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Analysis of an Electrical Transformer: A Bond Graph Approach

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***Abstract*— Bond graph models of an electrical transformer including the nonlinear saturation are presented. These models determine the relation between self and mutual inductances, and the leakage and magnetizing inductances of power transformers with two and three windings using the properties of a bond graph. The modelling and analysis using this methodology to three phase power transformers or transformers with internal incipient faults can be extended.**

Keywords– Bond graph, electrical transformer, nonlinear saturation.

I. Introduction

TRANSFORMERS make large power systems possible. To transmit hundreds of megawatts of power efficiently over long distances. The main uses of electrical transformers are for changing the magnitude of an AC voltage providing electrical isolation, and matching the load impedance to the source [1].

On the other hand, a bond graph is a model of a dynamic system where a collection of components interact with each other through energy ports. A bond graph consist of subsystems linked by lines to show the energetic connections. It can represent a variety of energy types and can describe how the power flows through the system [2], [3].

In [4] a magnetic circuit model of power transformer which takes into account the nonlinear hysteresis phenomenon is analyzed. However, this paper uses a special nonlinear function to introduce the hysteresis.

In [6] a bond graph model of a transformer based on a nonlinear conductive magnetic circuit is described. Here, the state space nonlinear magnetic model has to be known.

Therefore, in this paper bond graph models of a transformer with two windings using an I-field and TF element are proposed. The relationship between these models allow to determine the self and mutual inductances equations in terms of leakage and magnetizing inductances of each winding. Moreover, bond graph models with I-field and TF elements of a transformer with three windings in order to obtain the relation between both models are proposed. Also, a basic electromagnetic model for the magnetizing branch of a transformer with two or three windings in the physical domain is described. This magnetizing branch consists of a resistor and inductance. However, in order to introduce the magnetic saturation a nonlinear function is used.

The outline of the paper is as follows: Section II gives some basic elements of the modelling in bond graph. Section III summarizes the model of a two winding transformer including the flux linkage and voltage equations. A bond graph model of a transformer with two windings is proposed in section IV. The two winding transformer in the physical domain considering the linear and nonlinear core are presented in section V. Section VI proposes a bond graph model of a transformer with three windings including the core. Finally, section VII gives the conclusions.

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II. Bond Graph Model

Consider the following scheme of a multiport LTI system which includes the key vectors of Fig. 1 [2], [7].

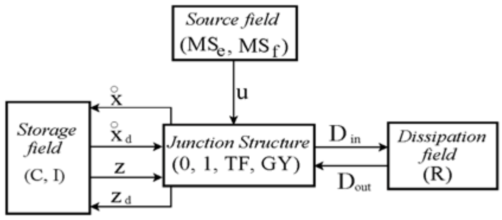

Fig. 1. Key vectors of a bond graph.

In Fig. 1, (MS_e, MS_f), (C, I) and (R) denote the source, the energy storage and the energy dissipation fields, and $(0, 1, TF, GY)$ the junction structure with transformers, TF, and gyrators, GY.

The state $x \in \Re^n$ and $x_d \in \Re^m$ are composed of energy variables p and q associated with I and C elements in integral and derivative causality, respectively, $u \in \Re^p$ denotes the plant input, $z \in \Re^n$ the co-energy vector, $z_d \in \Re^m$ the derivative co-energy and $D_{in} \in \Re^r$ and $D_{out} \in \Re^r$ are a mixture of e and f showing the energy exchanges between the dissipation field and the junction structure.

The relations of the storage and dissipation field are,

$$z = Fx \tag{1}$$
$$z_d = F_d x \tag{2}$$
$$D_{out} = LD_{in} \tag{3}$$

The relations of the junction structure are [2], [7],

$$\begin{bmatrix} \dot{x} \\ D_{in} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & 0 \end{bmatrix} \begin{bmatrix} z \\ D_{out} \\ u \\ \dot{x}_d \end{bmatrix}$$
$$z_d = -S_{14}^T z \tag{4}$$

