

Sliding Mode Control with Fuzzy Boundary Layer to Air-Air Interception Problem

Mustafa Resa Becan

Abstract—The performance of a type of fuzzy sliding mode control is researched by considering the nonlinear characteristic of a missile-target interception problem to obtain a robust interception process. The variable boundary layer by using fuzzy logic is proposed to reduce the chattering around the switching surface then is applied to the interception model which was derived. The performances of the sliding mode control with constant and fuzzy boundary layer are compared at the end of the study and the results are evaluated.

Keywords—Sliding Mode Control, Fuzzy, Boundary Layer, Interception Problem.

I. INTRODUCTION

THE purpose of this study is to research the performance of the fuzzy sliding mode control by considering the nonlinear characteristic of a missile-target interception process. Sliding mode controllers (SMC) belong to a wider class commonly referred to as variable-structure controllers can be applied effectively in the presence of model uncertainties, parameter variations, and disturbances. As well recognized, fuzzy logic controllers are effective robust controllers for various applications, however, the linguistic expression of the fuzzy controller makes it difficult to guarantee the stability and robustness of the control system [1]. Designing a fuzzy logic controller based on the sliding mode theory assures performance and stability, while simultaneously reducing the number of fuzzy rules. Furthermore, fuzzy partition of the manipulated variables avoids the chattering problem of the sliding mode control method [2]. Recently, some significant studied in the various forms of fuzzy sliding mode control have been achieved [3-9]. Besides the air-air interception problem has been studied by many authors, a guidance law was designed by adaptive fuzzy sliding mode control based on a Lyapunov function to guarantee the stability of the system[10] and an application of sliding mode control was developed to evaluate the performance of a robust control law which considers the nonlinear characteristics of a missile-target interception process by simulating a different mathematical model [11].

In this study, the sliding mode control is applied to the

aerodynamic missile-target model to obtain the robust interception based on the robustness property of the sliding mode control and a fuzzy boundary layer is proposed versus the conventional method. Firstly, interception model and control actuator dynamics are derived. After that, conventional and fuzzy sliding mode control theories are summarized. Finally, the results are presented by comparing the applications.

II. INTERCEPTION MODEL DYNAMICS

An interception model for air-air missile and point mass target was used [12]. Line of Sight (LOS) angle σ of the missile is expressed by

$$\sigma = a \tan(ey/ex) \quad (1)$$

$$ex = X_T - X_M \quad (2)$$

$$ey = Y_T - Y_M \quad (3)$$

The target range R (LOS distance), and the variation of the LOS angle $\dot{\sigma}$ are presented respectively

$$R = \sqrt{ex^2 + ey^2} \quad (4)$$

$$\dot{\sigma} = \frac{V_m \cdot (\sigma - \theta) - V_T \cdot (\sigma - \beta)}{R} \quad (5)$$

In these equations, θ is the pitch angle, β is the target angle, X_M , X_T , Y_M , Y_T are the missile-target trajectories.

III. CONTROL ACTUATOR DYNAMICS

The trajectory of the missile is determined by the fin angle which modifies the pitch angle θ . An electrical control signal δ_{zel} supplied by the controller is superimposed on a rate gyro feedback δ_{zd} which depends on the angular speed q for the missile (Fig.1).

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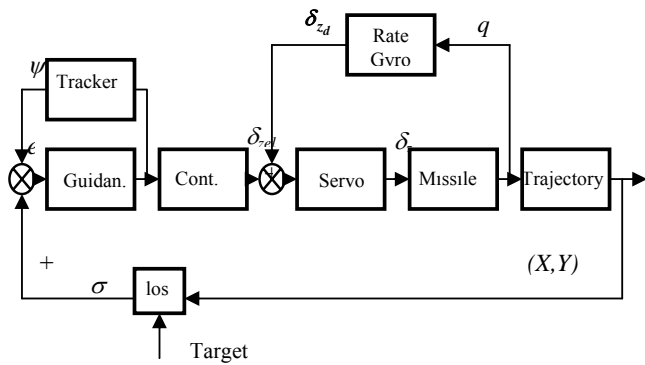


Fig.1 Block Diagram of Guidance Control System

A fin actuator servo, consisting of a first order element, filters the combined signal to generate the actual fin control δ_z . The dynamic equations are

$$\dot{\delta}_{zd} = \frac{\delta_{zd}}{\tau_g} + K_q \cdot \dot{q} \quad (6)$$

$$\dot{\delta}_z = -\frac{1}{\tau_s} (\delta_z + \delta_{zd} - \delta_{zel}) \quad (7)$$

where δ_{zd} is the rate gyro feedback K_q is rate gyro stabilization coefficient, τ_s and τ_g are servo and rate gyro time constants, δ_{zel} is the electrical fin signal as the control variable.

