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Establishing a New Simple Formula for Buckling Length Factor (K) of Rigid Frames Columns

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***Abstract*—**The calculation of buckling length factor (K) for steel frames columns is a major and governing processes to determine the dimensions steel frame columns cross sections during design. The buckling length of steel frames columns has a direct effect on the cost (weight) of using cross section. A new formula is required to determine buckling length factor (K) by simplified way. In this research a new formula for buckling length factor (K) was established to determine by accurate method for a limited interval of columns ends rigidity (GA, GB). The new formula can be used ease to evaluate the buckling length factor without needing to complicated equations or difficult charts.

***Keywords*—**Buckling length, New formula, Curve fitting, Simplification, Steel column design.

I. Introduction

SOME ways are used to evaluate the buckling length factor (K). Most codes use the alignment chart to get the buckling length factor (K) (AISC, 2008). [Egyptian Design Code of Steel Constructions (LRFD), 2008], (National Standard of Canada CAN/CSA- S16.1 –M89, 1989) , and (Egyptian Design Code of steel Constructions (ASD), 2009) as shown in figure (1). This chart is the graphic solution of the mathematically exact equation (chin, 1980) as shown in equations (1) and (2) . The formula proposed by the ACI for braced frames gives K = 0.7 for a beam fully fixed at both ends, instead of 0.5 as in (ACI, 200) . If GA = GB = 3.3, it yields K=1.0. instead of the expected 0.89. The equations for untraced frames are somewhat better: for GA = GB=2.0 for instanc,. they yield K = 1.56. instead of 1.61 as in (ACI. 2005). The accuracy of the alignment charts depends essentially on the size of the chart, and on the reader's sharpness of vision. Another approximate equation is established to be more easily in use than the exact equation, this equation is called French rule (Pierre Dumonteil, 1992), as shown in equations (3) and (4). All these methods depend on the rotational restraint at column ends (GA, GB). Consider a column AB elastically restraint at both ends, the rotational restraint at one end, A for instance, is represented by a restraint factor GA, expressing the relative stuffiness of the columns connected at A to that of all the beams framing into a [1], as shown in equation (5).

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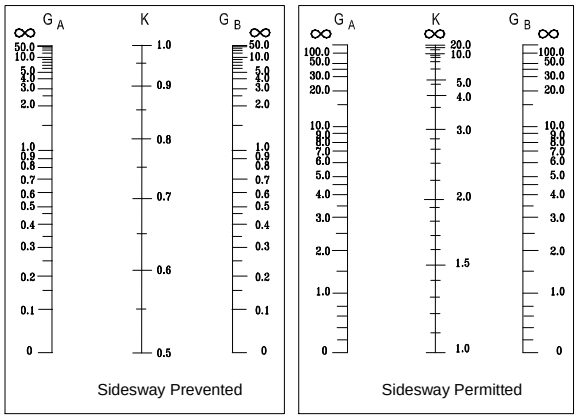

Fig. 1 Alignment Charts For Buckling Length Factor (K) Of Columns In Rigid Frames

$$\frac{G_A G_B}{4}(\pi/K)^2+\left(\frac{G_A+G_B}{2}\right)\left(1-\frac{\pi/K}{\tan\pi/K}\right)+2\frac{\tan\pi/2K}{\pi/K}=1 \quad (1)$$

(for side sway prevented)

$$\frac{G_A G_B(\pi/K)^2-36}{6(G_A+G_B)}=\frac{\pi/K}{\tan\pi/K} \quad (2)$$

(for side sway permitted)

$$K=\frac{3G_A G_B+1.4(G_A+G_B)+0.64}{3G_A G_B+2.0(G_A+G_B)+1.28} \quad (3)$$

(for side sway prevented)

$$K=\sqrt{\frac{1.6G_A G_B+4.0(G_A+G_B)+7.5}{G_A+G_B+7.5}} \quad (4)$$

(for side sway permitted)

$$G_A = \frac{\sum(I_c/L_c)}{\sum(I_b/L_b)} \quad (5)$$

Where
Ic = the moment of inertia of column cross section area
Lc = the column length
Ib = the moment of inertia of beam cross section area
Lb = the beam length

II. New Formula

From the graphic solution of exact equation the value of buckling length factors is estimated with respect to the rotational resistant at column ends (G_A, G_B). The estimation is done by fixing the rotational restraint at one end of column and changing the value of rotational restraint at the other end of column . Figure (2) represents the buckling length factors (K) as a function rotational restraint at column ends for prevented sway (braced) columns. Figure (3) represents the buckling length factors (K) as a function in rotational restraint at column ends for permitted sway (untraced) columns. The value of buckling length factors were tabled in tow tables. The first table represents prevented sway cases, and the other table represents permitted sway cases. As shown in table (1) and table (2). The estimated data of buckling length factors in table (1) and table (2) were fitted to a computer program soft ware to create a mathematic formulas presents the buckling length factors for both prevented and permitted sway cases.

The new formulas were extracted from the curve fitting. As shown in equations (6) and (7). The new formulas are simple and direct to apply by knowing the values of G_A and G_B. The new formulas depend only on the values of rotational restraint at column ends.

The equation of buckling length factors (K) for prevented sway columns:

$$K = 0.498 + 0.219\times G_A - 0.08935\times G_A^2 + 0.0153927\times G_A^3 - 0.000985\times G_A^4 + 0.00001422\times G_A^5 + 0.21769\times G_B - 0.0885\times G_B^2 + 0.0152\times G_B^3 - 0.0009713\times G_B^4 + 0.000014\times G_B^5 \quad (6)$$

Where
$0 \le G_A \le 10$
$0 \le G_B \le 10$

The equation of buckling length factors (K) for permitted sway columns:

$$K = 1.168 + 0.09634\times G_A + 0.09634\times G_B - 0.0022\times G_A^2 - 0.0022\times G_B^2 + 0.00212\times G_A\times G_B + 0.0000133\times G_A^3 + 0.0000133\times G_B^3 - 0.000007253\times G_A\times G_B^2 - 0.000007253\times G_A^2\times G_B \quad (7)$$

Where
$0 \le G_A \le 100$
$0 \le G_B \le 100$

The new formulas are limited for G_A and G_B values for prevented sway columns the values of G_A and G_B are limited from 0.0 to 10. While for permitted sway columns the values of G_A and G_B are limited from 0.0 to 100.