The entries of S take values inside the set $\{0, \pm 1, \pm m, \pm n\}$ where m and n are transformer and gyrator modules; S_{11} and S_{22} are square skew-symmetric matrices and S_{12} and S_{21} are matrices each other negative transpose. The state equation is,

$$\dot{x} = Ax + Bu \tag{5}$$

where

$$A = E^{-1}\left(S_{11} + S_{12}MS_{21}\right)F \quad (6)$$

$$B = E^{-1}\left(S_{13} + S_{12}MS_{23}\right) \quad (7)$$

being

$$E = I_n + S_{14}F_d^{-1}S_{14}^T F \quad (8)$$

$$M = \left(I_n - LS_{22}\right)^{-1}L \quad (9)$$

It is very common in electrical power systems to use the electrical current as state variable of this manner taking the derivative of (1) and (5), we have

$$\dot{z} = \overline{A}z + \overline{B}u \quad (10)$$

where

$$\overline{A} = FAF^{-1} \quad (11)$$

$$\overline{B} = FB \quad (12)$$

Next section summarizes the basic elements of an electrical transformer.

III. Model of a two-winding Transformer

Charles P. Steinmetz (1865-1923) developed the circuit model that is universally used for the analysis of iron core transformers at power frequencies. His model has many advantages over those resulting from straightforward application of linear circuit theory, primarily because the iron core exhibits saturation and hysteresis and is thus definitely nonlinear [1]. However it is good idea to consider transformers first from the point of view of basic linear circuit theory to better appreciate the Steinmetz model.

A. Flux Linkage Equations

Consider the magnetic coupling between the primary and secondary windings of a transformer shown in Fig. 2 [5].

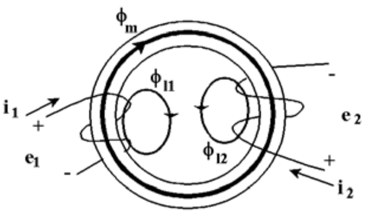

Fig. 2. Magnetic coupling of a two-winding transformer.

The total flux linked by each winding may be divided into two components: a mutual component, ϕ_m, that is common to both windings, and a leakage flux components that links only the winding itself. In terms of these flux components, the total flux by each of the windings can be expressed as,

$$\phi_1 = \phi_{l1} + \phi_m \quad (13)$$

$$\phi_2 = \phi_{l2} + \phi_m \quad (14)$$

where ϕ_{l1} and ϕ_{l2} are the leakage flux components of windings 1 and 2, respectively. Assuming that N_1 turns of winding 1 effectively link ϕ_m and ϕ_{l1}, the flux linkage of winding 1 is defined by,

$$\lambda_1 = N_1\phi_1 = N_1\left(\phi_{l1} + \phi_m\right) \quad (15)$$

the leakage and mutual fluxes can be expressed in terms of the winding currents using the magneto-motive forces (mmfs) and permeances. So, the flux linkage of winding 1 is,

$$\lambda_1 = N_1\left[N_1 i_1 P_{l1} + \left(N_1 i_1 + N_2 i_2\right)P_m\right] \quad (16)$$

where $P_{l1} = \dfrac{\phi_{l1}}{N_1 i_1}$ and $P_m = \dfrac{\phi_m}{N_1 i_1 + N_2 i_2}$.

Similarly, the flux linkage of winding 2 can be expressed as,

$$\lambda_2 = N_2\left(\phi_{l2} + \phi_m\right) \quad (17)$$

and using mmfs and permeances for this winding,

$$\lambda_2 = N_2\left[N_2 i_2 P_{l2} + \left(N_1 i_1 + N_2 i_2\right)P_m\right] \quad (18)$$

The resulting flux linkage equations for the two magnetically coupled windings, expressed in terms of the winding inductances are,

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (19)$$

where L_{11} and L_{22} are the self-inductances of the windings, and L_{12} and L_{21} are the mutual inductances between them.

Note that the self-inductance of the primary can be divided into two components, the primary leakage inductance, L_{l1} and the primary magnetizing inductance, L_{m1}, which are defined by,

$$L_{11} = L_{l1} + L_{m1} \quad (20)$$

where $L_{l1} = N_1^2 P_{l1}$ and $L_{m1} = N_1^2 i_1 P_m$.

Likewise, for winding 2

$$L_{22} = L_{l2} + L_{m2} \quad (21)$$

where $L_{l2} = N_2^2 P_{l2}$ and $L_{m2} = N_2^2 i_2 P_m$.

