# A convenient model for I-V characteristic of a solar cell generator as an active two-pole with self-limitation of current

A. A. Penin and A. S. Sidorenko

*Abstract*—**A convenient and physically sound mathematical model of the external or $I - V$ characteristic of solar cells generators is presented in this paper. This model is compared with the traditional model of $p-n$ junction. The direct analytical calculation of load regime leads to a quadratic equation, which is importantly to simplify the calculations in the real time.**

*Index Terms*—**a solar cell generator, $I-V$ characteristic, active two-pole.**

## I. INTRODUCTION

**F**OR the analysis and calculation of power supply systems based on solar cells generators mathematical model of the cell is needed. The traditional model or $I-V$ characteristic of $p-n$ junction [1], [2] requires iterative numerical calculation methods. Therefore, it might be valuable to use simpler models that allow carrying out direct or analytical computations.

The quadratic fractional expression of model allows calculating the point of maximum power through a quadratic equation [3]. But the calculation with the resistive load leads to the cubic equation, which complicates the calculations. One can also point out that this model is not justified physically, which complicates its development.

A more convenient and physically sound the mathematical model is presented in this paper. The model is also quadratic fractional expression and utilizes the $I - V$ characteristic of the active two-pole with self-limitation of the current [4]. The calculation with the resistive load leads to a quadratic equation, which is more important for the calculations of the regimes in the real time.

## II. EXTERNAL CHARACTERISTIC OF THE ACTIVE TWO-POLE WITH SELF-LIMITATION OF THE CURRENT

In the theory of the electric circuits idea of the ideal and the real voltage and current sources is developed. Such active two-poles have a linear external or $I - V$ characteristic. In turn, the author [4] examines the active two-pole with self-limitation of the current, which is characterized by a non-linear $I - V$ characteristic. An example of such two-pole is the quasi-resonant voltage converter $DC - DC$, which has the property of self-limitation of the load current. The $I - V$ characteristic of such active two-pole $i(u)$ has a characteristic species - curve 1 on Fig. 1.

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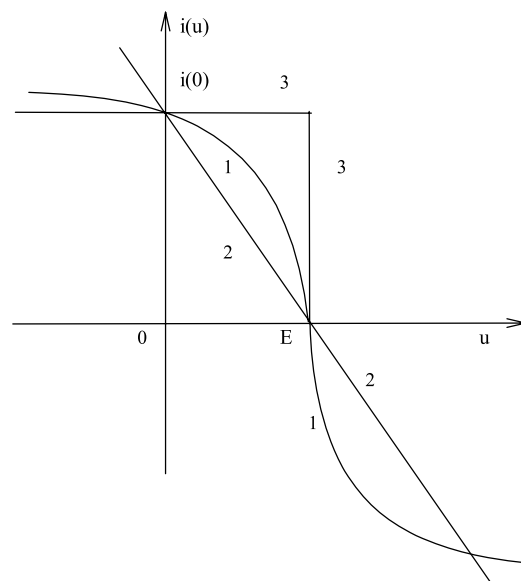
Fig. 1. The typical external or $I-V$ characteristics of the active two-poles: 1 convex characteristic of the active two-pole with self-limitation of the current, 2 linear characteristic of the real voltage or current source, 3 -rectangular characteristic of an ideal current source with the limiting maximum load voltage

In the first quadrant, the active two-pole delivers energy to a load, and its characteristic is already convex species as compared with the linear characteristic 2. The regime is changed from the short-circuit $SC$, when current is $i(0)$ , to the open-circuit $OC$ , when the load voltage is $E$ . In turn, the rectangular characteristic 3 corresponds to an ideal current source with the limiting maximum load voltage. Thus, depending on the degree of convexity, the curve 1 can move from the line 3 to the line 2.

In the second and fourth quadrants, the active two-pole consumes energy already, but there is a limitation of the current, even for the load voltage is greater of voltage $E$ .

