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# An Approach for Transient Response Calculation of large Nonproportionally Damped Structures using Component Mode Synthesis

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***Abstract*—**A minimal complexity version of component mode synthesis is presented that requires simplified computer programming, but still provides adequate accuracy for modeling lower eigenproperties of large structures and their transient responses. The novelty is that a structural separation into components is done along a plane/surface that exhibits rigid-like behavior, thus only normal modes of each component is sufficient to use, without computing any constraint, attachment, or residual-attachment modes. The approach requires only such input information as a few (lower) natural frequencies and corresponding undamped normal modes of each component. A novel technique is shown for formulation of equations of motion, where a double transformation to generalized coordinates is employed and formulation of nonproportional damping matrix in generalized coordinates is shown.

***Keywords*—**component mode synthesis, finite element models, transient response, nonproportional damping

## I. INTRODUCTION

FOR large structures, when it is necessary to modify the structure many times (like in optimization tasks) and run eigensolution each time, it is expedient to use a component mode synthesis (CMS) technique. This technique constructs a reduced eigenvalue problem that approximates the original (large) eigenvalue problem in terms of the lowest eigenvalues and normal modes. Upon solving this reduced eigenproblem, one can find the eigenvalues and construct the normal modes (pertaining to the whole structure) and they can be used further, for example in a transient analysis.

There are different CMS methods described in literature that can be classified on properties of the interface (the boundary between components), namely: 1) fixed interface methods [1], [2], where normal modes of constrained (at the interface d-o-fs) components are used; 2) free interface methods [3] - [9], where normal modes with free at the interface d-o-fs are used and 3) hybrid versions [10], [11], [12], where component's normal modes are obtained with some interface d-o-fs constrained and some free. In [13], all the versions are described and a method how to improve the accuracy by solving iteratively the condensed eigenvalue problem is shown. A free-interface method, where higher-order residual attachment modes are included is shown in [14]. A CMS version with a combination of interface types (free-free, fixed-fixed, fixed with overlapping elements) is considered in [15]. A fixed interface method was considered in [16], where of a residual term is employed (representing the effect of normal modes not retained) and where an iterative technique is employed for nonlinear equations of modal synthesis. Structures with general (nonproportional) damping were considered in [17], where complex residual flexibility modes, state-space free-free dynamic modes were used. Structures with incompatible substructure interfaces were considered in [18], where a hybrid Craig-Bampton method was described. In all these sources, along with the substructure's normal modes (eigenvectors), an additional set of modes (or basis vectors) was employed, namely either constraint modes, or attachment modes, or residual-attachment modes. These extra modes require knowledge of stiffness matrices (in physical coordinates) of each component, and additional static calculations using a finite element software, or analyst's own code are required to calculate those modes. It appears that there was no discussion in literature for what cases (structures), these extra modes can be safely neglected.

A novelty approach will be demonstrated in this paper, where there is no need to compute any constraint, attachment, or residual-attachment modes, and no need to have stiffness, or mass matrices of the components in physical coordinates. For brevity, a two-component approach is shown here. The 1st component embraces most of the structure (the large component) and 2nd component is a small size component – the component that embraces the area where all modifications are to be made, plus possibly some surrounding area. The interface between these two components is chosen along a plane/surface that is considered stiff enough to assume a rigid-like behavior (or very small deformation). In this case, the motion of the whole structure can be adequately spanned by the space of lower normal modes of these two components with the free interface (rigid-body modes of components are included). The feasibility of selection of such a rigid-like

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interface depends on the structure's stiffness distribution and it is up to the analyst's intuition to choose a stiff area where to split the structure into two components. For example, presence of some sort of stiffeners (stiffening rings, ribs, or plates) in the structure could a be good location for splitting.

The suggested here approach requires only such input information as a few lower natural frequencies of each component and corresponding mass-normalized normal undamped modes of each component. This input can come from an experimental modal survey data, or provided to the analyst from FE software normal mode analysis (like NASTRAN etc).

All the rest calculations can be done by either analyst's own code, or in MATLAB. Note that in all formulated equations, the values of component's normal mode at certain d-o-fs (only) will be needed, and not the whole component's normal mode. These d-o-fs are: 1) interface d-of-fs (where the components are split), 2) d-of-fs where the external forces are applied, 3) d-of-fs where the output of displacement/velocity/acceleration is requested, and 4) d-o-fs to which the dashpots are connected. For all other d-o-fs, the component's normal mode values are not needed and thus no need to be stored that leads to convenience in operating with small size files. As it will be seen later, the method will match the exact results (NASTRAN unsplitted model solution) very well, and as to the analyst's effort – it is less costly than all other CMS versions.

Following the CMS method presentation, a transient response solution technique is demonstrated, where a double transformation to the generalized coordinates is employed. Also a technique that creates a nonproportional damping matrix in the generalized coordinates for the structure with dashpots is presented. Numerical results (structure's natural frequencies, mode shapes, physical displacements/ accelerations in transient analysis) are obtained for an example of two-component structure.

## II. COMPONENT MODE SYNTHESIS TECHNIQUE

The CMS technique shown below is as follows in [4], except that the residual-attachment modes are not included and a state-space representation is not used. The state-space representation was used in [4], because the purpose was to conduct a complex eigenvalue analysis. In this paper, the objective is to obtain natural frequencies (not complex eigenvalues) and lower undamped normal modes of the whole structure, that will be used in a transient analysis (to approximate the solution) for the structure with nonproportional damping.

An equation of free vibration (in one of natural modes ) in physical coordinates pertaining to a FE model of undamped structure is written as

$$(-\omega^2 M + K)\varphi = 0 \tag{1}$$

that constitutes an eigenvalue problem. Let's split the structure into two substructures (components) that have mass and stiffness matrices $m_1$, $k_1$, $m_2$, $k_2$ respectively and there will be interface loads at the separation border. Then one can re-write the equation of free vibration (1) for the splitted up structure as shown below:

$$\left( -\omega^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \right) \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

or in abbreviated form

$$\left( -\omega^2 \hat{M} + \hat{K} \right) \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{2}$$

where $\varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$ is the normal mode (column) with related to physical displacements of the 1st component and related to physical displacements of the 2nd component; are columns of nodal loads at the interface between these two components (not to confuse with external loads). Note that since a free vibration motion is considered in this section, there are no external loads acting on the structure.

One can introduce an approximation by transformation to modal coordinates:

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \approx \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \left[\hat{\Phi}\right] p \tag{3}$$

where $\Phi_1, \Phi_2$ are mass-normalized normal modes of 1st and 2nd free-interface components (rigid body modes are included, if a component comes as free-free) and $p = [p_1 \; p_2]^T$ are modal coordinates.