The new formulas can be used easily in the computer programs as a direct mathematical equation. The design excel sheets of steel column in frames may be programmed by using the new formulas.

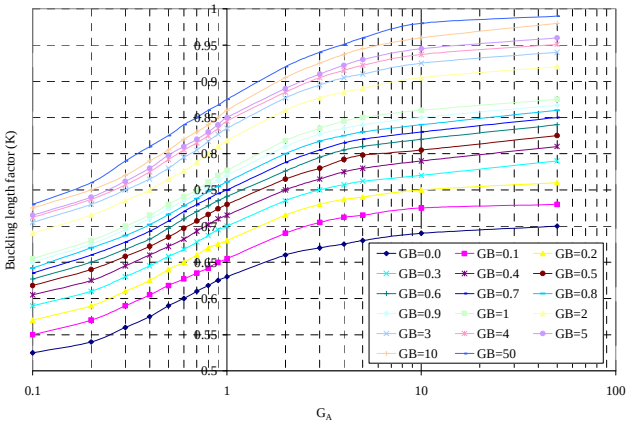

Fig. 2 Buckling Length Factor (K) for prevented of sway (Braced) columns in rigid frames

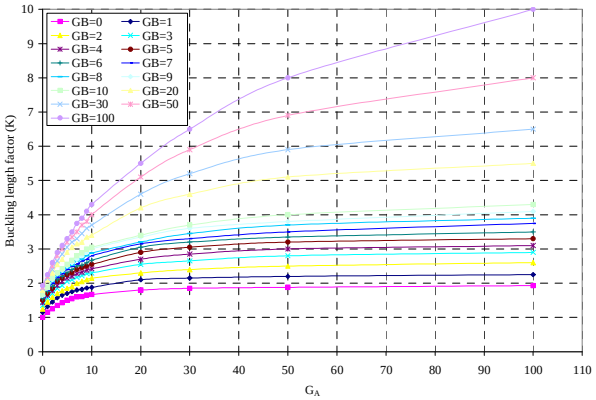

Fig. 3 Buckling Length Factor (K) for permitted of sway (Unbraced) columns

TABLE I
BUCKLING LENGTH FACTOR (K) FOR PREVENTED OF SWAY (BRACED) COLUMNS

G_A	G_B	K	G_A	G_B	K	G_A	G_B	K	G_A	G_B	K	G_A	G_B	K	G_A	G_B	K
0	0	0.5	50	0.1	0.73	10	0.3	0.77	5	0.5	0.798	4	0.7	0.815	3	0.9	0.827
0.1	0	0.525	0	0.2	0.54	50	0.3	0.79	10	0.5	0.805	5	0.7	0.82	4	0.9	0.835
0.2	0	0.54	0.1	0.2	0.57	0	0.4	0.575	50	0.5	0.825	10	0.7	0.83	5	0.9	0.84
0.3	0	0.56	0.2	0.2	0.59	0.1	0.4	0.605	0	0.6	0.6	50	0.7	0.85	10	0.9	0.85
0.4	0	0.575	0.3	0.2	0.61	0.2	0.4	0.625	0.1	0.6	0.627	0	0.8	0.618	50	0.9	0.867
0.5	0	0.59	0.4	0.2	0.625	0.3	0.4	0.645	0.2	0.6	0.65	0.1	0.8	0.642	0	1	0.63
0.6	0	0.6	0.5	0.2	0.64	0.4	0.4	0.66	0.3	0.6	0.668	0.2	0.8	0.67	0.1	1	0.655
0.7	0	0.61	0.6	0.2	0.65	0.5	0.4	0.672	0.4	0.6	0.682	0.3	0.8	0.687	0.2	1	0.68
0.8	0	0.618	0.7	0.2	0.66	0.6	0.4	0.682	0.5	0.6	0.697	0.4	0.8	0.702	0.3	1	0.7
0.9	0	0.625	0.8	0.2	0.67	0.7	0.4	0.693	0.6	0.6	0.71	0.5	0.8	0.716	0.4	1	0.715
1	0	0.63	0.9	0.2	0.675	0.8	0.4	0.702	0.7	0.6	0.72	0.6	0.8	0.728	0.5	1	0.73
2	0	0.66	1	0.2	0.68	0.9	0.4	0.71	0.8	0.6	0.728	0.7	0.8	0.738	0.6	1	0.741
3	0	0.67	2	0.2	0.715	1	0.4	0.715	0.9	0.6	0.735	0.8	0.8	0.748	0.7	1	0.75
4	0	0.675	3	0.2	0.73	2	0.4	0.75	1	0.6	0.741	0.9	0.8	0.755	0.8	1	0.761
5	0	0.68	4	0.2	0.737	3	0.4	0.765	2	0.6	0.776	1	0.8	0.761	0.9	1	0.77
10	0	0.69	5	0.2	0.74	4	0.4	0.775	3	0.6	0.795	2	0.8	0.8	1	1	0.777
50	0	0.7	10	0.2	0.75	5	0.4	0.78	4	0.6	0.805	3	0.8	0.817	2	1	0.818
0	0.1	0.525	50	0.2	0.76	10	0.4	0.79	5	0.6	0.81	4	0.8	0.825	3	1	0.835
0.1	0.1	0.55	0	0.3	0.56	50	0.4	0.81	10	0.6	0.82	5	0.8	0.83	4	1	0.845
0.2	0.1	0.57	0.1	0.3	0.59	0	0.5	0.59	50	0.6	0.84	10	0.8	0.84	5	1	0.85
0.3	0.1	0.59	0.2	0.3	0.61	0.1	0.5	0.618	0	0.7	0.61	50	0.8	0.86	10	1	0.86
0.4	0.1	0.605	0.3	0.3	0.63	0.2	0.5	0.64	0.1	0.7	0.635	0	0.9	0.625	50	1	0.875
0.5	0.1	0.618	0.4	0.3	0.645	0.3	0.5	0.658	0.2	0.7	0.66	0.1	0.9	0.65	0	2	0.66
0.6	0.1	0.627	0.5	0.3	0.658	0.4	0.5	0.672	0.3	0.7	0.678	0.2	0.9	0.675	0.1	2	0.69
0.7	0.1	0.635	0.6	0.3	0.668	0.5	0.5	0.685	0.4	0.7	0.693	0.3	0.9	0.695	0.2	2	0.715
0.8	0.1	0.642	0.7	0.3	0.678	0.6	0.5	0.697	0.5	0.7	0.707	0.4	0.9	0.71	0.3	2	0.735
0.9	0.1	0.65	0.8	0.3	0.687	0.7	0.5	0.707	0.6	0.7	0.72	0.5	0.9	0.724	0.4	2	0.75
1	0.1	0.655	0.9	0.3	0.695	0.8	0.5	0.716	0.7	0.7	0.73	0.6	0.9	0.735	0.5	2	0.765
2	0.1	0.69	1	0.3	0.7	0.9	0.5	0.724	0.8	0.7	0.738	0.7	0.9	0.745	0.6	2	0.776
3	0.1	0.705	2	0.3	0.735	1	0.5	0.73	0.9	0.7	0.745	0.8	0.9	0.755	0.7	2	0.788
4	0.1	0.712	3	0.3	0.75	2	0.5	0.765	1	0.7	0.75	0.9	0.9	0.763	0.8	2	0.8
5	0.1	0.715	4	0.3	0.757	3	0.5	0.78	2	0.7	0.788	1	0.9	0.77	0.9	2	0.81
10	0.1	0.725	5	0.3	0.762	4	0.5	0.792	3	0.7	0.805	2	0.9	0.81	1	2	0.818