Finally, the mutual inductance is given by,

$$L_{12} = N_1 N_2 i_2 P_m \quad (22)$$

$$L_{21} = N_1 N_2 i_1 P_m \quad (23)$$

Taking the ratio of L_{m2} a L_{m1},

$$L_{m2} = \frac{N_2\phi_m}{i_2} = \frac{N_2 L_{12}}{N_1} = N_2^2 P_m = \left(\frac{N_2}{N_1}\right)^2 L_{m1} \quad (24)$$

B. Voltage Equations

The induced voltage in winding 1 is given by,

$$e_1 = \frac{d\lambda_1}{dt} = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} \quad (25)$$

replacing L_{11} by $L_{l1} + L_{m1}$ and $L_{12}i_2$ by $N_2 L_{m1} i_2/N_1$, we obtain

$$e_1 = L_{l1}\frac{di_1}{dt} + L_{m1}\frac{d\left(i_1 + \left(N_2/N_1\right)i_2\right)}{dt} \quad (26)$$

Similarly, the induced voltage of winding 2 is written by,

$$e_2 = L_{l2}\frac{di_2}{dt} + L_{m2}\frac{d\left(i_2 + \left(N_1/N_2\right)i_1\right)}{dt} \quad (27)$$

Finally, the terminal voltage of a winding is the sum of the induced voltage and the resistive drop in the winding, the complete equations of the two windings are,

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 i_1 \\ r_2 i_2 \end{bmatrix} + \begin{bmatrix} L_{l1} + L_{m1} & a^{-1} L_{m1} \\ a L_{m2} & L_{l2} + L_{m2} \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix} \tag{28}$$

where $a = N_1/N_2$.

Next section a bond graph model of a transformer with two windings is proposed.

IV. Bond Graph Model of a Transformer with two Windings

The bond graph methodology allows to model a system in a simple and direct manner. Using fields and junction structures, one may conveniently study systems containing complex multiport components using bond graphs. In fact, bond graphs with fields prove to be a most effective way to handle the modeling of complex multiport systems [2]. In Fig 3 shows a equivalent circuit for a two winding transformer using an I-field that represents a transformer with two windings and taking account the self and mutual inductances.

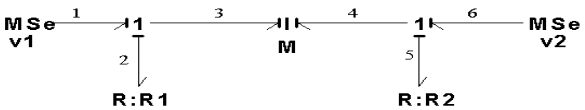

Fig. 3. Bond Graph of a two windings transformer using I-field.

The key vectors of the bond graph are,

$$\begin{aligned} x &= \begin{bmatrix} p_3 \\ p_4 \end{bmatrix}; \dot{x} = \begin{bmatrix} e_3 \\ e_4 \end{bmatrix}; z = \begin{bmatrix} f_3 \\ f_4 \end{bmatrix} \qquad (29) \\ D_{in} &= \begin{bmatrix} f_2 \\ f_5 \end{bmatrix}; D_{out} = \begin{bmatrix} e_2 \\ e_5 \end{bmatrix}; u = \begin{bmatrix} e_1 \\ e_6 \end{bmatrix} \end{aligned}$$

the constitutive relations of the fields are,

$$L = diag\left\{R_1, R_2\right\} \tag{30}$$

$$F^{-1} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \tag{31}$$

and the junction structure is,

$$S_{21} = S_{13} = -S_{12} = I_2; S_{11} = S_{22} = S_{23} = 0 \tag{32}$$

From (11), (12), (30), (31) and (32) the state space representation is,

$$\dot{z} = \frac{1}{\Delta} \begin{bmatrix} L_{22} & -L_{12} \\ -L_{12} & L_{11} \end{bmatrix} \left\{ \begin{bmatrix} -R_1 & 0 \\ 0 & -R_2 \end{bmatrix} z + \begin{bmatrix} e_1 \\ e_6 \end{bmatrix} \right\} \tag{33}$$

Now, a bond graph model of a transformer with two windings using leakage inductance, L_l and the magnetizing inductance, L_m in each winding is proposed in Fig 4.