The specific family of the characteristics for the first quadrant with different periods of switching $T_K$ is represented on Fig. 2. For example, the natural oscillations period is $T_0 = 43.4\mu S$. The switching period $T_K = 44\mu S$ is the
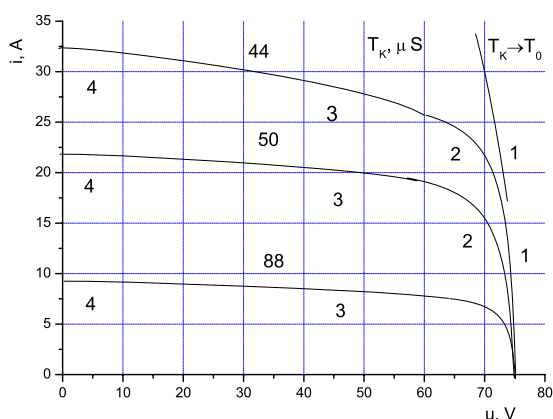
Fig. 2. The typical parts of the $I-V$ characteristics of the quasi-resonant voltage converter $DC-DC$ with different period of switching $T_K$ : 1-characteristic of the voltage source, 2- start of the current limitation and the maximum load power, 3-characteristic of the current source, 4- short circuit point .

minimal working and $T_K = 88 \mu S$ is the twice period.

The part 1 for all curves corresponds to the characteristic of the voltage source when the load voltage is depended little from its current in view of the high efficiency. Point 2 corresponds to the start of the current limitation and the maximum load power. The part 3 in the form of an inclined line corresponds to further current limitation up to the $SC$ point 4. It is worth noting the similarity of the reviewed characteristics and the $I-V$ characteristics of non linear elements such as transistors, vacuum tubes, solar cells.

The represented specific characteristics allow carrying out the analysis and justifying equivalent generator for such active two-pole. Such points of the characteristic regimes as $SC$
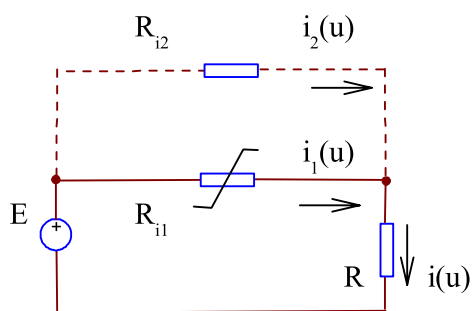
on Fig. 3. The $OC$ voltage determines the voltage source $E$ and the $SC$ current - the value of the internal resistance for this regime:

$$R_{i1}(0) = E/i_1(0) = E/I_1$$

Then, it is necessary to determine the dependence of the internal resistance on the voltage. Therefore it is worth noting three areas of the family of the characteristics.

For the first area, where $T_K \geq 2T_0$ after the point $P_M$ until the $SC$ point and further in part $u \leq 0$ the current is increasing little. Therefore, the equivalent generator characteristic is similar to the current source.

For the third area, where $T_K = T_0$ -it is a resonance, the characteristic is a linear form. Therefore, the $SC$ current is defined a loss small resistance. Thus, the equivalent generator is the voltage source.

For the second area, where $T_0 < T_K \leq 2T_0$ there is an interim case - the equivalent generator type is changed from the voltage source to the current source. Therefore, the characteristic is changed from the curve similar to a vertical line, to the curve similar to a horizontal line.

*The first area characteristic.* Let we consider at greater length one curve of the characteristics family for a given value of the switching period. To establish the dependence of the internal resistance on the voltage, we define the resistance value $R_{i1}(u)$ for actual data for the current and voltage and we build a plot of the linear relationship (Fig. 4, line 1). The