At the interface between two components, a condition of compatibility is used, i.e. the equality of physical displacements. Assigning all interface degrees of freedom in matrices $\Phi_1, \Phi_2$ as with subscript "*B*" (meaning "border") and internal degrees of freedom with subscript "*i*", one can re-write matrix $\Phi_1$ as

$$\Phi_1 = \begin{bmatrix} \Phi_{1B} \\ \Phi_{1i} \end{bmatrix}$$

Thus the compatibility equation can be written as

$$\Phi_{1B}\, p_1 = \Phi_{2B}\, p_2$$

Let's say, there are *B* degrees-of-freedom at the interface, then, for example for 2nd component, one can select *B* dependent coordinates (among those $p_2$ coordinates) and re-write the above expression in other form (indeed it is assumed that the size of column $p_2 > B$ ).

$$\Phi_{1B}\, p_1 = [\Phi_{2B}^{dep}\; \Phi_{2B}^{ind}] \begin{bmatrix} p_2^{dep} \\ p_2^{ind} \end{bmatrix}$$

or

$$\Phi_{1B}\, p_1 = \Phi_{2B}^{dep}\, p_2^{dep} + \Phi_{2B}^{ind}\, p_2^{ind} \tag{4}$$

where superscript "*dep*" stands for dependent coordinates and "*ind*" stands for independent ones. Note that matrix $\Phi_{2B}^{dep}$ will be a square matrix. Now one can express the dependent ones from (4) as

$$p_2^{dep} = [\Phi_{2B}^{dep}]^{-1} (\Phi_{1B} p_1 - \Phi_{2B}^{ind} \, p_2^{ind}) \tag{5}$$

Therefore the column of generalized coordinates $p = [p_1 \; p_2]^T$ can be expressed (taking into account the compatibility equation (5)) as

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} I & | & 0 \\ [\Phi_{2B}^{dep}]^{-1}\Phi_{1B} & | & -[\Phi_{2B}^{dep}]^{-1}\Phi_{2B}^{ind} \\ 0 & | & I \end{bmatrix} \begin{bmatrix} p_1 \\ p_2^{ind} \end{bmatrix}$$

or introducing a new matrix notation $\beta$, it can be written in abbreviated form

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \beta \begin{bmatrix} p_1 \\ p_2^{ind} \end{bmatrix}$$

Let's denote this reduced (final) column of generalized coordinates as $q$

$$\begin{bmatrix} p_1 \\ p_2^{ind} \end{bmatrix} = q$$

then one can re-write (3) as

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \approx \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = [\hat{\Phi}] p = [\hat{\Phi}] \beta q \tag{6}$$

Now substituting (6) in (2) and pre-multiplying by $\beta^T \hat{\Phi}^T$ one obtains

$$\beta^T \hat{\Phi}^T (-\omega^2 \hat{M} + \hat{K}) \hat{\Phi} \beta q = 0 \tag{7}$$

Notice that due to equality of displacements and opposite forces at the interface, the right-hand side of equation (7) is 0 (see [4] for details). Introducing new matrix notations *A* and *B*, the equation (7) can be re-written as a reduced eigenvalue problem:

$$(-\lambda B + A) q = 0 \tag{8}$$

where $\beta^T \hat{\Phi}^T \hat{M} \hat{\Phi} \beta = B$, $\beta^T \hat{\Phi}^T \hat{K} \hat{\Phi} \beta = A$, $\lambda = \omega^2$ is an eigenvalue and $q$ is the corresponding eigenvector. Matrices A and B are real symmetric matrices, the eigenvalues $\lambda$ will be real and positive and eigenvectors will be all real. The size of this egenvalue problem (8) is equal to the number of modes chosen for the 1st component plus the number of independent modes chosen for the 2nd component.

Upon modifying 2nd component - that was chosen as small one - no need to solve the large eigenvalue problem (1), but instead just find new $\Phi_2$ (normal modes of 2nd component), substitute them in (3) and (6) and then solve the reduced eigenvalue problem (8). Notice that in matrices $\Phi_1$ and $\Phi_2$ (components' normal modes) only the values for certain physical degrees of freedom are necessary to know, namely, for interface d-of-fs (where the components are split), and (as it will be seen later) for d-of-fs where the external forces are applied; where the response (displacements, velocities, or accelerations) need to be determined and where dashpots are present (connecting the specified degrees of freedom).

Also notice that in (7), the following holds

$$\begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

since normal modes of each component are assumed mass-normalized and

$$\begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} = \begin{bmatrix} [\omega_k^2]_1 & 0 \\ 0 & [\omega_k^2]_2 \end{bmatrix}$$

where $[\omega_k^2]_1$ and $[\omega_k^2]_2$ are diagonal sub-matrices with eigenvalues at the diagonal for the 1st and 2nd components. Therefore no need to know the mass and stiffness matrices of each component in physical coordinates.

## III. Transient Response Calculation

After the eigenproblem (8) solved, i.e. all eigenvalues and eigenvectors are found, one can begin a transient analysis. The transient response is due to some external forces (not to confuse with the interface forces). Let's denote these external forces with capital *F(t)* and let's write the equation of motion of two components in splitted up form again, where the external forces have been added:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} + \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \tag{9}$$

Now, one can introduce a transformation using a 2nd set of coordinates (one can call them as final generalized coordinates) $\alpha_i$, $i = 1,2,...,n$

$$X_{phys}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \approx \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \beta Q \begin{bmatrix} \alpha_1(t) \\ \vdots \\ \alpha_n(t) \end{bmatrix} \tag{10}$$

where matrix $Q$ is a square matrix consisting from eigenvectors $q_k$ found in (8). Substituting (10) in (9) and then pre-multiplying (9) by basis functions

$$Q^T \beta^T \begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix}$$

which is actually a Galerkin's method application, one obtains

---- see formula (*) in Appendix ----

which is (using notations adopted in (7) and (8)) will be

$$Q^T B Q \begin{bmatrix} \ddot{\alpha}_1(t) \\ \vdots \\ \ddot{\alpha}_n(t) \end{bmatrix} + Q^T A Q \begin{bmatrix} \alpha_1(t) \\ \vdots \\ \alpha_n(t) \end{bmatrix} = Q^T \beta^T \begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \tag{11}$$

If eigenvectors $Q$ are B-normalized (one can always arrange this normalization), then matrix $Q^T BQ$ will be a unit matrix and matrix $Q^T AQ$ will be a diagonal with eigenvalues $\lambda_k$ from (8) at the diagonal. Therefore, equation (11) can be written as

$$\begin{bmatrix} \ddot{\alpha}_1(t) \\ \vdots \\ \ddot{\alpha}_n(t) \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & \lambda_n \end{bmatrix} \begin{bmatrix} \alpha_1(t) \\ \vdots \\ \alpha_n(t) \end{bmatrix} = Q^T \beta^T \begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \quad (12)$$

Integrating (12) with initial conditions $\alpha(0) = 0, \dot{\alpha}(0) = 0$, one can find these final generalized coordinates $\alpha_1(t),....,\alpha_n(t)$. Then one can recover the physical displacements of the system by using (10).

Addition of modal global damping to each of the uncoupled equation in (12) can be done by adding a viscous term:

---- see formula (13) in Appendix ----

where $\omega_i = \sqrt{\lambda_i}$ and $\xi_i$ are damping coefficients.

In case, if there are local dashpots in the system, then the following term needs to be added to the left side of equation (13):

$$Q^T \beta^T \begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \beta Q \begin{bmatrix} \dot{\alpha}_1(t) \\ \vdots \\ \dot{\alpha}_n(t) \end{bmatrix} \quad (14)$$

where $c_1, c_2$ are damping matrices (non-diagonal) of the 1st and 2nd components due to presence of dashpots. In this case the system of differential equations becomes coupled. Formulation of the product

$$\begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \quad (15)$$

will be shown in the next section. In this case, one need additionally to know the values of eigenvectors $\Phi_1, \Phi_2$ at the corresponding pairs of degrees of freedom where dashpots are present and values of dashpots [force/velocity], indeed. In case, if in addition to the global modal damping and discrete dashpots, there are some viscoelastic elements in the structure (not to confuse with regular viscous dashpots), then equations of motion (13) can be re-written using a space-state presentation, like for example shown in [19].