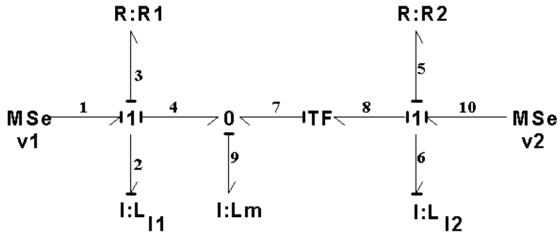

Fig. 4. Bond graph of a two windings transformer using leakage and magnetizing inductances.

The key vectors of the bond graph are,

$$\begin{aligned} x &= \begin{bmatrix} p_2 \\ p_6 \end{bmatrix}; \dot{x} = \begin{bmatrix} e_2 \\ e_6 \end{bmatrix}; z = \begin{bmatrix} f_2 \\ f_6 \end{bmatrix} \qquad (34) \\ D_{in} &= \begin{bmatrix} f_3 \\ f_5 \end{bmatrix}; D_{out} = \begin{bmatrix} e_3 \\ e_5 \end{bmatrix}; u = \begin{bmatrix} e_1 \\ e_{10} \end{bmatrix} \\ x_d &= p_9; \dot{x}_d = p_9; z_d = f_9 \end{aligned}$$

the constitutive relations of the fields are,

$$L = diag\left\{R_1, R_2\right\} \tag{35}$$

$$F^{-1} = diag\left\{L_{l1}, L_{l2}\right\} \tag{36}$$

$$F_d^{-1} = L_m \tag{37}$$

and the junction structure,

$$\begin{aligned} S_{21} &= -S_{12} = S_{13} = I_2; S_{14}^T = \begin{bmatrix} -1 & -a^{-1} \end{bmatrix} \\ S_{11} &= S_{22} = S_{23} = 0 \qquad (38) \end{aligned}$$

From (8), (36), (37) and (38) the relationship between the storage field in integral causality and the storage element in derivative causality is,

$$E = \begin{bmatrix} 1 + \frac{L_m}{L_{l1}} & \frac{L_m}{aL_{l2}} \\ \frac{L_m}{aL_{l1}} & 1 + \frac{L_m}{a^2 L_{l2}} \end{bmatrix} \tag{39}$$

The state matrix of this system is given by,

$$\overline{A} = \frac{1}{\Delta_1} \begin{bmatrix} L_{l2} + \frac{L_m}{a^2} & \frac{-L_m}{a} \\ \frac{-L_m}{a} & L_{l1} + L_m \end{bmatrix} \begin{bmatrix} -R_1 & 0 \\ 0 & -R_2 \end{bmatrix} \tag{40}$$

where $\Delta_1 = L_{l1} L_{l2} + L_{l1} L_m a^{-2} + L_m L_{l2}$ and

$$\overline{B} = E^{-1} F I_2 \tag{41}$$

Note that (40) is the same result considering the self and mutual inductances.

If we use the following numerical values of the parameters of the transformer, the simulation of the two bond graph models can be compared.

The numerical values of the parameters of the bond graph of Fig. 4 are $L_{l1} = 1.59mH$, $L_{l2} = 6.34mH$, $L_m = 3.19mH$, $R_1 = 4\Omega$, $R_2 = 16\Omega$, $a = 10$, and comparing (33) with (40) and (41) yields $L_1 = 4.78mH$, $L_2 = 19.1mH$ and $L_{12} = 6.38mH$. Also, the simulation of both bond graph is shown in Fig. 5.

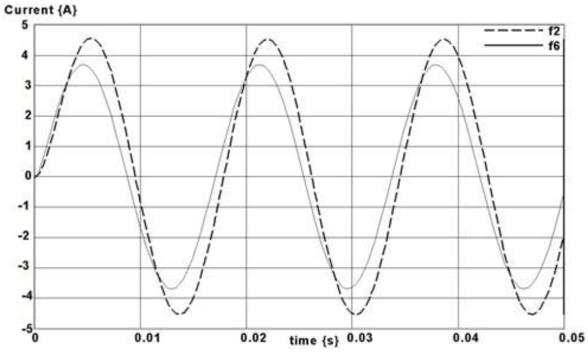

Fig. 5. Simulation of a transformer with two windings.