Fig. 3. The equivalent generator of the active two-pole with self-limitation of the current with the non linear and linear internal resistances

and $OC$ are suggesting the possibility of using the equivalent generator (circuit) with the non linear internal resistance $R_{i1}$
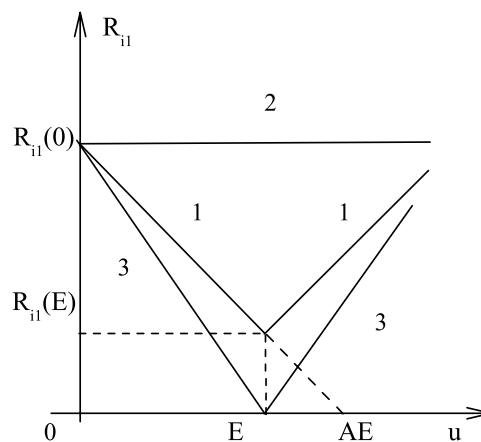


Fig. 4. The dependence of the internal resistance on the load voltage of the typical active two-poles: 1-active two-pole with self-limitation of the current, 2- real voltage source, 3- ideal current source with the limiting maximum load voltage

minimum value of the resistance $R_{i1}(E)$ corresponds to the voltage $E$ . If the resistance $R_{i1}(u)$ is not dependent on the voltage (line 2), we get a voltage source with the internal resistance and the linear characteristic (Fig. 1, line 2). If the resistance goes through the point - straight line 3, then we get a current source (Fig. 1, lines 3).

The given case presents then an interim version where the straight line 1 passes through the point $(AE, 0)$. The equation of the line 1 has the appearance for the work area $u \leq E$ :

$$R_{i1} = \frac{1}{I_1} \left( E - \frac{u}{A} \right)$$

On the other hand, we receive from the equivalent generator circuit:

$$R_{i1} = \frac{E - u}{i_1(u)} \qquad (1)$$

Eliminating value $R_{i1}$ , we get the equation:

$$i_1(u) = I_1 A \frac{1 - u}{A - u} \qquad (2)$$

Here and further the voltage $u$ is normalized by the $E$ . This expression defines a hyperbole. The parameter $A$ determines the degree of convexity. If $A \to \infty$ , then curve is degenerated into a straight line ((Fig. 1, line 2). ). If , then we get a rectangular characteristic the hyperbole is merged with the asymptotes (Fig. 1, lines 3).

*The second area characteristic.* In the characteristics in this area at the parts 3 a linear part is shown increasingly, which is tended to a vertical line when $T_K \to T_0$. Therefore, the characteristics are a linear hyperbolic. Then we can make an assumption about the introduction of the equivalent generator scheme on Fig. 3 of another, but a linear resistor $R_{i2}$ with a value that is specified the period $T_K$. Physical support of this element is that in the third area the value $R_{i2}$ is so small that the load current is set by this resistance to a greater extent. Let we obtain the expression of the characteristic for this case. Under the equivalent generator circuit, taking into account (1), (2) the load current is given:

$$i(u) = i_1(u) + i_2(u) = I_1 A \frac{1 - u}{A - u} + I_2(1 - u) \qquad (3)$$

where the linear component of the current in the $SC$ regime is:

$$I = i(0) = E/R_{i2}$$

### III. I-V CHARACTERISTIC OF A SOLAR CELL GENERATOR

Let we consider the calculated characteristic family on Fig. 5 according [3] . Numbers denote the typical parts, similar to Fig. 2. Then part 3 corresponds to a current source with nearly horizontal straight line, 2 is the point of the maximum power and part 1 as an inclined line corresponds to the voltage source with a relatively high internal resistance. Hence the similarity follows with the curves on Fig. 2 - inclined part 1 on Fig. 5 corresponds to the part 3 on Fig. 2.