## IV. Formulation of Nonproportional Damping Matrix in Case of Dashpots Presence

Creation of the modal damping matrix $\tilde{c}_1 = \Phi_1^T c_1 \Phi_1$ for the 1st component (in equation (15)) is done upon given pairs of physical degrees of freedom connected by the corresponding dashpots. For 2nd component, the procedure is exactly similar. Let's assume that we have retained p lower modes and total number of physical degrees of freedom of the component is n. Let's introduce one dashpot of value c that connects the physical degrees of freedom "*i*" and "*k*". So to begin with, the matrix in physical coordinates is everywhere 0, except locations (*i*,*i*), (*i*,*k*), (*k*,*i*) and (*k*,*k*) as shown below by " " and " " (under the matrix and to the right of it). As a result of multiplication, the output matrix (size *p* x *p*) will have form as shown below

---- see Formula (**) in Appendix ----

Introduction of another dashpot will produce a similar matrix as above. These two matrices then need to be added to each other. Therefore introduction of arbitrary number of dashpots can be done in such way. Thus in order to formulate a product in (15) - and consequently the term (14) - one need to know the values of components' normal modes at degrees of freedom where dashpots are connected.

Integration of equations (13) (with (14) added to the left side in general) then can be done. The structure's physical response is recovered by using (10) afterwards.

## V. Method Implementation and Numerical Results

A computer code was written that implements the approach described in the previous sections. It was termed as "CMSDYN" code and it can be run on Unix, or PC. On Unix, this code (a shell script file) executes a series of commands, and on PC, this code is a batch file that executes analogous commands. One of the commands launches a commercial program (NASTRAN) to compute the normal modes and natural frequencies for the 1st (large) component (just one time solution) and 2nd (small) component (can be multiple solutions). Another command launches a FORTRAN code that contains the calculations described in the above sections. For the matrix inversion in equation (5), the subroutine "INV" from [20] was used and for eigenproblem solution (8), "DQZ" subroutine was used, also from [20].

For illustration of numerical results and CMSDYN code's validity, a free-free large FE model of Ares I structure (a future space exploration vehicle) was considered (Figure 1), where three concentrated masses (chute masses) are attached by linear springs and dashpots to the vehicle's body as shown in Figure 2. The node "A" in Figure 2 is an interface node, where the structure will be split. As a 2nd (modifiable) component, those three springs with concentrated masses (chute masses) were chosen, since their stiffness was a parameter of optimization. All the rest of the structure was selected as the 1st (large) component. NASTRAN program was used as an eigensolver to provide the natural frequencies and values of normal modes at the required degrees of freedom for the 1st and 2nd components. For the 1st component (about 700,000 degrees of freedom), it was necessary to perform a normal mode analysis only once, since this component was not a subject to change. For the 2nd component (small size component), the NASTRAN normal mode analysis can be done numerous times (no computer time issue).

The goal of optimization was *to reduce the acceleration level at the astronaut seat* (see Figure 1) *by only varying the connection (stiffness and dashpot) of the main chutes to the vehicle*. Notice that these two locations (astronaut seat and main chute connections) are quite far apart. Indeed, one may suggest an alternative option, like *introducing dampers at the seats directly, however for this study it was not specified as the objective.*

It was known that the acceleration level at the seat location was sensitive to the axial loading at the frequency coinciding with the 2nd axial natural frequency of the vehicle (around 12.45 Hz). Two axial loads, resulting from 1 psi pressure at the forward and aft domes (due to final seconds of propellant burn out), were applied and they are as shown in Figure 1 (red arrows). These forces follow one sinusoidal loading at frequency of 12.45 Hz (exciting the 2nd axial normal mode) and act for about 2 sec. The same direction of these loads was specified, because it corresponded to the 1st acoustic mode data.

At first, a normal mode analysis was performed for the whole (unsplit) structure by using NASTRAN. The obtained natural frequencies are shown in Figure 3 for two configurations, namely when the chute masses are connected by stiff (rigid) elements and when they are connected to the vehicle by flexible springs that have 12.45 Hz natural frequency (for the given chute mass). One can see from Figure 3, that a difference in natural frequencies begins from frequency # 33 (12.5 Hz vs 13.2 Hz). Also note that the first 6 natural frequencies are zeros, because the structure is free-free (not constrained).

Then CMSDYN code was run for these two configurations. A comparison between the natural frequencies obtained by NASTRAN (exact solution) and CMSDYN code is shown in Figures 4 and 5. NASTRAN produced natural frequencies for the whole (unsplit) structure. One can see a very good match for the first 90 natural frequencies between these two programs that validates the assumption for the case of rigid-like interfaces: using only lower normal modes of each component is sufficient, thus no need to introduce any additional constraint, attachment, or residual-attachment modes. In terms of computer time, the NASTRAN eigensolution for unsplit model would take about 2 hours each time, while CMSDYN eigensolution takes only 1 minute each time (upon changing the 2nd component for this example).

After the CMS part (normal mode analysis) was finished, a transient analysis can be done by integrating of equations (13) with the specified load history applied to the structure and with zero initial conditions. A Runge-Kutta 4th order of accuracy integrator was used. A comparison of NASTRAN (unsplit model) transient results with CMSDYN code results was done as well. Note that numerous values of stiffness for springs (in Figure 2) can be chosen and structure's natural frequencies and transient solution results can be obtained quickly. Here, for the sake of brevity, the results are shown for only two configurations: stiff connection and 12.45 Hz springs connection. In both calculations (NASTRAN and CMSDYN code), a modal damping of 1 % was assumed for all modes. In all graphs below by "acceleration" it is implied the axial (along the vehicle) component of acceleration (the dominant one), since the other two are much smaller.

For the case of stiff connection, the acceleration (in terms of g) vs time at the astronaut seat is plotted in Figure 6. One can see a perfect match of the results between NASTRAN transient solution and CMSDYN solution, since the graphs are indiscernible.

The acceleration of the main chute node is plotted in Figure 7. One can see a good match of the results between NASTRAN transient solution and CMSDYN solution. For this case, the chute acceleration is quite low, since the chute masses are connected by stiff springs to the vehicle.

For the 12.45 Hz springs connection configuration, the acceleration at the astronaut seat is plotted in Figure 8. One can see a good match of the results between NASTRAN and CMSDYN solutions, since the graphs are basically on top of each other.

The acceleration of the main chute node is plotted in Figure 9 for the 12.45 Hz springs connection configuration. The acceleration level becomes large, since the chute masses are connected by 12.45 Hz springs that are excited by the external force frequency of 12.45 Hz. One can see a good match of the results between NASTRAN and CMSDYN solutions.

For the case of 12.45 Hz springs, when the chute mass acceleration becomes large (Figure 9), it may be expedient to introduce dashpots connecting the chutes to the vehicle. Note that it is done in addition to the global modal damping of 1 % assumed earlier for the whole structure. The value of dashpot was chosen as C = 5, or 10, or 100 lbf/(inch/sec). The formulation of non-diagonal (nonproportional) damping matrix was shown in section 4. Comparison of CMSDYN code with NASTRAN (unsplit structure) results is shown for the case of dashpot C=100 lbf/(inch/sec) in Figure 10 (seat node) and Figure 11 (chute node). One can see good matching of results, those graphs are basically on top of each other. For other cases of dashpots C=5; 10 lbf/(inch/sec), matching was good as well, though the graphs for these cases are omitted here for the sake of brevity.

The results (transient response graphs) of CMSDYN code are shown below in Figure 12 and 13 for different values of dashpot. One can see that the acceleration at the astronaut seat depends on the dashpots introduced at the main chute connections. It is obvious that the greater damping at the chute connections, the lesser accelerations/velocities of these chutes (see Figure 13), thus the lesser kinetic energy they (main chutes) will absorb. Therefore, the seat acceleration goes up with the increase of damping at the chute connections, as one can see in Figure 12. The reason for introducing of dashpots at the chute connections can be only if there is a need to decrease the level of vibration of the chutes.