G_A	G_B	K
2	2	0.86
3	2	0.877
4	2	0.885
5	2	0.89
10	2	0.905
50	2	0.92
0	3	0.67
0.1	3	0.705
0.2	3	0.73
0.3	3	0.75
0.4	3	0.765
0.5	3	0.78
0.6	3	0.795
0.7	3	0.805
0.8	3	0.817
0.9	3	0.827

G_A	G_B	K
1	3	0.835
2	3	0.877
3	3	0.895
4	3	0.905
5	3	0.91
10	3	0.925
50	3	0.94
0	4	0.675
0.1	4	0.712
0.2	4	0.737
0.3	4	0.757
0.4	4	0.775
0.5	4	0.792
0.6	4	0.805
0.7	4	0.815
0.8	4	0.825

G_A	G_B	K
0.9	4	0.835
1	4	0.845
2	4	0.885
3	4	0.905
4	4	0.915
5	4	0.922
10	4	0.937
50	4	0.951
0	5	0.68
0.1	5	0.715
0.2	5	0.74
0.3	5	0.762
0.4	5	0.78
0.5	5	0.798
0.6	5	0.81
0.7	5	0.82

G_A	G_B	K
0.8	5	0.83
0.9	5	0.84
1	5	0.85
2	5	0.89
3	5	0.91
4	5	0.922
5	5	0.93
10	5	0.945
50	5	0.96
0	10	0.69
0.1	10	0.725
0.2	10	0.75
0.3	10	0.77
0.4	10	0.79
0.5	10	0.805
0.6	10	0.82

G_A	G_B	K
0.7	10	0.83
0.8	10	0.84
0.9	10	0.85
1	10	0.86
2	10	0.905
3	10	0.925
4	10	0.937
5	10	0.945
10	10	0.96
50	10	0.98
0	50	0.7
0.1	50	0.73
0.2	50	0.76
0.3	50	0.79
0.4	50	0.81
0.5	50	0.825

G_A	G_B	K
0.6	50	0.84
0.7	50	0.85
0.8	50	0.86
0.9	50	0.867
1	50	0.875
2	50	0.92
3	50	0.94
4	50	0.951
5	50	0.96
10	50	0.98
50	50	0.99

TABLE II
BUCKLING LENGTH FACTOR (K) FOR PERMITTED OF SWAY (UNBRACED) COLUMNS

G_A	G_B	K
0	0	1
1	0	1.15
2	0	1.25
3	0	1.35
4	0	1.43
5	0	1.5
6	0	1.55
7	0	1.6
8	0	1.61
9	0	1.65
10	0	1.67
20	0	1.8
30	0	1.85
50	0	1.88
100	0	1.93

G_A	G_B	K
0	1	1.15
1	1	1.32
2	1	1.45
3	1	1.57
4	1	1.65
5	1	1.7
6	1	1.75
7	1	1.8
8	1	1.82
9	1	1.86
10	1	1.88
20	1	2.1
30	1	2.15
50	1	2.2
100	1	2.25

G_A	G_B	K
0	2	1.25
1	2	1.45
2	2	1.59
3	2	1.7
4	2	1.78
5	2	1.85
6	2	1.9
7	2	2
8	2	2.05
9	2	2.1
10	2	2.15
20	2	2.3
30	2	2.4
50	2	2.5
100	2	2.6

G_A	G_B	K
0	3	1.35
1	3	1.57
2	3	1.7
3	3	1.85
4	3	1.93
5	3	2.05
6	3	2.1
7	3	2.17
8	3	2.22
9	3	2.28
10	3	2.3
20	3	2.55
30	3	2.65
50	3	2.8
100	3	2.9

G_A	G_B	K
0	4	1.43
1	4	1.65
2	4	1.78
3	4	1.93
4	4	2.05
5	4	2.15
6	4	2.2
7	4	2.28
8	4	2.32
9	4	2.38
10	4	2.42
20	4	2.7
30	4	2.85
50	4	3
100	4	3.1

G_A	G_B	K
0	5	1.5
1	5	1.7
2	5	1.85
3	5	2.05
4	5	2.15
5	5	2.25
6	5	2.3
7	5	2.4
8	5	2.45
9	5	2.5
10	5	2.55
20	5	2.9
30	5	3.05
50	5	3.2
100	5	3.3

G_A	G_B	K
0	6	1.55
1	6	1.75
2	6	1.9
3	6	2.1
4	6	2.2
5	6	2.3
6	6	2.4
7	6	2.5
8	6	2.53
9	6	2.6
10	6	2.67
20	6	3.05
30	6	3.2
50	6	3.35
100	6	3.5
0	7	1.6
1	7	1.8
2	7	2
3	7	2.17
4	7	2.28
5	7	2.4
6	7	2.5
7	7	2.56

G_A	G_B	K
8	7	2.65
9	7	2.7
10	7	2.8
20	7	3.15
30	7	3.3
50	7	3.5
100	7	3.75
0	8	1.61
1	8	1.82
2	8	2.05
3	8	2.22
4	8	2.32
5	8	2.45
6	8	2.53
7	8	2.65
8	8	2.7
9	8	2.8
10	8	2.88
20	8	3.2
30	8	3.45
50	8	3.7
100	8	3.9
0	9	1.65