Then one can draw up a dual equivalent generator circuit on Fig. 6. The current generator $I$ and the internal non linear conductivity $Y_{i1}$ define a hyperbolic component, and the linear conductance $Y_{i2}$ - a linear component of the load voltage. One can write the equation immediately, using expression (3) and changing formally currents $i$ by voltages $u$ and resistances $R_i$ by conductivities $Y_i$ :

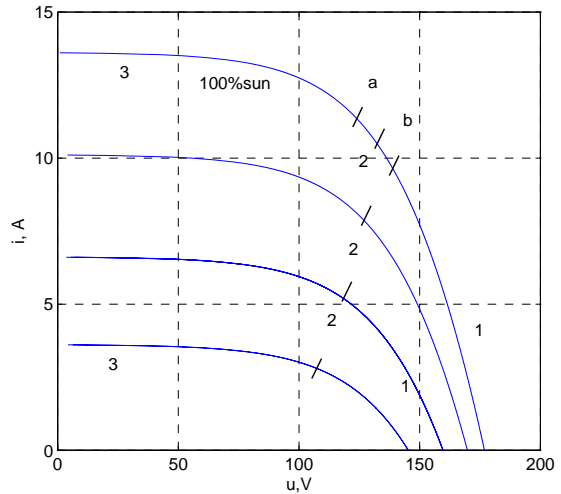$$u(i) = u_1(i) + u_2(i) = E_1 A \frac{1 - i/I}{A - i/I} + E_2(1 - i/I) \qquad (4)$$



Fig. 5. The typical parts of the calculated family of the $I - V$ characteristic with different values of insulation or $SC$ current: 1-characteristic of the voltage source with a high internal resistance, 2- maximum load power point, 3- characteristic of the current source
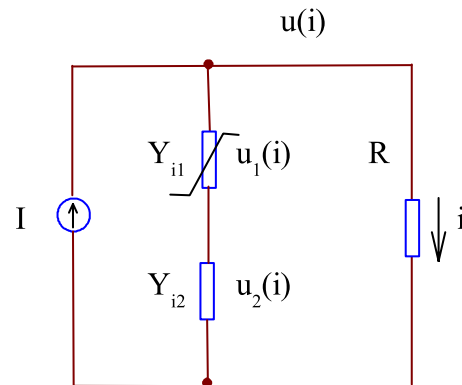


Fig. 6. The equivalent generator of the solar cell with the non linear and linear internal conductivities

where a hyperbolic component of the voltage in the $OC$ regime is:

$$E_1 = u_1(0) = I/Y_{i1}(0)$$

and linear component of the voltage is:

$$E_2 = u_2(0) = I/Y_{i2}$$

In turn, Fig. 4 also corresponds to the dependence of the non linear conduction $Y_{i1}$ on the current $i$. For a given family curve on Fig. 5 it is need to find the settings $I, A, E_1, E_2$ using the $SC, OC$ points and two points $a, b$ are on each side near of the maximum power point with the $i^a, u^a, i^b, u^b$ coordinates.

Let we give the necessary design parameters:

$$A = \frac{i^a \cdot \alpha}{\alpha - \beta}$$

where

$$\alpha = \frac{i^b}{i^a - i^b}, \quad \beta = \frac{1}{\beta_{12}} \left( \frac{u^b}{1 - i^b} - E \right),$$

$$\beta_{12} = \frac{u^a}{1 - i^a} - \frac{u^b}{1 - i^b},$$

$$E_1 = \frac{\beta_{12}/A}{1/(A - i^a) - 1/(A - i^b)}$$

and the currents $i^a, i^b$ are normalized by the current .

The proposed expression (4) allows direct computation of a current $i$ at a given load resistance $R$ . In this case, the load voltage $u = R \cdot i$ . Substituting this expression in (4), we get the following equation:

$$i^2 - i \left( AI + \frac{AE_1 + E_2}{R + E_2/I} \right) + AI \frac{E_1 + E_2}{R + E_2/I} = 0 \quad (5)$$

In the practical realization of the power supply an array of pre-calculated parameters is set to the memory of the micro controller, including the coordinates of the maximum power for different insulation levels. The current regime is calculated by the expressions (4), (5). The parameters of the maximum power point are found from expression of the power $P = u \cdot i$ , taking into account (4). In this case, it is needed to find the maximum of function $P(i)$ by well-known numerical algorithms. One can also carry out an analysis of the curve, using derivative $dP(i)/dt$ .
The cubic equation is obtained:

$$2i^3 - (1 + 3A + AB) i^2 + 2A (1 + AB) i - A^2 B = 0$$

where $B = 1 + E_1/E_2$. This equation is also solved with the help of well-known algorithms.