Next section proposes a two windings transformer including the linear and nonlinear core in the physical domain.

V. Bond Graph of a Two Windings Transformer with Core

The final concept involved in the Steinmetz transformer model is a scheme for handling the nonlinearity of the core and the core losses. The Steinmetz model approaches the problem of representing core excitation by first dividing it into two parts: magnetization and core losses. In order to consider the core losses of a transformer a bond graph model is presented in Fig. 6.

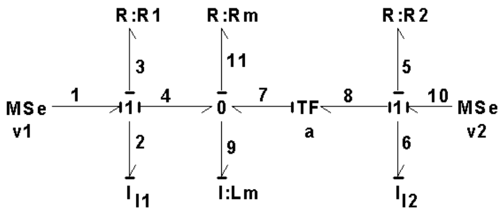

Fig. 6. Bond graph of a complete transformer.

By using the same numerical values of the transformer parameters of previous section with $R_m = 80\Omega$ and considering a linear performance of the core, the Fig. 7 shows the simulation of this transformer.

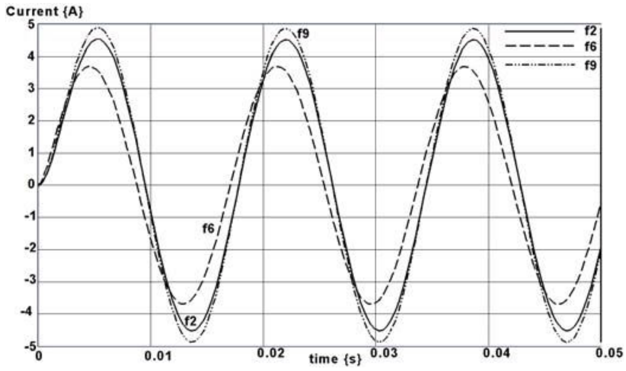

Fig. 7. Simulation of a transformer with linear core.

The incorporation of nonlinear effects such as magnetic saturation and hysteresis is achieved in the transformer model with the appropriate modification of the inductance L_m in the bond graph of Fig. 8.

In Fig. 8 the saturation curve is illustrated and this curve is approximated with the equation [8],

$$i_{Lm} = \tan\left(\frac{\lambda_{Lm}}{0.0319}\right) \qquad (42)$$

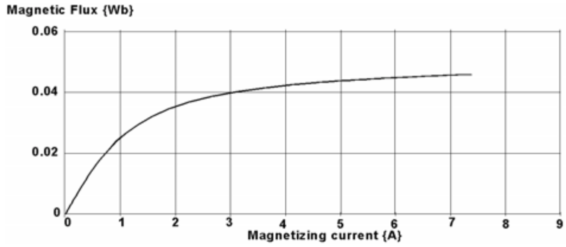

Fig. 8. Saturation curve of equation (42) .

If we introduce (42) to the bond graph model of Fig. 6 the nonlinear phenomena is incorporated. Fig. 9 shows the saturation performance in the bond graph model of the transformer.

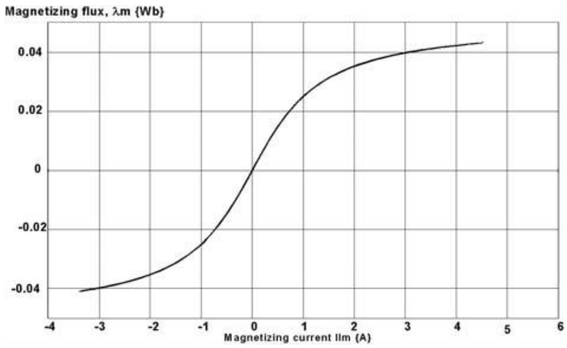

Fig. 9. Nonlinear performance of the transformer of Fig. 6.

The hysteresis losses and the nonlinear magnetizing inductance performance of the proposed bond graph model is shown in Fig. 10.