G_A	G_B	K
1	9	1.86
2	9	2.1
3	9	2.28
4	9	2.38
5	9	2.5
6	9	2.6
7	9	2.7
8	9	2.8
9	9	2.9
10	9	3
20	9	3.35
30	9	3.6
50	9	3.8
100	9	4.1
0	10	1.67
1	10	1.88
2	10	2.15
3	10	2.3
4	10	2.42
5	10	2.55
6	10	2.67
7	10	2.8
8	10	2.88

G_A	G_B	K
9	10	3
10	10	3.05
20	10	3.4
30	10	3.7
50	10	4
100	10	4.3
0	20	1.8
1	20	2.1
2	20	2.3
3	20	2.55
4	20	2.7
5	20	2.9
6	20	3.05
7	20	3.15
8	20	3.2
9	20	3.35
10	20	3.4
20	20	4.2
30	20	4.6
50	20	5.1
100	20	5.5
0	30	1.85
1	30	2.15

G_A	G_B	K
2	30	2.4
3	30	2.65
4	30	2.85
5	30	3.05
6	30	3.2
7	30	3.3
8	30	3.45
9	30	3.6
10	30	3.7
20	30	4.6
30	30	5.2
50	30	5.9
100	30	6.5
0	50	1.88
1	50	2.2
2	50	2.5
3	50	2.8
4	50	3
5	50	3.2
6	50	3.35
7	50	3.5
8	50	3.7
9	50	3.8

G_A	G_B	K
10	50	4
20	50	5.1
30	50	5.9
50	50	6.9
100	50	8
0	100	1.93
1	100	2.25
2	100	2.6
3	100	2.9
4	100	3.1
5	100	3.3
6	100	3.5
7	100	3.75
8	100	3.9
9	100	4.1
10	100	4.3
20	100	5.5
30	100	6.5
50	100	8
100	100	10

III. Analysis Of Results Comparison

Tables (3) and (4) present the comparison the values of the buckling length factors which are obtained from the exact equation by using alignment charts and the values which are calculated from the new formulas. The difference between these values and its percentage are also shown in these tables.

In case of prevented sway columns, the percentage of difference ranged from – 3% to +5%. And in case of permitted sway columns, the percentage of difference ranged from -8% to + 31%. From the tables it could be noted that the most difference percentage values were very small that the exact value and the values of new formulas are very close except at a few values of GA and GB.

The deviation between values of exact equation and values of new formula in case of prevented sway columns was less than the deviation between values of exact equation and values of new formula in case of permitted sway columns.

In general, the new formulas are suitable for most values of GA and GB specially for prevented sway columns cases.

TABLE III
Comparison Between Exact Equation and New Formula for The Buckling Length Factor of Prevented Sway Columns

G_A	G_B	K exact (from alignment chart)	K (from new formula)	Difference	% of Difference
0	0	0.5	0.498	-0.002	0%
0.1	0	0.525	0.519	-0.006	-1%
0.2	0	0.54	0.538	-0.002	0%
0.3	0	0.56	0.556	-0.004	-1%
0.4	0	0.575	0.572	-0.003	0%
0.5	0	0.59	0.587	-0.003	-1%
0.6	0	0.6	0.600	0.000	0%
0.7	0	0.61	0.613	0.003	0%