## IV. EXAMPLE

Let we consider a specific example. Let the family curve corresponds to the maximal insulation and is set the following expression:

$$u = -0.9i + \frac{1}{0.042} \ln \frac{13.615 - i + 0.0081}{0.0081} \quad (6)$$

Then the $OC$ voltage is $E = E_1 + E_2 = 176$, and the $SC$ current is $I = 13.608$ . Here and further values dimension are not indicated for simplifying of the record. To the point of load maximum power values $I_M = 11.32, U = 123.6$ , value of the load resistor $R_M = 10.92$ and the power equal to 1400. Let $i^a = 12, u^a = 114.76; i^b = 10, u^b = 135.58$
Then

$$\alpha = \frac{10}{12 - 10} = 5, \ \beta_{12} = \frac{114.76}{1 - 0.881} - \frac{135.58}{1 - 0.734} = 459.8$$

$$\beta = \frac{1}{459.8} \left( \frac{135.58}{1 - 0.734} - 176 \right) = 0.7293, \ A = 1.0324$$

and

$$E_1 = \frac{459.81/1.0324}{1/(1.0324 - 0.881) - 1/(1.0324 - 0.7293)} = 134.7$$

Ultimately, we get the approximate expression for the characteristic model:

$$u(i) = 139.06 \frac{1 - i/13.608}{1.0324 - i/13.608} + 41.3(1 - i/13.608) \quad (7)$$
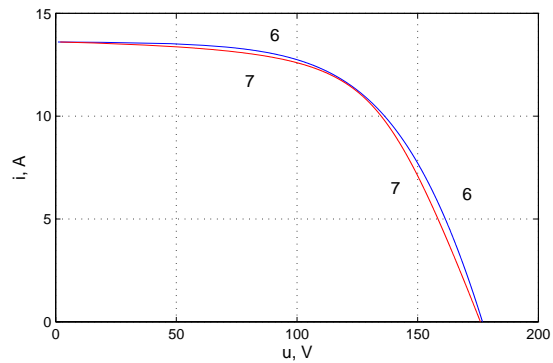


Fig. 7.   Comparison characteristics $I - V$

The curves $i(u)$ under expressions (6), (7) are shown on Fig. 7. In turn, the curves of powers $P = i \cdot u$ are shown on Fig. 8.
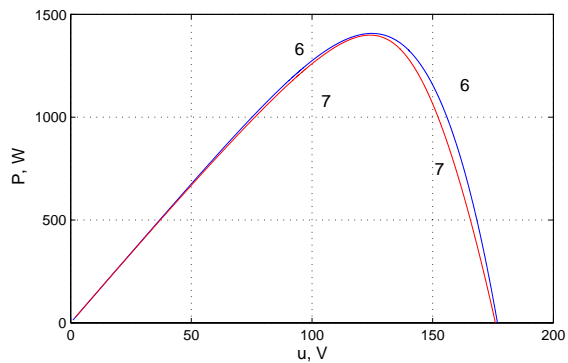


Fig. 8.   Comparison characteristics pow

The curves of these expressions are marked the same numerals $6, 7$. Then, if $R = 10.92$ we get:

$$i^2 - 26.97i + 177.2 = 0$$

The decision gives the value of the root $i = 11.33$ . We find from the expression (4) $u_M = 123.36$ . In turn, the power is equal to 1398, which coincides with the maximum load power.

## V. Conclusion

The example shows a reliable approximation of the proposed expression to the $p - n$ junction characteristic.

This model of the characteristics allows calculating the load regime analytically and can be used in the implementation of the power supply with the direct digital control.

One can further develop the model through the introduction, for example, the series or shunt resistances to the equivalent generator.

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