The summary of results in terms of maximum accelerations (taken from graphs in Figures 6-13) at the seat and chute locations is shown in Table 1 (CMSDYN code) and Table 2 (NASTRAN, unsplit model) for two configurations (main chutes connected by rigid and 12.45 Hz springs). One can see a good correspondence of accelerations between these two solvers. Some difference is explained by the fact that NASTRAN (unsplit model) solution has more high modes included which contribute to the response (and it is more visible at the lower values of dashpots). One can see that a stiff connection of main chutes gives 7.698 g maximum acceleration at the astronaut seat, while introduction of flexible (12.45 Hz springs) main chutes connection decreases it to 4.654 g (Table 2) and at the same time increases the acceleration of chutes to 7.098 g (with no dashpots). If the chute acceleration needs to be lower than 7.098 g, then one can select, for example, a configuration with 12.45 Hz springs, and dashpot C = 5, 10, or 100. The value of critical damping coefficients ξ (corresponding to dashpot values C) is provided in Tables 1, 2, just to show the damping level for those 12.45 Hz springs (do not confuse with the global modal damping of 1 % that exists in addition). Note that producing NASTRAN (unsplit model) transient solution each time (upon main chute springs modification) would take 2 hours, while CMSDYN code produces a similar result in a matter of 1-2 minutes.

## VI. Conclusion

A minimal complexity version of CMS that requires simplified computer programming (saving analyst's time and effort) and provides accurate determination of lower natural frequencies/mode shapes of large structures and their transient responses was presented. A novel technique for formulation of equations of motion for transient response was demonstrated, where a double transformation to generalized coordinates was employed. Also a technique that creates a nonproportional damping matrix in the generalized coordinates for the structure with dashpots was shown.

The presented method was implemented as a computer code (UNIX shell script, or PC batch file) that yields significant computer time savings compared with the solution for unsplit FE models. The results of this code were compared with NASTRAN simulation results (for unsplit models) for several cases and they matched them very well.

## Appendix

Referenced Equations (*), (13), (**) and all Figures are presented here.

$$Q^T \beta^T \begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \left( \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \beta Q \begin{bmatrix} \ddot{\alpha}_1(t) \\ \vdots \\ \ddot{\alpha}_n(t) \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \beta Q \begin{bmatrix} \alpha_1(t) \\ \vdots \\ \alpha_n(t) \end{bmatrix} \right) = \tag{*}$$

$$= Q^T \beta^T \begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \left( \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \right) = Q^T \beta^T \begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\alpha}_1(t) \\ \vdots \\ \ddot{\alpha}_n(t) \end{bmatrix} + \begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & 2\xi_n\omega_n \end{bmatrix} \begin{bmatrix} \dot{\alpha}_1(t) \\ \vdots \\ \dot{\alpha}_n(t) \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & \lambda_n \end{bmatrix} \begin{bmatrix} \alpha_1(t) \\ \vdots \\ \alpha_n(t) \end{bmatrix} = Q^T \beta^T \begin{bmatrix} \Phi_1^T & 0 \\ 0 & \Phi_2^T \end{bmatrix} \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \tag{13}$$

$$\tilde{c}_1 = \Phi_1^{\ T} c_1 \Phi_1 = \begin{bmatrix} \varphi_{11} & \varphi_{21} & \cdots & \varphi_{n1} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1p} & \varphi_{2p} & \cdots & \varphi_{np} \end{bmatrix} \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ \cdots & c & \cdots & -c & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & -c & \cdots & c & \cdots \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix} \begin{matrix} "i" \\ \\ "k" \end{matrix} \begin{bmatrix} \varphi_{11} & \cdots & \varphi_{1p} \\ \varphi_{21} & \cdots & \varphi_{2p} \\ \cdots & \cdots & \cdots \\ \varphi_{n1} & \cdots & \varphi_{np} \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & \cdots & c\varphi_{i1} - c\varphi_{k1} & \cdots & 0 & \cdots & -c\varphi_{i1} + c\varphi_{k1} & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & c\varphi_{ip} - c\varphi_{kp} & \cdots & 0 & \cdots & -c\varphi_{ip} + c\varphi_{kp} & \cdots & 0 \end{bmatrix} \begin{bmatrix} \varphi_{11} & \cdots & \varphi_{1p} \\ \varphi_{21} & \cdots & \varphi_{2p} \\ \cdots & \cdots & \cdots \\ \varphi_{n1} & \cdots & \varphi_{np} \end{bmatrix} =$$

$$= \begin{bmatrix} c(\varphi_{i1} - \varphi_{k1})^2 & \cdots & c(\varphi_{i1} - \varphi_{k1})(\varphi_{ip} - \varphi_{kp}) \\ \cdots & \cdots & \cdots \\ c(\varphi_{ip} - \varphi_{kp})(\varphi_{i1} - \varphi_{k1}) & \cdots & c(\varphi_{ip} - \varphi_{kp})^2 \end{bmatrix} \qquad (**)$$