G_A	G_B	K exact (from alignment chart)	K (from new formula)	Difference	% of Difference
0.8	0	0.618	0.623	0.005	1%
0.9	0	0.625	0.633	0.008	1%
1	0	0.63	0.642	0.012	2%
2	0	0.66	0.686	0.026	4%
3	0	0.67	0.690	0.020	3%
4	0	0.675	0.692	0.017	3%
5	0	0.68	0.712	0.032	5%
10	0	0.69	0.718	0.028	4%
50	0	0.7	-0.340	-1.040	-149%
0	0.1	0.525	0.519	-0.006	-1%
0.1	0.1	0.55	0.540	-0.010	-2%
0.2	0.1	0.57	0.559	-0.011	-2%
0.3	0.1	0.59	0.577	-0.013	-2%
0.4	0.1	0.605	0.593	-0.012	-2%
0.5	0.1	0.618	0.608	-0.010	-2%
0.6	0.1	0.627	0.621	-0.006	-1%
0.7	0.1	0.635	0.633	-0.002	0%
0.8	0.1	0.642	0.644	0.002	0%
0.9	0.1	0.65	0.654	0.004	1%
1	0.1	0.655	0.663	0.008	1%
2	0.1	0.69	0.707	0.017	3%
3	0.1	0.705	0.711	0.006	1%
4	0.1	0.712	0.713	0.001	0%
5	0.1	0.715	0.733	0.018	3%
10	0.1	0.725	0.739	0.014	2%
50	0.1	0.73	-0.319	-1.049	-144%
0	0.2	0.54	0.538	-0.002	0%
0.1	0.2	0.57	0.559	-0.011	-2%
0.2	0.2	0.59	0.578	-0.012	-2%
0.3	0.2	0.61	0.596	-0.014	-2%
0.4	0.2	0.625	0.612	-0.013	-2%
0.5	0.2	0.64	0.627	-0.013	-2%
0.6	0.2	0.65	0.641	-0.009	-1%
0.7	0.2	0.66	0.653	-0.007	-1%
0.8	0.2	0.67	0.664	-0.006	-1%
0.9	0.2	0.675	0.673	-0.002	0%
1	0.2	0.68	0.682	0.002	0%
2	0.2	0.715	0.727	0.012	2%
3	0.2	0.73	0.730	0.000	0%
4	0.2	0.737	0.732	-0.005	-1%
5	0.2	0.74	0.752	0.012	2%
10	0.2	0.75	0.758	0.008	1%
50	0.2	0.76	-0.299	-1.059	-139%
0	0.3	0.56	0.556	-0.004	-1%
0.1	0.3	0.59	0.577	-0.013	-2%
0.2	0.3	0.61	0.596	-0.014	-2%
0.3	0.3	0.63	0.614	-0.016	-3%
0.4	0.3	0.645	0.630	-0.015	-2%
0.5	0.3	0.658	0.645	-0.013	-2%
0.6	0.3	0.668	0.658	-0.010	-1%
0.7	0.3	0.678	0.670	-0.008	-1%
0.8	0.3	0.687	0.681	-0.006	-1%
0.9	0.3	0.695	0.691	-0.004	-1%
1	0.3	0.7	0.700	0.000	0%
2	0.3	0.735	0.744	0.009	1%
3	0.3	0.75	0.748	-0.002	0%
4	0.3	0.757	0.750	-0.007	-1%
5	0.3	0.762	0.770	0.008	1%
10	0.3	0.77	0.775	0.005	1%
50	0.3	0.79	-0.282	-1.072	-136%
0	0.4	0.575	0.572	-0.003	-1%
0.1	0.4	0.605	0.593	-0.012	-2%
0.2	0.4	0.625	0.612	-0.013	-2%
0.3	0.4	0.645	0.630	-0.015	-2%
0.4	0.4	0.66	0.646	-0.014	-2%
0.5	0.4	0.672	0.661	-0.011	-2%
0.6	0.4	0.682	0.674	-0.008	-1%
0.7	0.4	0.693	0.686	-0.007	-1%
0.8	0.4	0.702	0.697	-0.005	-1%
0.9	0.4	0.71	0.707	-0.003	0%
1	0.4	0.715	0.716	0.001	0%
2	0.4	0.75	0.760	0.010	1%
3	0.4	0.765	0.764	-0.001	0%
4	0.4	0.775	0.766	-0.009	-1%
5	0.4	0.78	0.786	0.006	1%
10	0.4	0.79	0.792	0.002	0%
50	0.4	0.81	-0.266	-1.076	-133%
0	0.5	0.59	0.587	-0.003	-1%
0.1	0.5	0.618	0.608	-0.010	-2%
0.2	0.5	0.64	0.627	-0.013	-2%
0.3	0.5	0.658	0.645	-0.013	-2%
0.4	0.5	0.672	0.661	-0.011	-2%
0.5	0.5	0.685	0.676	-0.009	-1%
0.6	0.5	0.697	0.689	-0.008	-1%
0.7	0.5	0.707	0.701	-0.006	-1%
0.8	0.5	0.716	0.712	-0.004	-1%
0.9	0.5	0.724	0.722	-0.002	0%
1	0.5	0.73	0.731	0.001	0%
2	0.5	0.765	0.775	0.010	1%
3	0.5	0.78	0.779	-0.001	0%
4	0.5	0.792	0.780	-0.012	-1%
5	0.5	0.798	0.801	0.003	0%
10	0.5	0.805	0.806	0.001	0%
50	0.5	0.825	-0.251	-1.076	-130%
0	0.6	0.6	0.600	0.000	0%
0.1	0.6	0.627	0.621	-0.006	-1%
0.2	0.6	0.65	0.640	-0.010	-1%
0.3	0.6	0.668	0.658	-0.010	-2%
0.4	0.6	0.682	0.674	-0.008	-1%
0.5	0.6	0.697	0.689	-0.008	-1%
0.6	0.6	0.71	0.702	-0.008	-1%
0.7	0.6	0.72	0.714	-0.006	-1%
0.8	0.6	0.728	0.725	-0.003	0%
0.9	0.6	0.735	0.735	0.000	0%
1	0.6	0.741	0.744	0.003	0%
2	0.6	0.776	0.788	0.012	2%

G_A	G_B	K exact (from alignment chart)	K (from new formula)	Difference	% of Difference
3	0.6	0.795	0.792	-0.003	0%
4	0.6	0.805	0.794	-0.011	-1%
5	0.6	0.81	0.814	0.004	1%
10	0.6	0.82	0.820	0.000	0%
50	0.6	0.84	-0.238	-1.078	-128%
0	0.7	0.61	0.612	0.002	0%
0.1	0.7	0.635	0.633	-0.002	0%
0.2	0.7	0.66	0.652	-0.008	-1%
0.3	0.7	0.678	0.670	-0.008	-1%
0.4	0.7	0.693	0.686	-0.007	-1%
0.5	0.7	0.707	0.701	-0.006	-1%
0.6	0.7	0.72	0.714	-0.006	-1%
0.7	0.7	0.73	0.727	-0.003	0%
0.8	0.7	0.738	0.737	-0.001	0%
0.9	0.7	0.745	0.747	0.002	0%
1	0.7	0.75	0.756	0.006	1%
2	0.7	0.788	0.800	0.012	2%
3	0.7	0.805	0.804	-0.001	0%
4	0.7	0.815	0.806	-0.009	-1%
5	0.7	0.82	0.826	0.006	1%
10	0.7	0.83	0.832	0.002	0%
50	0.7	0.85	-0.225	-1.075	-127%
0	0.8	0.618	0.623	0.005	1%
0.1	0.8	0.642	0.644	0.002	0%
0.2	0.8	0.67	0.663	-0.007	-1%
0.3	0.8	0.687	0.681	-0.006	-1%
0.4	0.8	0.702	0.697	-0.005	-1%
0.5	0.8	0.716	0.712	-0.004	-1%
0.6	0.8	0.728	0.725	-0.003	0%
0.7	0.8	0.738	0.737	-0.001	0%
0.8	0.8	0.748	0.748	0.000	0%
0.9	0.8	0.755	0.758	0.003	0%
1	0.8	0.761	0.767	0.006	1%
2	0.8	0.8	0.811	0.011	1%
3	0.8	0.817	0.815	-0.002	0%
4	0.8	0.825	0.817	-0.008	-1%
5	0.8	0.83	0.837	0.007	1%
10	0.8	0.84	0.843	0.003	0%
50	0.8	0.86	-0.215	-1.075	-125%
0	0.9	0.625	0.633	0.008	1%
0.1	0.9	0.65	0.654	0.004	1%
0.2	0.9	0.675	0.673	-0.002	0%
0.3	0.9	0.695	0.691	-0.004	-1%
0.4	0.9	0.71	0.707	-0.003	0%
0.5	0.9	0.724	0.722	-0.002	0%
0.6	0.9	0.735	0.735	0.000	0%
0.7	0.9	0.745	0.747	0.002	0%
0.8	0.9	0.755	0.758	0.003	0%
0.9	0.9	0.763	0.768	0.005	1%
1	0.9	0.77	0.777	0.007	1%
2	0.9	0.81	0.821	0.011	1%
3	0.9	0.827	0.825	-0.002	0%
4	0.9	0.835	0.827	-0.008	-1%
5	0.9	0.84	0.847	0.007	1%
10	0.9	0.85	0.852	0.002	0%
50	0.9	0.867	-0.205	-1.072	-124%
0	1	0.63	0.641	0.011	2%
0.1	1	0.655	0.662	0.007	1%
0.2	1	0.68	0.682	0.002	0%
0.3	1	0.7	0.699	-0.001	0%
0.4	1	0.715	0.716	0.001	0%
0.5	1	0.73	0.730	0.000	0%
0.6	1	0.741	0.744	0.003	0%
0.7	1	0.75	0.756	0.006	1%
0.8	1	0.761	0.767	0.006	1%
0.9	1	0.77	0.777	0.007	1%
1	1	0.777	0.786	0.009	1%
2	1	0.818	0.830	0.012	1%
3	1	0.835	0.834	-0.001	0%
4	1	0.845	0.835	-0.010	-1%
5	1	0.85	0.856	0.006	1%
10	1	0.86	0.861	0.001	0%
50	1	0.875	-0.196	-1.071	-122%
0	2	0.66	0.686	0.026	4%
0.1	2	0.69	0.707	0.017	2%
0.2	2	0.715	0.726	0.011	2%
0.3	2	0.735	0.744	0.009	1%
0.4	2	0.75	0.760	0.010	1%
0.5	2	0.765	0.775	0.010	1%
0.6	2	0.776	0.788	0.012	2%
0.7	2	0.788	0.800	0.012	2%
0.8	2	0.8	0.811	0.011	1%
0.9	2	0.81	0.821	0.011	1%
1	2	0.818	0.830	0.012	1%
2	2	0.86	0.874	0.014	2%
3	2	0.877	0.878	0.001	0%
4	2	0.885	0.880	-0.005	-1%
5	2	0.89	0.900	0.010	1%
10	2	0.905	0.906	0.001	0%
50	2	0.92	-0.152	-1.072	-116%
0	3	0.67	0.690	0.020	3%
0.1	3	0.705	0.711	0.006	1%
0.2	3	0.73	0.730	0.000	0%
0.3	3	0.75	0.748	-0.002	0%
0.4	3	0.765	0.764	-0.001	0%
0.5	3	0.78	0.779	-0.001	0%
0.6	3	0.795	0.792	-0.003	0%
0.7	3	0.805	0.804	-0.001	0%
0.8	3	0.817	0.815	-0.002	0%
0.9	3	0.827	0.825	-0.002	0%
1	3	0.835	0.834	-0.001	0%
2	3	0.877	0.878	0.001	0%
3	3	0.895	0.882	-0.013	-1%
4	3	0.905	0.884	-0.021	-2%
5	3	0.91	0.904	-0.006	-1%
10	3	0.925	0.909	-0.016	-2%

