1.15	0.69	0.07	25.99	0.000	0.00	0.00	0.000
22	1.15	0.69	0.07	25.41	0.950	0.07	0.20	0.989
23	0.83	0.69	0.09	25.49	2.040	0.13	0.39	2.146
24	0.69	0.69	0.10	25.15	3.210	0.21	0.58	3.356
25	0.56	0.69	0.14	25.44	4.630	0.26	0.78	5.093
26	0.62	0.68	0.12	24.47	5.080	0.31	0.96	5.375
27	0.53	0.69	0.16	24.58	6.330	0.33	1.15	6.678
28	0.54	0.69	0.15	24.42	7.590	0.34	1.29	8.132
29	0.47	0.69	0.19	24.94	8.840	0.46	1.46	9.795
30	0.49	0.69	0.18	24.69	10.20	0.49	1.58	10.80
31	0.47	0.68	0.19	23.75	10.90	0.47	1.62	11.79
32	0.87	0.68	0.08	24.94	0.000	0.00	0.00	0.000
33	0.77	0.68	0.09	25.11	0.250	0.02	0.04	0.263
34	0.73	0.68	0.10	25.13	0.480	0.03	0.07	0.510
35	0.62	0.68	0.12	25.01	0.840	0.05	0.11	0.895
36	0.58	0.69	0.13	25.32	1.520	0.08	0.18	1.625
37	0.53	0.69	0.15	24.95	1.720	0.11	0.22	1.788
38	0.54	0.69	0.15	24.49	1.700	0.11	0.22	1.605
39	0.59	0.69	0.13	23.79	2.390	0.15	0.30	2.194
40	0.53	0.69	0.15	23.89	2.300	0.16	0.31	2.144

TABLE II
CALCULATION OF RICHARDSON NUMBER FROM VANONI'S (1946)
EXPERIMENTAL DATA

Run	β	η_{dip} (m)	λ	B	$C_{0.05}$ (%)	C_1 (%)	C_m (%)	R_i
1	0.95	0.932	0.09	20.24	0.000	0.000	0.000	0.000
2	0.83	0.927	0.12	19.35	0.000	0.000	0.000	0.000
3	1.08	0.931	0.09	19.50	0.000	0.000	0.000	0.000
1	0.88	0.930	0.14	19.56	0.098	0.001	0.048	0.598
2	1.07	0.984	0.11	19.22	0.088	0.001	0.043	0.565
3	1.07	0.927	0.10	19.39	0.219	0.001	0.099	1.325
4	0.96	0.928	0.12	19.02	0.196	0.000	0.081	1.197
5	0.96	0.927	0.12	19.32	0.313	0.001	0.139	1.894
6	0.85	0.918	0.12	20.68	0.349	0.002	0.157	2.099
7	0.75	0.925	0.14	19.84	0.317	0.001	0.139	1.929
8	0.81	0.932	0.11	20.72	0.315	0.001	0.137	1.923
9	0.76	0.987	0.14	20.70	0.294	0.000	0.122	1.903
10	0.94	0.933	0.11	20.91	0.294	0.000	0.122	1.815
11	0.70	0.935	0.15	21.32	0.740	0.001	0.304	4.543
12	0.70	0.988	0.12	20.73	0.487	0.000	0.193	3.170
13	0.66	0.933	0.16	20.77	0.528	0.002	0.232	3.241
14	1.03	0.904	0.07	18.35	0.000	0.000	0.000	0.000
14	1.07	0.904	0.07	19.56	0.000	0.000	0.000	0.000
21	1.01	0.997	0.10	16.23	0.000	0.000	0.000	0.000
14	1.02	0.910	0.09	18.70	0.343	0.001	0.147	2.050
15	0.74	0.991	0.10	18.91	0.198	0.001	0.079	1.304
16	1.02	0.910	0.09	20.69	0.114	0.000	0.046	1.371
17	1.01	0.993	0.08	19.08	0.014	0.000	0.005	0.184
18	0.80	0.939	0.13	19.67	0.181	0.004	0.098	2.209
19	0.84	0.996	0.13	20.24	0.049	0.000	0.024	0.645
20	0.97	0.939	0.11	18.77	0.161	0.015	0.103	0.903
21	0.73	0.997	0.18	17.53	0.129	0.008	0.077	0.808
22	0.80	0.988	0.14	18.82	0.255	0.004	0.126	1.639

Hydrodynamics Lab, MIT. 1961.

[15] N. L. Coleman, "Effects of suspended sediment on the open-channel velocity distribution," *Water Resources Research*, vol. 22, no. 10, pp. 1377-1384, 1986.

[16] G. Parker, and N. L. Coleman, "Simple model of sediment-laden flows," *Journal of Hydraulic Engineering*, ASCE, vol. 112, no. 5, pp. 356-375, 1986.

[17] F. Cioffi, and F. Gallerano, "Velocity and concentration profiles of solid particles in a channel with movable and erodible bed," *Journal of Hydraulic Engineering*, ASCE,, vol. 129, no. 3, pp. 387-401, 1991.

[18] J. Guo, "Turbulent velocity profiles in clear water and sediment-laden flows," Ph.D. dissertation, Colorado State University, Fort Collins, CO 1998.

[19] X. Wang, and N. Qian, "Turbulence Characteristics of Sediment-Laden Flow," *Journal Hydraulic Engineering*, vol. 115, no. 6, pp. 781-800, 1989.

[20] S. Q. Yang, S. K. Tan, and S. Y. Lim, "Velocity Distribution and Dip-Phenomenon in Smooth Uniform Open Channel Flows," *Journal Hydraulic Engineering*, vol. 130, no. 12, pp. 1179-1186, December 2004.

[21] X. Wang, Z. Y. Wang, M. Yu, and D. Li, "Velocity profile of sediment suspensions and comparison of log-law and wake-law," *Journal Hydraulic Research*, vol. 39, no. 2, pp. 211-217, 2001.

[22] J. Guo, and P. Y. Julien, "Turbulent velocity profiles in sediment-laden flows," *Journal Hydraulic Research*, vol. 39, no. 1, pp. 11-23, 2001.

Transaction of ASCE, vol. 111, pp. 67-133, 1946.

[12] H. A. Einstein, and N. Chien, "Effects of heavy sediment concentration near the bed on velocity and sediment distribution," U. S. Army Corps of Engineers, Missouri River Division Rep. No.8, 1955.

[13] V. A. Vanoni, and G. N. Nomicos, "Resistance properties in sediment-laden streams," *Trans.*, ASCE, vol. 125, pp. 1140-1175, 1960.

[14] C. Elata, and A. T. Ippen, "The dynamics of open channel flow with suspensions of neutrally buoyant particles," Technical Report No.45,