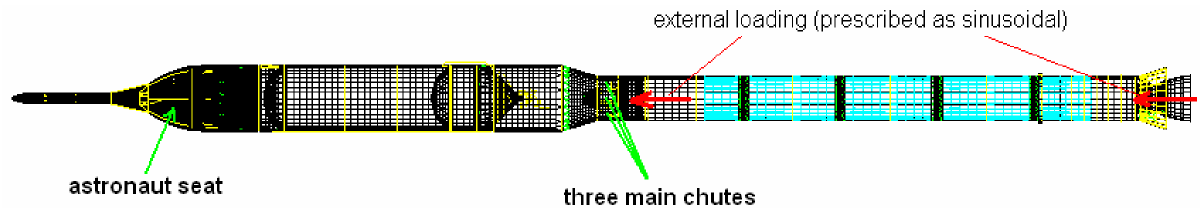


Fig. 1 FE model (unsplit) of the vehicle

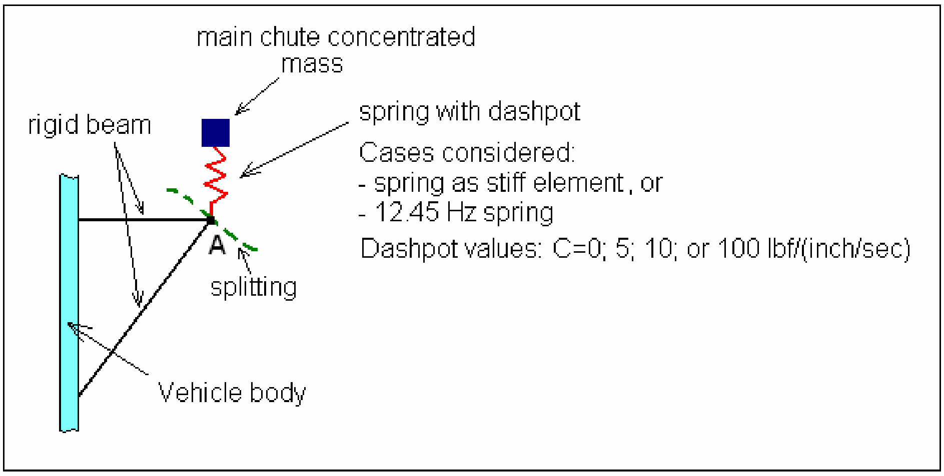


Fig. 2 Connection of main chutes to the vehicle

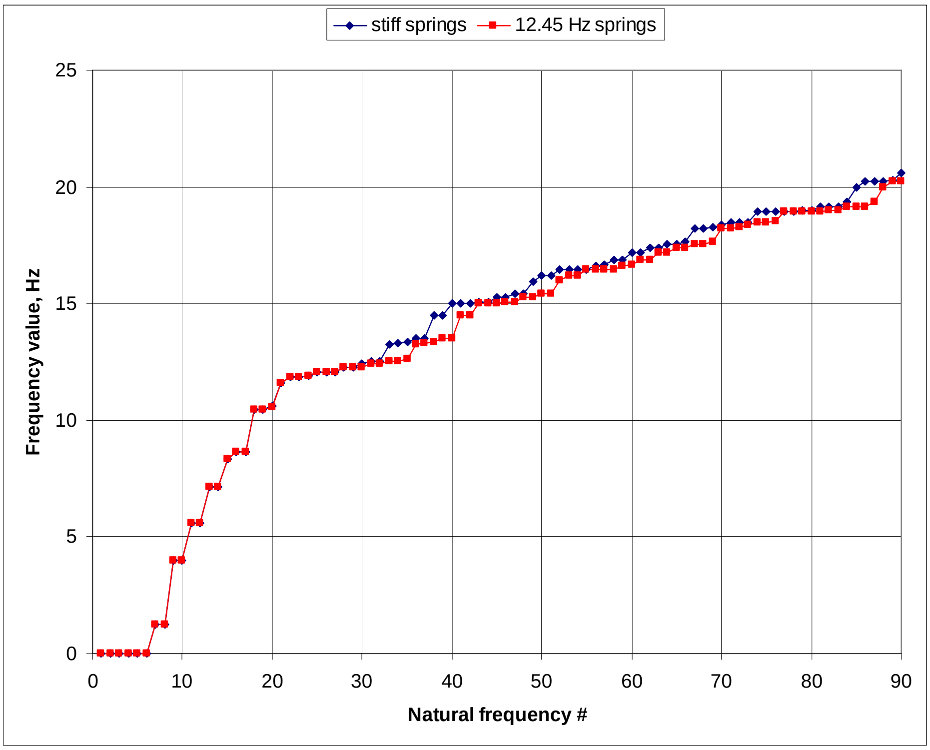


Fig. 3 NASTRAN (unsplit model) natural frequencies for two configurations: chutes connected by stiff springs and by 12.45 Hz springs

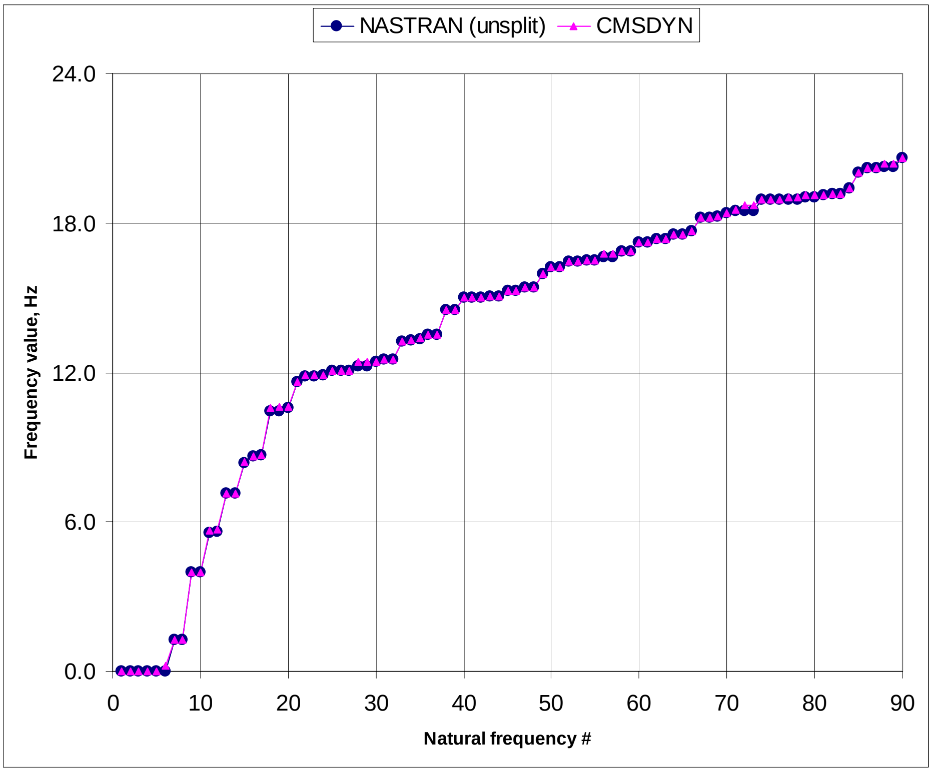


Fig. 4 Comparison of natural frequencies, stiff spring connection configuration

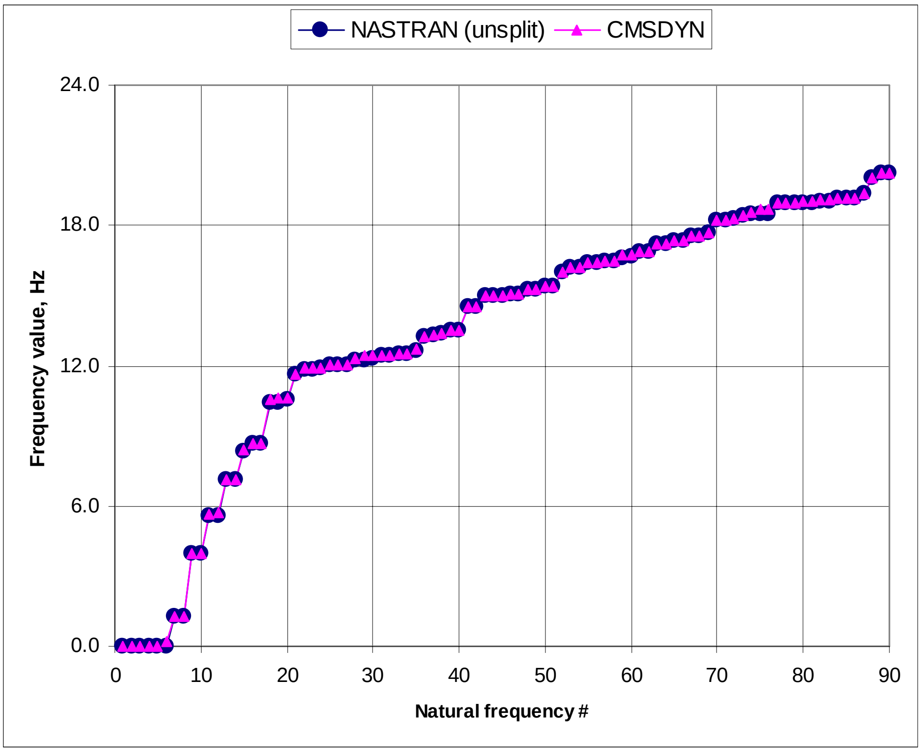


Fig. 5 Comparison of natural frequencies, 12.45 Hz springs connection configuration