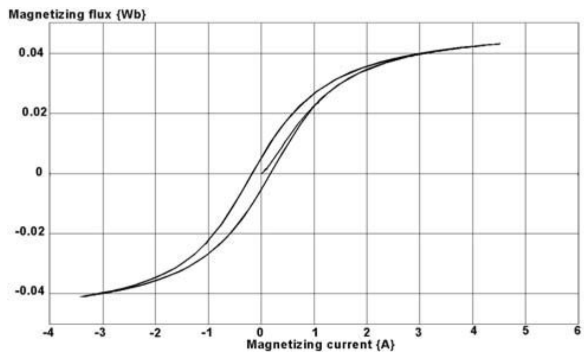

Fig. 10. Hysteresis curve of the transformer.

The exciting and secondary current of the bond graph model of the transformer are shown in Fig. 11.

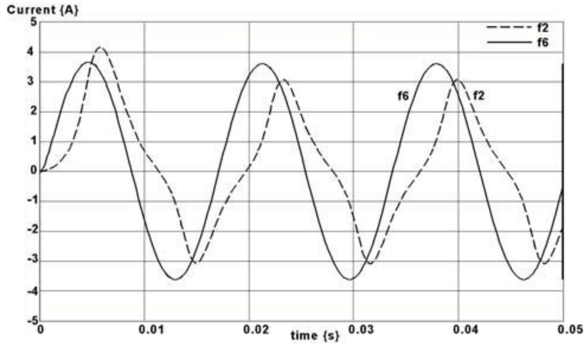

Fig. 11. Exciting and secondary current of a transformer with nonlinear core.

The analysis of a transformer with two windings can be generalized. Thus, next section presents a bond graph model of a transformer with three windings.

VI. Bond Graph Model of a Transformer with three Windings

An electrical transformer has been considered as a device of two windings and a core. The relationship between these windings is defined by the ratio of the numbers of turns in the two windings, $a = N_1/N_2$. However, in some cases it is interesting and necessary to change the number of the windings in a transformer. Fig. 12 shows a bond graph model of a transformer with three windings using an I-field.

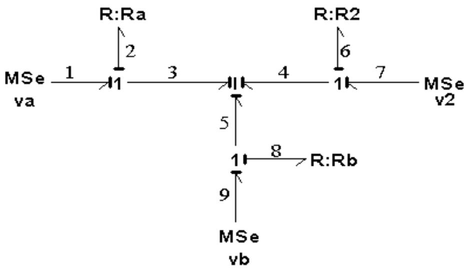

Fig. 12. Bond graph of a transformer with three windings using an I-field.

The key vectors of the bond graph are,

$$x = \begin{bmatrix} p_3 \\ p_5 \\ p_4 \end{bmatrix}; \dot{x} = \begin{bmatrix} e_3 \\ e_5 \\ e_4 \end{bmatrix}; z = \begin{bmatrix} f_3 \\ f_5 \\ f_4 \end{bmatrix} \quad (43)$$

$$D_{in} = \begin{bmatrix} f_2 \\ f_8 \\ f_6 \end{bmatrix}; D_{out} = \begin{bmatrix} e_2 \\ e_8 \\ e_6 \end{bmatrix}; u = \begin{bmatrix} e_1 \\ e_9 \\ e_7 \end{bmatrix}$$

the constitutive relations of the fields are,

$$L = diag\{R_a, R_b, R_2\} \quad (44)$$

$$F^{-1} = \begin{bmatrix} L_3 & M_{35} & M_{34} \\ M_{35} & L_5 & M_{54} \\ M_{34} & M_{54} & L_4 \end{bmatrix} \quad (45)$$

and the junction structure is,

$$S_{12} = -S_{21} = S_{13} = I_3, S_{11} = S_{22} = S_{23} = 0 \quad (46)$$

From (11), (12), (44), (45) and (46) the matrices of the state space are,

$$\bar{A}_p = F \begin{bmatrix} -R_a & 0 & 0 \\ 0 & -R_b & 0 \\ 0 & 0 & -R_c \end{bmatrix}; \ \bar{B}_p = F \begin{bmatrix} e_1 \\ e_9 \\ e_7 \end{bmatrix} \quad (47)$$

Also, a bond graph model using the leakage and magnetizing inductances in each winding is proposed in Fig 13.