G_A	G_B	K exact (from alignment chart)	K (from new formula)	Difference	% of Difference
50	3	0.94	-0.148	-1.088	-116%
0	4	0.675	0.691	0.016	2%
0.1	4	0.712	0.712	0.000	0%
0.2	4	0.737	0.732	-0.005	-1%
0.3	4	0.757	0.749	-0.008	-1%
0.4	4	0.775	0.766	-0.009	-1%
0.5	4	0.792	0.780	-0.012	-1%
0.6	4	0.805	0.794	-0.011	-1%
0.7	4	0.815	0.806	-0.009	-1%
0.8	4	0.825	0.817	-0.008	-1%
0.9	4	0.835	0.827	-0.008	-1%
1	4	0.845	0.835	-0.010	-1%
2	4	0.885	0.880	-0.005	-1%
3	4	0.905	0.883	-0.022	-2%
4	4	0.915	0.885	-0.030	-3%
5	4	0.922	0.905	-0.017	-2%
10	4	0.937	0.911	-0.026	-3%
50	4	0.951	-0.146	-1.097	-115%
0	5	0.68	0.711	0.031	5%
0.1	5	0.715	0.732	0.017	2%
0.2	5	0.74	0.751	0.011	1%
0.3	5	0.762	0.769	0.007	1%
0.4	5	0.78	0.785	0.005	1%
0.5	5	0.798	0.800	0.002	0%
0.6	5	0.81	0.813	0.003	0%
0.7	5	0.82	0.825	0.005	1%
0.8	5	0.83	0.836	0.006	1%
0.9	5	0.84	0.846	0.006	1%
1	5	0.85	0.855	0.005	1%
2	5	0.89	0.899	0.009	1%
3	5	0.91	0.903	-0.007	-1%
4	5	0.922	0.905	-0.017	-2%
5	5	0.93	0.925	-0.005	-1%
10	5	0.945	0.930	-0.015	-2%
50	5	0.96	-0.127	-1.087	-113%
0	10	0.69	0.712	0.022	3%
0.1	10	0.725	0.733	0.008	1%
0.2	10	0.75	0.752	0.002	0%
0.3	10	0.77	0.770	0.000	0%
0.4	10	0.79	0.786	-0.004	0%
0.5	10	0.805	0.801	-0.004	-1%
0.6	10	0.82	0.814	-0.006	-1%
0.7	10	0.83	0.826	-0.004	0%
0.8	10	0.84	0.837	-0.003	0%
0.9	10	0.85	0.847	-0.003	0%
1	10	0.86	0.856	-0.004	0%
2	10	0.905	0.900	-0.005	-1%
3	10	0.925	0.904	-0.021	-2%
4	10	0.937	0.906	-0.031	-3%
5	10	0.945	0.926	-0.019	-2%
10	10	0.96	0.932	-0.028	-3%
50	10	0.98	-0.126	-1.106	-113%
0	50	0.7	-5.493	-6.193	-885%
0.1	50	0.73	-5.471	-6.201	-850%
0.2	50	0.76	-5.452	-6.212	-817%
0.3	50	0.79	-5.434	-6.224	-788%
0.4	50	0.81	-5.418	-6.228	-769%
0.5	50	0.825	-5.403	-6.228	-755%
0.6	50	0.84	-5.390	-6.230	-742%
0.7	50	0.85	-5.378	-6.228	-733%
0.8	50	0.86	-5.367	-6.227	-724%
0.9	50	0.867	-5.357	-6.224	-718%
1	50	0.875	-5.348	-6.223	-711%
2	50	0.92	-5.304	-6.224	-677%
3	50	0.94	-5.300	-6.240	-664%
4	50	0.951	-5.299	-6.250	-657%
5	50	0.96	-5.278	-6.238	-650%
10	50	0.98	-5.273	-6.253	-638%
50	50	0.99	-6.330	-7.320	-739%