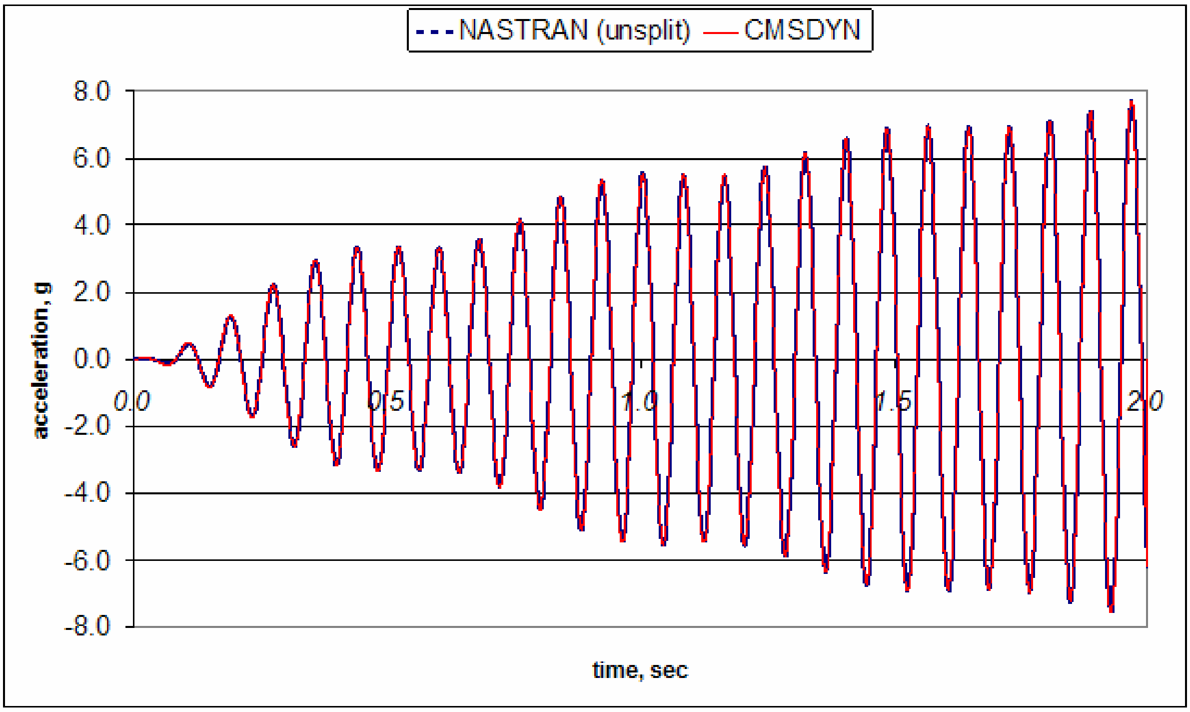


Fig. 6 Seat acceleration, stiff springs connection configuration

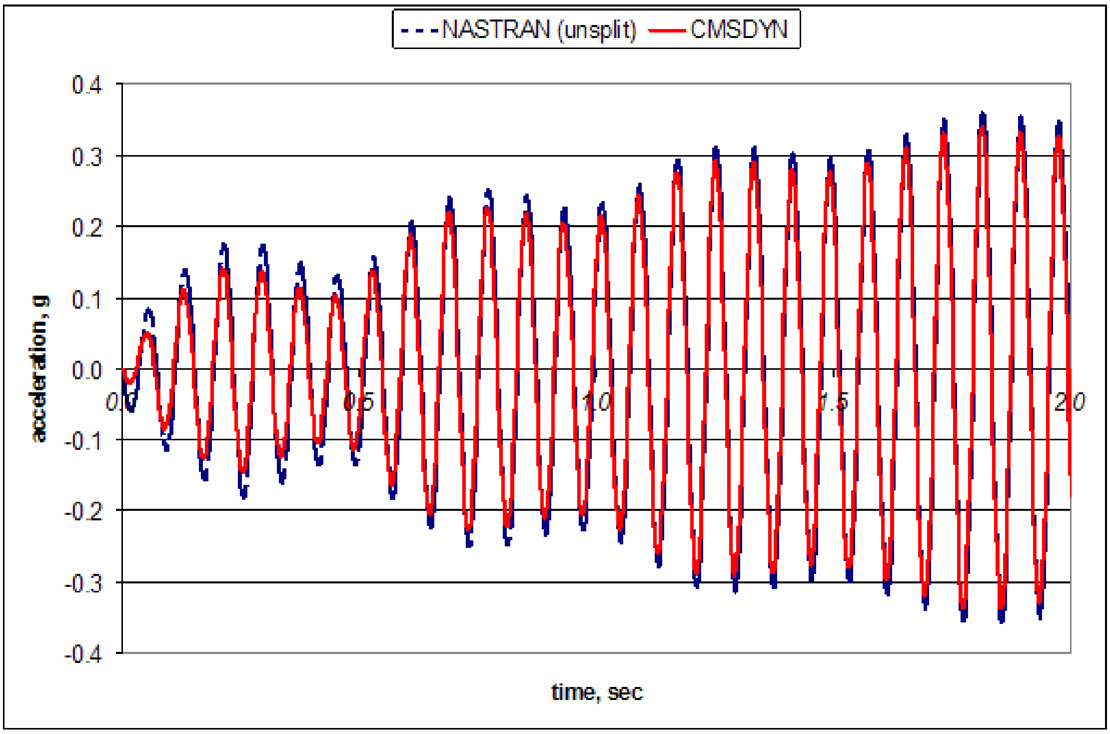


Fig. 7 Chute acceleration, stiff spring connection configuration

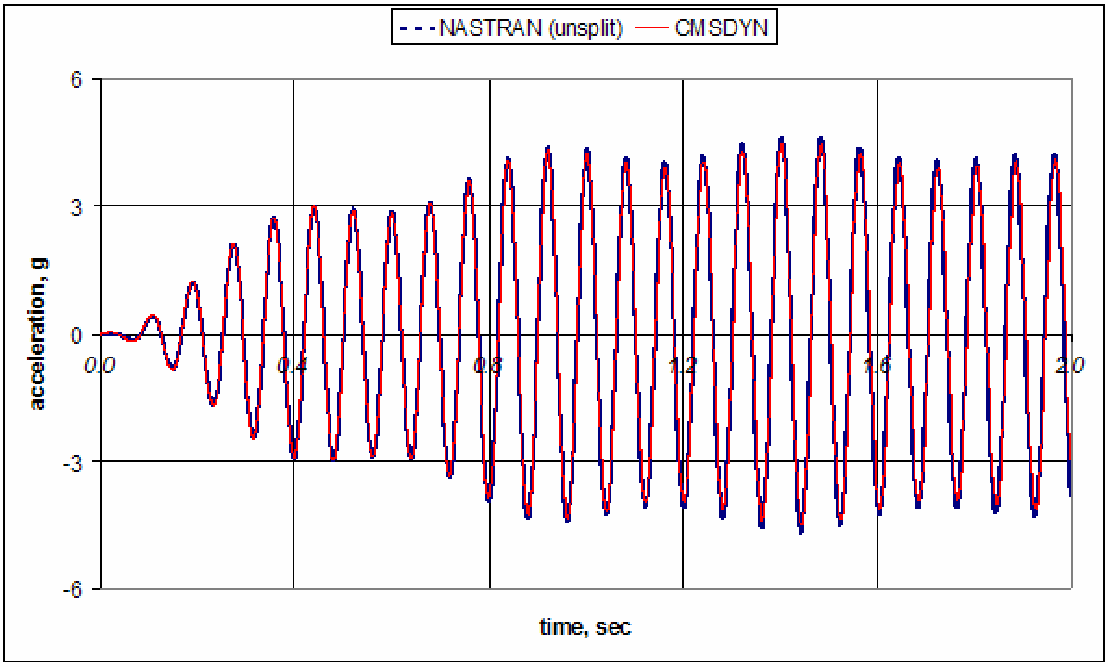


Fig. 8 Seat acceleration, 12.45 Hz springs connection configuration

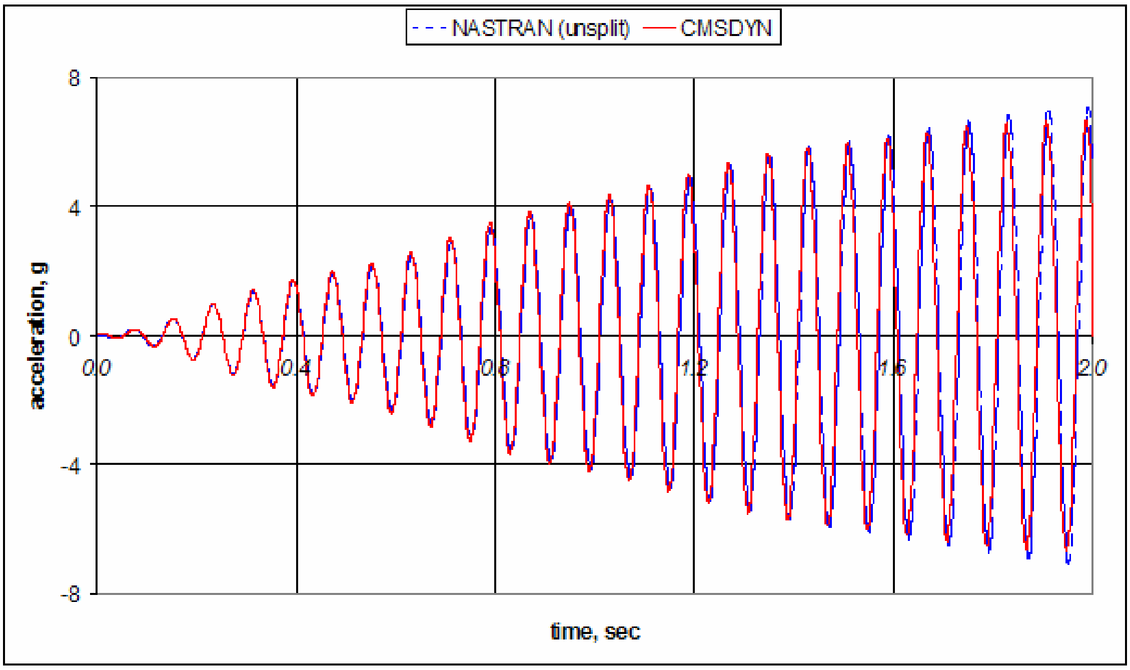