IV. APPLICATION OF THE SLIDING MODE CONTROL TO THE INTERCEPTION PROBLEM

Using a sign function in the well-known conventional sliding mode theory [13], often causes chattering in practice. Therefore a boundary layer around the switching surface is introduced to solve the chattering problem [14].

$$u = u_{eq} + ks \text{sat}\left(\frac{s}{\phi}\right) \quad (8)$$

where ϕ is the thickness of the boundary layer. $\text{sat}\left(\frac{s}{\phi}\right)$ is a saturation function which is defined as:

$$\text{sat}\left(\frac{s}{\phi}\right) = \begin{cases} \frac{s}{\phi} & \text{if } \left|\frac{s}{\phi}\right| \leq 1 \\ \text{sgn}\left(\frac{s}{\phi}\right) & \text{if } \left|\frac{s}{\phi}\right| > 1 \end{cases} \quad (9)$$

The proportional navigation guidance was chosen as the switching function [11]. Therefore, the surface can be written as

$$s = G_{sm} \dot{\sigma} - \delta_z \quad (10)$$

where G_{sm} is the navigation constant for sliding mode control. So,

$$\dot{s} = G_{sm} \ddot{\sigma} - \dot{\delta}_z \quad (11)$$

can be written. Substitution of the expression (7) into (11) yields

$$s\dot{s} = s\left(-G_{sm} \ddot{\sigma} + \frac{\delta_z + \delta_{zd}}{\tau_s} + \frac{\delta_{zel}}{\tau_s}\right) \quad (12)$$

δ_{zel} has to be chosen to satisfy the sliding mode condition $s\dot{s} < 0$. A possible choice is proposed

$$\delta_{zel} = \varepsilon \text{sign}(s) \quad (13)$$

ε is a constant. This choice corresponds to a switching of the controller. Substitution of (13) into (12) gives

$$s\dot{s} = s\left(-G_{sm} \ddot{\sigma} + \frac{\varepsilon \text{sgn } s}{\tau_s} + \frac{\delta_z + \delta_{zd}}{\tau_s}\right) \quad (14)$$

Thus $s\dot{s} < 0$ if

$$\varepsilon > \left| -G_{sm} \ddot{\sigma} + \delta_z + \delta_{zd} \right| \quad (15)$$

Note that the right-hand side of (15) is the equivalent control.

V. INTRODUCTION OF THE FUZZY BOUNDARY LAYER

The introduction of the boundary layer around the switching surfaces reduces chattering at the cost of increased tracking error. The variable boundary layer is a popular solution to this problem. In this paper, the fuzzy logic is proposed in order to improve the performance of the sliding mode controller. The fuzzy boundary layer leads a strategy to adjust the thickness of the boundary automatically and this fuzzy system adopts the sliding surface s as input and the thickness ϕ of boundary layer as output. The triangle membership functions for input s and output ϕ are used in the fuzzification procedure [13].

The single input-single output rule base is presented below:

- IF s is NE THEN ϕ is NE
- IF s is ME THEN ϕ is ME
- IF s is ZE THEN ϕ is ZE
- IF s is PO THEN ϕ is PO

Where NE: Negative, ME: Medium, ZE: Zero, SM: Small, PO: Positive.

VI. THE PERFORMANCE OF THE SLIDING MODE CONTROL

In this section, the simulation results based on sliding mode control are presented. Conventional and Fuzzy Sliding Mode Controllers are tested under the same initial conditions.

A. Conventional Sliding Mode Control

Firstly, the simulation results for the conventional sliding mode control are presented in Fig.2. To determine the value of the control gain ε is important because of the limitations of the guidance system. On the other hand, increasing this gain results in a more direct interception trajectory. Therefore, $\varepsilon = 0.9$ rad was taken as the optimal gain value by using the trial and error method. The other significant parameter is the sliding mode navigation constant G_{sm} . Simulation studies showed that if this parameter is chosen large enough the smooth trajectory at the interception process can be obtained as required. The results given in below were provided as the final LOS distance value $R_f = 0.3$ m was performed within simulation time $T_f = 8.72$ s, then the chattering problem exists in this case.