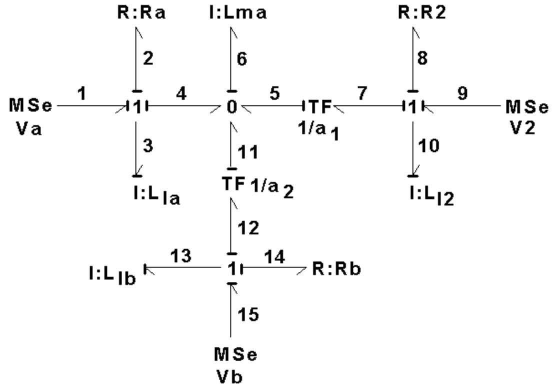

Fig. 13. Bond graph of a transformer with three windings using leakage and magnetizing inductances.

The key vectors of the bond graph are,

$$x = \begin{bmatrix} p_3 \\ p_{10} \\ p_{13} \end{bmatrix}; \dot{x} = \begin{bmatrix} e_3 \\ e_{10} \\ e_{13} \end{bmatrix}; z = \begin{bmatrix} f_3 \\ f_{10} \\ f_{13} \end{bmatrix}; \ \begin{matrix} x_d = p_6 \\ \dot{x}_d = e_6 \\ z_d = f_6 \end{matrix}$$

$$D_{in} = \begin{bmatrix} f_2 \\ f_8 \\ f_{14} \end{bmatrix}; D_{out} = \begin{bmatrix} e_2 \\ e_8 \\ e_{14} \end{bmatrix}; u = \begin{bmatrix} e_1 \\ e_9 \\ e_{13} \end{bmatrix} \quad (48)$$

the constitutive relations are,

$$F = diag\left\{\frac{1}{L_{la}}, \frac{1}{L_{l2}}, \frac{1}{L_{lb}}\right\} \quad (49)$$

$$F_d = \frac{1}{L_{ma}} \quad (50)$$

$$L = diag\{R_a, R_2, R_b\} \quad (51)$$

and the junction structure is,

$$S_{21} = -S_{12} = S_{13} = I_3; S_{11} = S_{22} = S_{23} = 0 \quad (52)$$

$$S_{14} = -\begin{bmatrix} 1 & a_1^{-1} & a_2^{-1} \end{bmatrix}^T$$

where $a_1 = N_a/N_2$ and $a_2 = N_b/N_2$.

The submatrix S_{14} gives the relation between the storage elements in an integral and derivative causality assignment. In this case, between leakage and magnetizing inductances, respectively.

Therefore, from (11) and (12) the state space representation is,

$$\overline{A} = -FE^{-1}L; \quad (53)$$

$$\overline{B} = Fu \quad (54)$$

where

$$E = \begin{bmatrix} 1 + \frac{L_{ma}}{L_{la}} & \frac{a_1^{-1}L_{ma}}{L_{l2}} & \frac{a_2^{-1}L_{ma}}{L_{lb}} \\ \frac{a_1^{-1}L_{ma}}{L_{la}} & 1 + \frac{a_1^{-2}L_{ma}}{L_{l2}} & \frac{a_1^{-1}a_2^{-1}L_{ma}}{L_{lb}} \\ \frac{a_2^{-1}L_{ma}}{L_{la}} & \frac{a_1^{-1}a_2^{-1}L_{ma}}{L_{l2}} & 1 + \frac{a_2^{-1}L_{ma}}{L_{lb}} \end{bmatrix} \quad (55)$$

By comparing (45) with (55) the self-inductances are given by,

$$L_3 = L_{la} + L_{ma} \quad (56)$$
$$L_5 = L_{lb} + a_1^{-2} L_{ma} \quad (57)$$
$$L_4 = L_{l2} + a_2^{-2} L_{ma} \quad (58)$$

and the mutual inductances are,

$$M_{35} = a_1^{-1} L_{ma} \quad (59)$$
$$M_{34} = a_2^{-1} L_{ma} \quad (60)$$
$$M_{45} = a_1^{-1} a_2^{-1} L_{ma} \quad (61)$$

The relationship between the bond graph with three windings and two windings shows that, $N_1 = N_a + N_b$, $R_1 = R_a + R_b$ and $L_{l1} = L_{la} + L_{lb}$ which yields $R_a = \frac{N_a}{N_1} R_1$; $R_b = \frac{N_b}{N_1} R_1$, $R_{ma} = \left(\frac{N_a}{N_1}\right)^2 R_m$, $L_{la} = \frac{N_a^2}{N_a^2 + N_b^2} L_{l1}$, $L_{lb} = \frac{N_b^2}{N_a^2 + N_b^2} L_{l1}$ and $L_{ma} = \left(\frac{N_a}{N_1}\right)^2 L_{m1}$.