Fig. 10 Chute acceleration, 12.45 Hz springs connection configuration

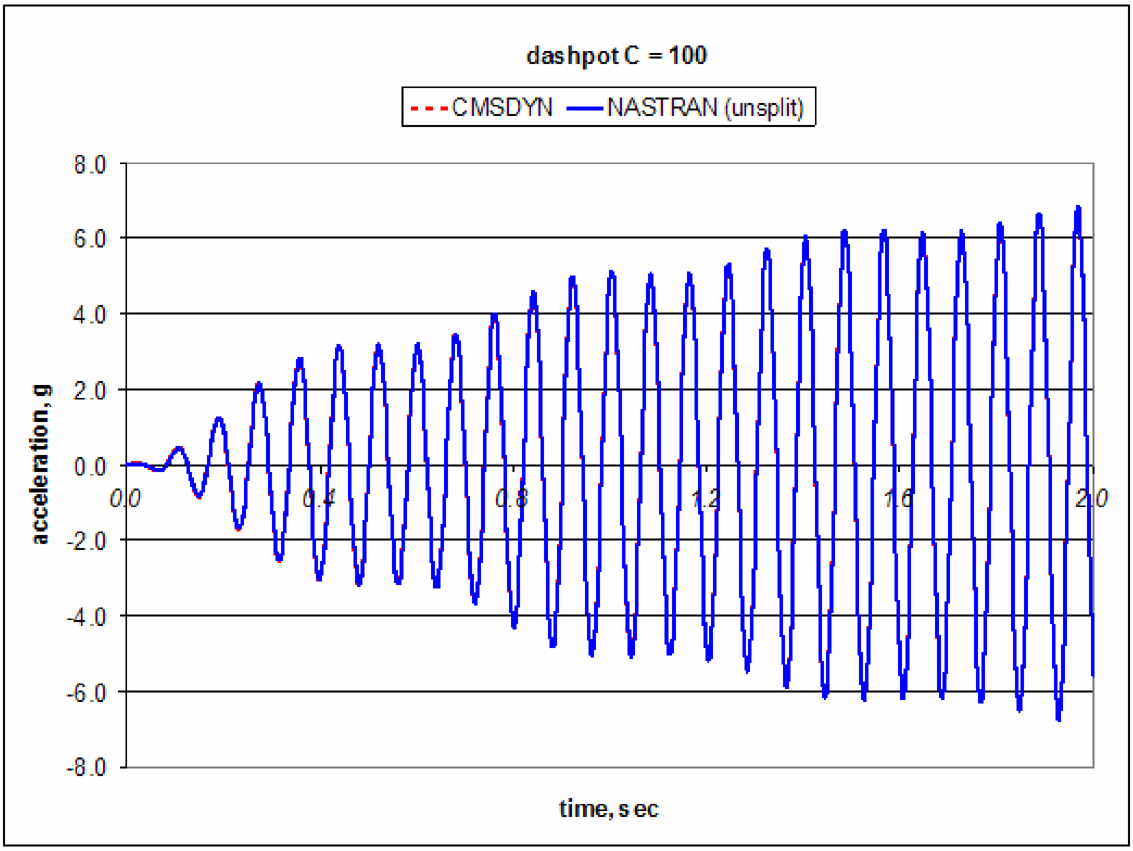


Fig. 11 Seat acceleration, 12.45 Hz springs configuration with dashpot C=100 lbf/(inch/sec), CMSDYN result vs NASTRAN (unsplit) result

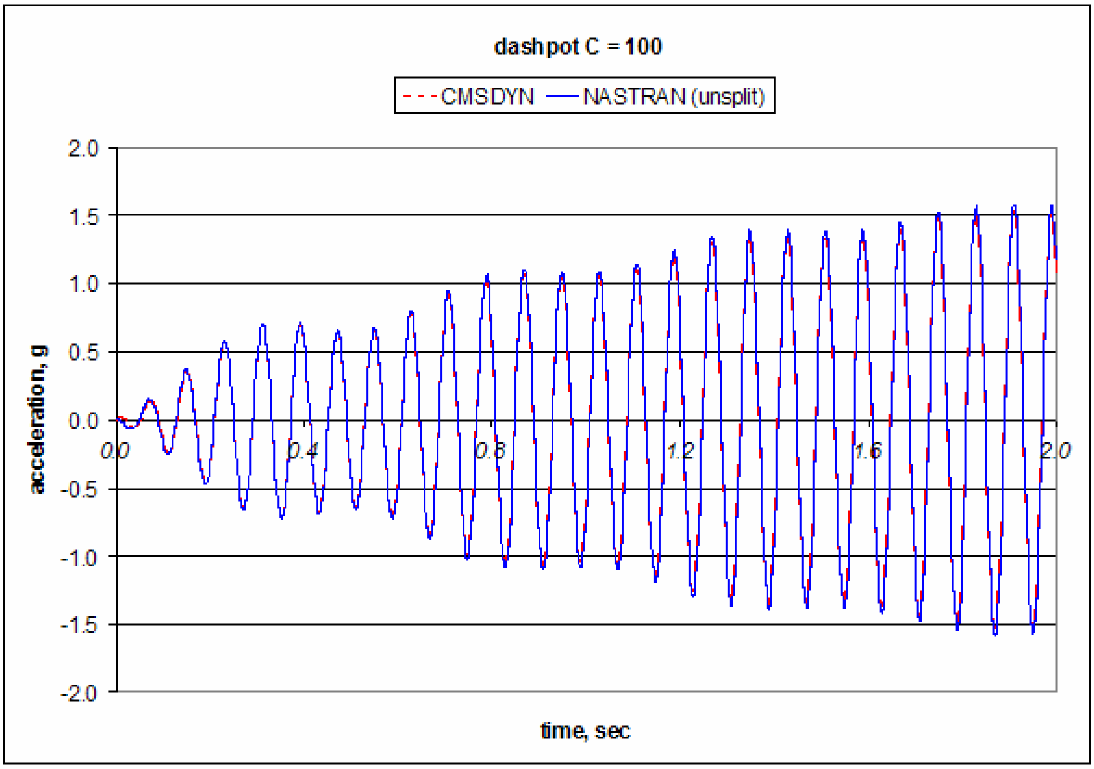


Fig. 12 Chute acceleration, 12.45 Hz springs configuration with dashpot C=100 lbf/(inch/sec), CMSDYN result vs NASTRAN (unsplit) result