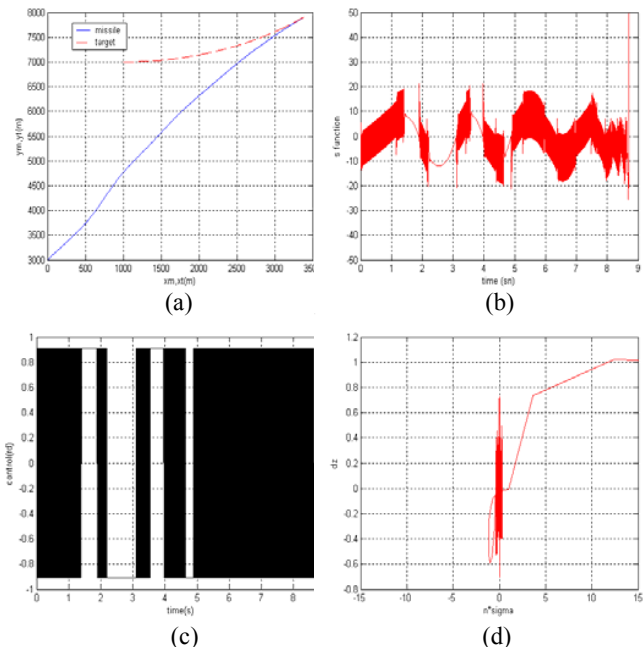


Fig. 2 Conventional SMC a) Interception Trajectory b) Control Signal c) Function s d) Switching Surface

B. Sliding Mode Control with Fuzzy Boundary Layer

The system responses via fuzzy logic are shown in Fig. 3. The best results shown below are for the scaling factors $K_\phi = 3.3$ and $K_s = 0.001$. Final LOS distance value $R_f = 0.0257$ m was obtained within simulation time $T_f = 8.72$ s. This interception performance is better than the previous application and the chattering is entirely rejected as can be seen. It means that the fuzzy boundary layer successfully addressed to the system requirements.

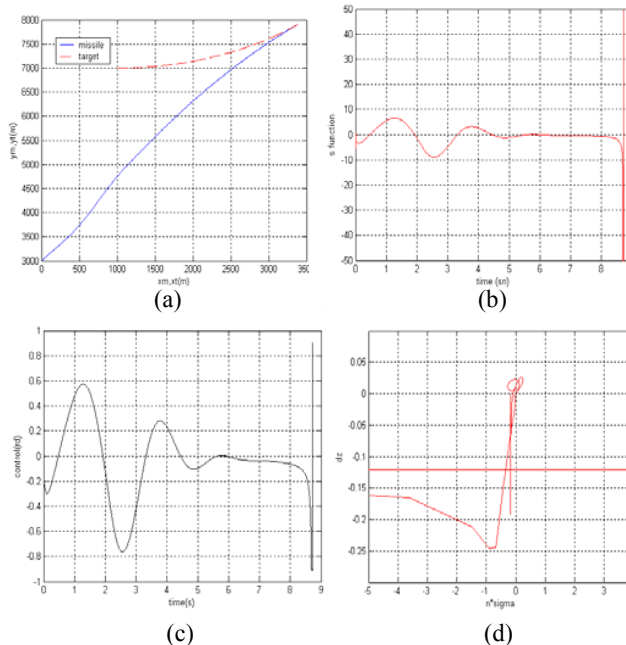


Fig.3 SMC with fuzzy boundary layer a) Interception Trajectory b) Control Signal c) Function s d) Switching Surface

VII. CONCLUSION

A fuzzy boundary layer was introduced versus the conventional sliding mode control to air-air interception problem. Simulation studies showed that the proposed method achieved to reduce chattering around the switching surface within the much better interception time than the conventional case. Because of the limitations of the homing missile characteristics, the control gain can be selected between a limited range in the conventional sliding mode control. The results indicate that the fuzzy application can compensate this limitation by supplying more direct trajectory. That's why, a fuzzy control gain can be improved besides the fuzzy boundary layer for future studies in this area.

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