TABLE IV
COMPARISON BETWEEN EXACT EQUATION AND NEW FORMULA FOR THE BUCKLING LENGTH FACTOR OF PERMITTED SWAY COLUMNS

G_A	G_B	K exact (from alignment chart)	K (from new formula)	Difference	% of Difference
0	0	1	1.168	0.168	17%
1	0	1.15	1.262	0.112	10%
2	0	1.25	1.352	0.102	8%
3	0	1.35	1.438	0.088	6%
4	0	1.43	1.519	0.089	6%
5	0	1.5	1.596	0.096	6%
6	0	1.55	1.670	0.120	8%
7	0	1.6	1.739	0.139	9%
8	0	1.61	1.805	0.195	12%
9	0	1.65	1.867	0.217	13%
10	0	1.67	1.925	0.255	15%
20	0	1.8	2.321	0.521	29%
30	0	1.85	2.437	0.587	32%
50	0	1.88	2.148	0.268	14%
100	0	1.93	2.102	0.172	9%
0	1	1.15	1.262	0.112	10%
1	1	1.32	1.358	0.038	3%
2	1	1.45	1.450	0.000	0%
3	1	1.57	1.538	-0.032	-2%
4	1	1.65	1.621	-0.029	-2%
5	1	1.7	1.701	0.001	0%
6	1	1.75	1.776	0.026	2%
7	1	1.8	1.848	0.048	3%
8	1	1.82	1.915	0.095	5%
9	1	1.86	1.979	0.119	6%
10	1	1.88	2.039	0.159	8%
20	1	2.1	2.455	0.355	17%
30	1	2.15	2.588	0.438	20%

G_A	G_B	K exact (from alignment chart)	K (from new formula)	Difference	% of Difference
50	1	2.2	2.329	0.129	6%
100	1	2.25	2.335	0.085	4%
0	2	1.25	1.352	0.102	8%
1	2	1.45	1.450	0.000	0%
2	2	1.59	1.544	-0.046	-3%
3	2	1.7	1.634	-0.066	-4%
4	2	1.78	1.720	-0.060	-3%
5	2	1.85	1.801	-0.049	-3%
6	2	1.9	1.878	-0.022	-1%
7	2	2	1.952	-0.048	-2%
8	2	2.05	2.021	-0.029	-1%
9	2	2.1	2.087	-0.013	-1%
10	2	2.15	2.149	-0.001	0%
20	2	2.3	2.584	0.284	12%
30	2	2.4	2.735	0.335	14%
50	2	2.5	2.506	0.006	0%
100	2	2.6	2.562	-0.038	-1%
0	3	1.35	1.438	0.088	6%
1	3	1.57	1.538	-0.032	-2%
2	3	1.7	1.634	-0.066	-4%
3	3	1.85	1.726	-0.124	-7%
4	3	1.93	1.813	-0.117	-6%
5	3	2.05	1.897	-0.153	-7%
6	3	2.1	1.976	-0.124	-6%
7	3	2.17	2.052	-0.118	-5%
8	3	2.22	2.123	-0.097	-4%
9	3	2.28	2.191	-0.089	-4%
10	3	2.3	2.255	-0.045	-2%
20	3	2.55	2.708	0.158	6%
30	3	2.65	2.876	0.226	9%
50	3	2.8	2.677	-0.123	-4%
100	3	2.9	2.783	-0.117	-4%
0	4	1.43	1.519	0.089	6%
1	4	1.65	1.621	-0.029	-2%
2	4	1.78	1.720	-0.060	-3%
3	4	1.93	1.813	-0.117	-6%
4	4	2.05	1.903	-0.147	-7%
5	4	2.15	1.988	-0.162	-8%
6	4	2.2	2.070	-0.130	-6%
7	4	2.28	2.147	-0.133	-6%
8	4	2.32	2.221	-0.099	-4%
9	4	2.38	2.290	-0.090	-4%
10	4	2.42	2.356	-0.064	-3%
20	4	2.7	2.828	0.128	5%
30	4	2.85	3.013	0.163	6%
50	4	3	2.844	-0.156	-5%
100	4	3.1	2.999	-0.101	-3%
0	5	1.5	1.596	0.096	6%
1	5	1.7	1.701	0.001	0%
2	5	1.85	1.801	-0.049	-3%
3	5	2.05	1.897	-0.153	-7%
4	5	2.15	1.988	-0.162	-8%
5	5	2.25	2.076	-0.174	-8%
6	5	2.3	2.159	-0.141	-6%
7	5	2.4	2.239	-0.161	-7%
8	5	2.45	2.314	-0.136	-6%
9	5	2.5	2.386	-0.114	-5%
10	5	2.55	2.454	-0.096	-4%
20	5	2.9	2.943	0.043	1%
30	5	3.05	3.146	0.096	3%
50	5	3.2	3.006	-0.194	-6%
100	5	3.3	3.210	-0.090	-3%
0	6	1.55	1.670	0.120	8%
1	6	1.75	1.776	0.026	2%
2	6	1.9	1.878	-0.022	-1%
3	6	2.1	1.976	-0.124	-6%
4	6	2.2	2.070	-0.130	-6%
5	6	2.3	2.159	-0.141	-6%
6	6	2.4	2.245	-0.155	-6%
7	6	2.5	2.326	-0.174	-7%
8	6	2.53	2.403	-0.127	-5%
9	6	2.6	2.477	-0.123	-5%
10	6	2.67	2.547	-0.123	-5%
20	6	3.05	3.055	0.005	0%
30	6	3.2	3.274	0.074	2%
50	6	3.35	3.163	-0.187	-6%
100	6	3.5	3.414	-0.086	-2%
0	7	1.6	1.739	0.139	9%
1	7	1.8	1.848	0.048	3%
2	7	2	1.952	-0.048	-2%
3	7	2.17	2.052	-0.118	-5%
4	7	2.28	2.147	-0.133	-6%
5	7	2.4	2.239	-0.161	-7%
6	7	2.5	2.326	-0.174	-7%
7	7	2.56	2.409	-0.151	-6%
8	7	2.65	2.488	-0.162	-6%
9	7	2.7	2.564	-0.136	-5%
10	7	2.8	2.636	-0.164	-6%
20	7	3.15	3.162	0.012	0%
30	7	3.3	3.397	0.097	3%
50	7	3.5	3.316	-0.184	-5%
100	7	3.75	3.614	-0.136	-4%
0	8	1.61	1.805	0.195	12%
1	8	1.82	1.915	0.095	5%
2	8	2.05	2.021	-0.029	-1%
3	8	2.22	2.123	-0.097	-4%
4	8	2.32	2.221	-0.099	-4%
5	8	2.45	2.314	-0.136	-6%
6	8	2.53	2.403	-0.127	-5%
7	8	2.65	2.488	-0.162	-6%
8	8	2.7	2.570	-0.130	-5%
9	8	2.8	2.647	-0.153	-5%
10	8	2.88	2.721	-0.159	-6%
20	8	3.2	3.265	0.065	2%
30	8	3.45	3.517	0.067	2%
50	8	3.7	3.464	-0.236	-6%