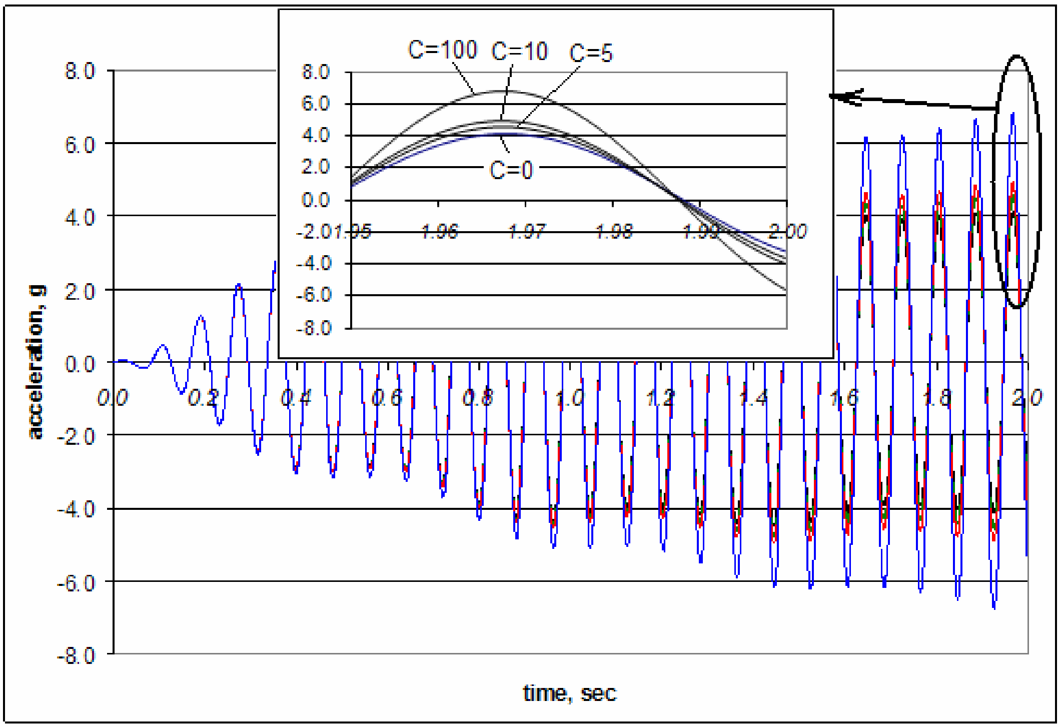


Fig. 13 Seat acceleration, 12.45 Hz springs configuration with different dashpot values C, CMSDYN result

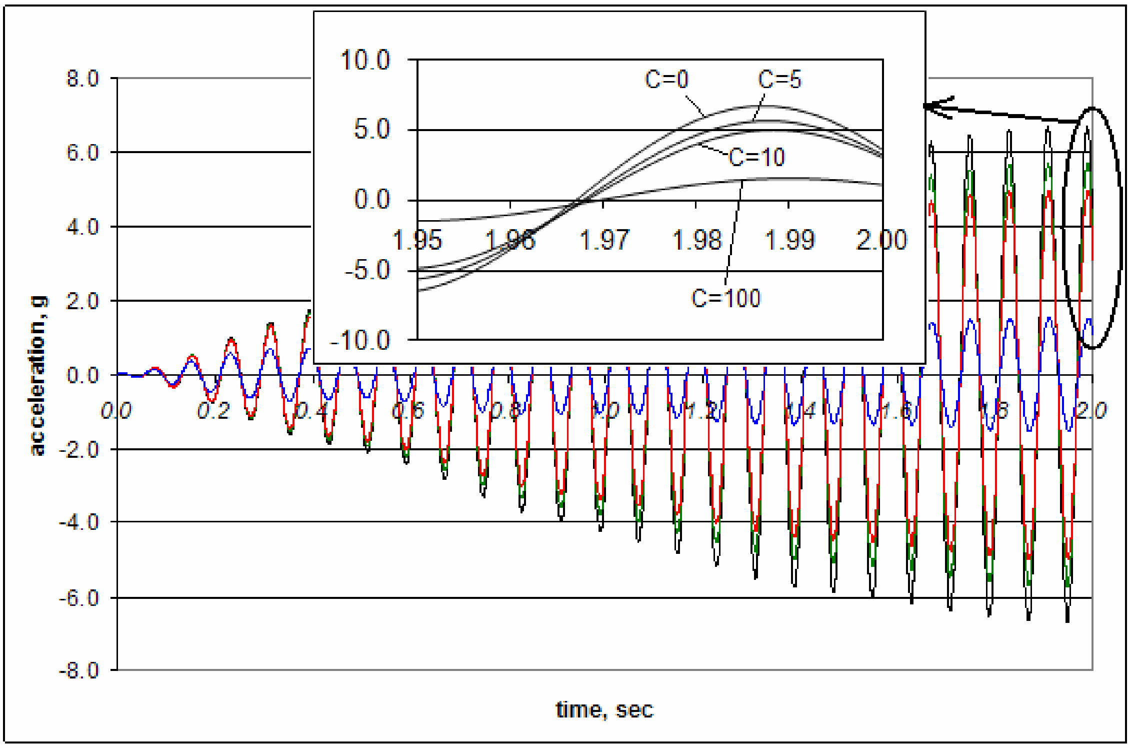


Fig. 14 Chute acceleration, 12.45 Hz springs configuration with different dashpot values C, CMSDYN result

TABLE I
CMSDYN TRANSIENT SOLUTION RESULTS FOR TWO CONFIGURATIONS

| | *12.45 Hz springs connection configuration* | | *Stiff springs connection configuration* | |
|---|---|---|---|---|
| Dashpot [lbf/(inch/sec)] | *Max Seat acceleration, g* | *Max Chute acceleration, g* | *Max Seat acceleration, g* | *Max Chute acceleration, g* |
| C = 0 (ξ = 0.) | 4.493 | 6.710 | 7.698 | 0.338 |
| C = 5 (ξ = 0.0044) | 4.744 | 5.721 | | |
| C = 10 (ξ = 0.0088) | 4.951 | 4.985 | | |
| C = 100 (ξ = 0.088) | 6.827 | 1.532 | | |

TABLE II
NASTRAN (UNSPLIT MODEL) TRANSIENT SOLUTION RESULTS FOR TWO CONFIGURATIONS

<table>
<tr><th></th><th colspan="2">12.45 Hz springs connection configuration</th><th colspan="2">Stiff springs connection configuration</th></tr>
<tr><th>Dashpot [lbf/(inch/sec)]</th><th>Max Seat acceleration, g</th><th>Max Chute acceleration, g</th><th>Max Seat acceleration, g</th><th>Max Chute acceleration, g</th></tr>
<tr><td>C = 0 (ξ = 0.)</td><td>4.654</td><td>7.098</td><td rowspan="4">7.698</td><td rowspan="4">0.358</td></tr>
<tr><td>C = 5 (ξ = 0.0044)</td><td>4.870</td><td>6.040</td></tr>
<tr><td>C = 10 (ξ = 0.0088)</td><td>5.050</td><td>5.245</td></tr>
<tr><td>C = 100 (ξ = 0.088)</td><td>6.825</td><td>1.584</td></tr>
</table>

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