The numerical parameters of the bond graph models of Fig. 12 and 13 are: $L_3 = 3.538mH$, $L_4 = 6.4676mH$, $L_5 = 0.1254mH$, $M_{35} = 0.2552mH$, $M_{34} = 0.5104mH$, $M_{45} = 0.0638mH$, $R_a = 3.2\Omega$, $R_b = 0.8\Omega$, $R_2 = 16\Omega$, $a_1 = 8$, $a_2 = 4$, $V_a = 20\sin(377t)\,V$, $V_b = 80\sin(377t)V$ and $V_2 = 60\sin(377t)\,V$. From (56) to (61) yields $L_{la} = 1.4964mH$, $L_{lb} = 0.09352mH$, $L_{l2} = 6.34mH$, $L_{ma} = 2.0416mH$. The response of the both equivalent transformers is shown in Fig. 14.

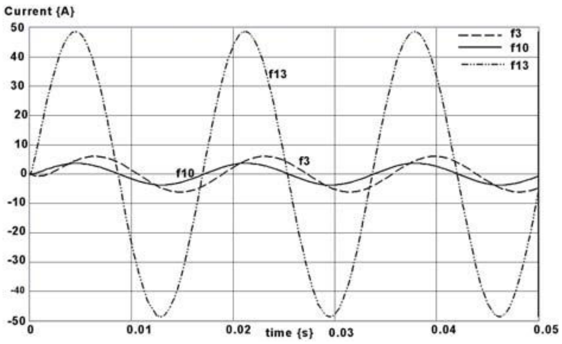

Fig. 14. Electrical currents of the storage elements of Fig. 12 and 13.

By incorporating the core losses to the bond graph model of Fig. 13, the inductance L_{ma} is nonlinear and linearly independent which is shown in Fig. 15.

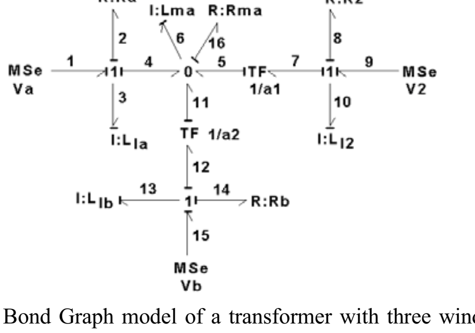

Fig. 15. Bond Graph model of a transformer with three windings.

Finally, including nonlinear magnetization characteristics given by (42), the transformer response with three windings is shown in Fig. 16.

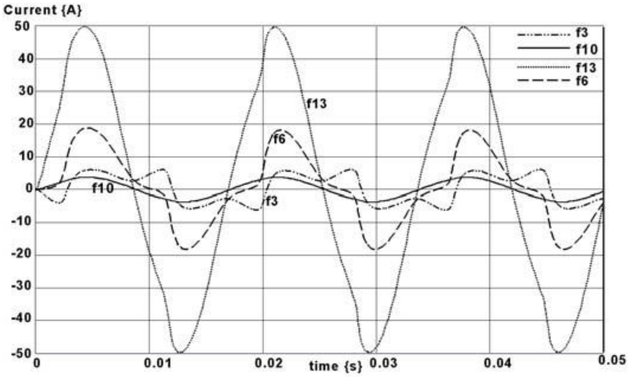

Fig. 16. Transformer response with three windings.

Note that the Fig. 16 shows the typical exciting currents of a transformer with a nonlinear core. In addition, this analysis to obtain the bond graph model of a three phase transformer can be applied.

VII. Conclusions

Bond graph models of a power transformer incorporating the nonlinear saturation are presented. These models allow to obtain relations between the self and mutual inductances, and the leakage and magnetizing inductances in a simple and direct manner using the derivative causality assignment of a bond graph. In order to prove the results the graphical simulation are shown. These models can be extended to transformers with internal incipient faults and three phase power transformers.

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