G_A	G_B	K exact (from alignment chart)	K (from new formula)	Difference	% of Difference
100	8	3.9	3.808	-0.092	-2%
0	9	1.65	1.867	0.217	13%
1	9	1.86	1.979	0.119	6%
2	9	2.1	2.087	-0.013	-1%
3	9	2.28	2.191	-0.089	-4%
4	9	2.38	2.290	-0.090	-4%
5	9	2.5	2.386	-0.114	-5%
6	9	2.6	2.477	-0.123	-5%
7	9	2.7	2.564	-0.136	-5%
8	9	2.8	2.647	-0.153	-5%
9	9	2.9	2.726	-0.174	-6%
10	9	3	2.802	-0.198	-7%
20	9	3.35	3.363	0.013	0%
30	9	3.6	3.632	0.032	1%
50	9	3.8	3.607	-0.193	-5%
100	9	4.1	3.997	-0.103	-3%
0	10	1.67	1.925	0.255	15%
1	10	1.88	2.039	0.159	8%
2	10	2.15	2.149	-0.001	0%
3	10	2.3	2.255	-0.045	-2%
4	10	2.42	2.356	-0.064	-3%
5	10	2.55	2.454	-0.096	-4%
6	10	2.67	2.547	-0.123	-5%
7	10	2.8	2.636	-0.164	-6%
8	10	2.88	2.721	-0.159	-6%
9	10	3	2.802	-0.198	-7%
10	10	3.05	2.879	-0.171	-6%
20	10	3.4	3.458	0.058	2%
30	10	3.7	3.743	0.043	1%
50	10	4	3.747	-0.253	-6%
100	10	4.3	4.181	-0.119	-3%
0	20	1.8	2.321	0.521	29%
1	20	2.1	2.455	0.355	17%
2	20	2.3	2.584	0.284	12%
3	20	2.55	2.708	0.158	6%
4	20	2.7	2.828	0.128	5%
5	20	2.9	2.943	0.043	1%
6	20	3.05	3.055	0.005	0%
7	20	3.15	3.162	0.012	0%
8	20	3.2	3.265	0.065	2%
9	20	3.35	3.363	0.013	0%
10	20	3.4	3.458	0.058	2%
20	20	4.2	4.206	0.006	0%
30	20	4.6	4.645	0.045	1%
50	20	5.1	4.913	-0.187	-4%
100	20	5.5	5.754	0.254	5%
0	30	1.85	2.437	0.587	32%
1	30	2.15	2.588	0.438	20%
2	30	2.4	2.735	0.335	14%
3	30	2.65	2.876	0.226	9%
4	30	2.85	3.013	0.163	6%
5	30	3.05	3.146	0.096	3%
6	30	3.2	3.274	0.074	2%
7	30	3.3	3.397	0.097	3%
8	30	3.45	3.517	0.067	2%
9	30	3.6	3.632	0.032	1%
10	30	3.7	3.743	0.043	1%
20	30	4.6	4.645	0.045	1%
30	30	5.2	5.223	0.023	0%
50	30	5.9	5.726	-0.174	-3%
100	30	6.5	6.903	0.403	6%
0	50	1.88	2.148	0.268	14%
1	50	2.2	2.329	0.129	6%
2	50	2.5	2.506	0.006	0%
3	50	2.8	2.677	-0.123	-4%
4	50	3	2.844	-0.156	-5%
5	50	3.2	3.006	-0.194	-6%
6	50	3.35	3.163	-0.187	-6%
7	50	3.5	3.316	-0.184	-5%
8	50	3.7	3.464	-0.236	-6%
9	50	3.8	3.607	-0.193	-5%
10	50	4	3.747	-0.253	-6%
20	50	5.1	4.913	-0.187	-4%
30	50	5.9	5.726	-0.174	-3%
50	50	6.9	6.614	-0.286	-4%
100	50	8	8.242	0.242	3%
0	100	1.93	2.102	0.172	9%
1	100	2.25	2.335	0.085	4%
2	100	2.6	2.562	-0.038	-1%
3	100	2.9	2.783	-0.117	-4%
4	100	3.1	2.999	-0.101	-3%
5	100	3.3	3.210	-0.090	-3%
6	100	3.5	3.414	-0.086	-2%
7	100	3.75	3.614	-0.136	-4%
8	100	3.9	3.808	-0.092	-2%
9	100	4.1	3.997	-0.103	-3%
10	100	4.3	4.181	-0.119	-3%
20	100	5.5	5.754	0.254	5%
30	100	6.5	6.903	0.403	6%
50	100	8	8.242	0.242	3%
100	100	10	9.730	-0.270	-3%

IV. Conclusion

Most codes for design depend on the alignment chart for determination of buckling length factor where the alignment chart itself depends on the size of the chart, and on the reader's sharpness of vision. In this work, a new formula has been presented to calculate the buckling length efficiently, the main calculation in this work can be summarized in the following points:

- The created formulas are limited to values of G_A and G_B not greater than 50 for prevented sway columns and 100 for permitted sway columns.

- The values of new formulas are generally suitable for the known values of G_A and G_B except some values, which need to correction.
- The new formulas are distinguished by direct substitution to get the buckling length factor.
- The new formulas can be programmed easily using design excel sheets and the other design computer programs.

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