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Chaos Synchronization Using Sliding Mode Technique

Behzad Khademian, and Mohammad Haeri, *Member, IEEE*

***Abstract*—**In this paper, an effective sliding mode design is applied to chaos synchronization. The proposed controller can make the states of two identical modified Chua's circuits globally asymptotically synchronized. Numerical results are provided to show the effectiveness and robustness of the proposed method.

***Keywords*—**Sliding mode, Chaos synchronization, Modified Chua's circuit.

I. Introduction

CHAOS control and synchronization have been intensively studied during the last decade [1]. Chaos synchronization is closely related to observer problem in control theory [2]. Generally speaking, chaos synchronization can be thought as the design problem of a feedback law for full observer using the known information of the plant, so as to ensure that the controlled receiver synchronizes with the plant. Given a chaotic system, which is considered as the master system, and another identical system, which is considered as the slave system, the dynamical behaviors of these two systems may be identical after a transient when the slave system is driven by a control input. In this paper, the goal is to force the master–slave n-dimensional chaotic systems to be synchronized even if they have differences in initial conditions or expose to external disturbances such as channel noise. In order to increase the robustness of the closed loop systems, the key idea is that a sliding mode type of controller is employed. Based on this proposed method, a switching surface is designed for chaos synchronization.

The paper is organized as follows. In Section II, the sliding mode controller and the switching surface is designed. In Section III, the proposed controller is applied to a coupled modified Chua's circuits. Numerical simulations are carried out in Section IV for illustration and verification of the presented methodology. Finally some concluding remarks are given in Section V.

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Behzad Khademian is with the Electrical Engineering Department, Sharif University of Technology, Tehran, Iran, (e-mail: khademian@mehr.sharif.edu).

Dr. M. Haeri is with the Electrical Engineering Department, Sharif University of Technology, and the head of the Advanced Control System Lab. Iran (phone: 98-21-616-5964, fax: +98-21-602-3261; e-mail: haeri@sina.sharif.edu).

II. Sliding Mode Controller

Consider the following two n-dimensional chaotic systems:

$$\dot{x} = Ax + f(x) \tag{1}$$

and

$$\dot{y} = Ay + f(y) + u(t) \tag{2}$$

$A \in R^{n\times n}$ is a constant matrix, $f : R^n \to R^n$ is a continuous nonlinear function and $u \in R^n$ is the control input. The control problem considered in this paper is that for different initial conditions of systems (1) and (2), the two coupled system, i.e. the master system (1) and the slave system (2), to be synchronized by designing an appropriate sliding mode control $u(t)$ which is attached to the slave system (2) such that:

$$\lim_{t\to\infty}\left\|x(t) - y(t)\right\| \to 0$$

where $\|*\|$ the Euclidean norm of a vector.

Considering $e = y - x \in R^n$ and $f(y) - f(x) = M_{x,y}$, error dynamics can be written as:

$$\dot{e} = Ae + M_{x,y} + u(t) \tag{3}$$

Following the active control approach of Bai and Lonngren [3], to eliminate the nonlinear part of the error dynamics, we can choose $u(t) = Bv(t) - M_{x,y}$, where B is a constant gain vector which is selected such that (A, B) be controllable and then (3) becomes:

$$\dot{e} = Ae + Bv(t) \tag{4}$$

Now the original synchronization problem can be replaced by the equivalent problem of stabilizing the zero solution of the system (4) by a suitable choice of the sliding mode control. In the following, the sliding mode controller will be designed using variable structure control [4] and sliding mode control [5] methods. Let us introduce $s = s(e)$ as the sliding surface which can be defined as:

$$s(e) = Ce \tag{5}$$

where C is a constant vector.

When in sliding surface, the controlled system satisfies the following conditions:

$$s(e) = 0 \tag{6}$$

and

$$\dot{s}(e) = 0 \tag{7}$$

Notice that the second one is the necessary condition for the state trajectory to stay on the switching surface $s(e) = 0$.

Using (4) in (5) we can rewrite equation (7) as:

$$\dot{s}(e) = C(Ae + Bv(t)) = 0 \tag{8}$$

Solving equation (8) for $v(t)$ yields the equivalent control $v_{eq}(t)$:

$$v_{eq}(t) = -(CB)^{-1}CAe \tag{9}$$

where existence of the $(CB)^{-1}$ is the necessary condition.

By using (9) in (5), the error dynamics in sliding mode is given as follows:

$$\dot{e} = [I - B(CB)^{-1}C]Ae \tag{10}$$

The vector C is selected such that all the eigenvalues of $[I - B(CB)^{-1}C]A$ have negative real parts, so the controlled system is asymptotically stable.

To design the sliding mode controller we use the constant plus proportional rate reaching law [4]:

$$\dot{s} = -q\,\mathrm{sgn}(s) - ks \tag{11}$$

where $\mathrm{sgn}(\cdot)$ denotes the sign function, and the gains $q > 0$ and $k > 0$ is determined such that the sliding condition is satisfied and sliding mode motion will occur.

From equations (8) and (11), we can obtain $v(t)$:

$$v(t) = -(CB)^{-1}[C(kI + A)e + q\,\mathrm{sgn}(s)] \tag{12}$$

which yields:

$$v(t) = \begin{cases} -(CB)^{-1}[C(kI + A)e + q] & s(e) > 0 \\ -(CB)^{-1}[C(kI + A)e - q] & s(e) < 0 \end{cases} \tag{13}$$

In order to observe the stability of the error dynamics with the forgoing controller, we choose $V = s^2/2$, as a possible Lyapunov function. Then the derivative of V becomes:

$$\begin{aligned} \dot{V} &= \dot{s}s = C(Ae + Bv)s \\ &= C(Ae - B(CB)^{-1}[C(kI + A)e + q\,\mathrm{sgn}(s)])s \\ &= -ks^2 - q\,\mathrm{sgn}(s)s < 0 \end{aligned} \tag{14}$$

Notice that $\mathrm{sgn}(s)s$ is always positive when $e \neq 0$ and $q, k > 0$.

Since V is a positive and decrescent function and $\dot{V}$ is negative semidefinite, it follows that the equilibrium point $(e_i = 0, i = 1, \ldots, n)$ of the systems (4) is uniformly stable, i.e. $e_i(t) \in L_\infty$. From (14), we can easily show that the squares of $e_i(t), i = 1, \ldots, n$ are integrable with respect to time, i.e. $e_i \in L_2$. Based on Barbalat's lemma and for any initial condition (4) implies that, $\dot{e}_i(t) \in L_\infty$, which in turn implies $e_i(t) \to 0$ as $t \to \infty$. Thus in the closed-loop system $x(t) \to y(t)$, as $t \to \infty$. This implies that two chaotic systems have been synchronized through the sliding mode control.

III. Synchronization of Modified Chua's Circuit

In this section we apply the above techniques to modified Chua's circuit described by [6]:

$$\begin{aligned} \dot{x} &= p(y - f(x)), \quad f(x) = 2x^3 - x/7 \\ \dot{y} &= x - y + z \\ \dot{z} &= -qy \end{aligned} \tag{15}$$

which has a chaotic attractor as shown in Fig. 1 when $p = 10$ and $q = 100/7$.

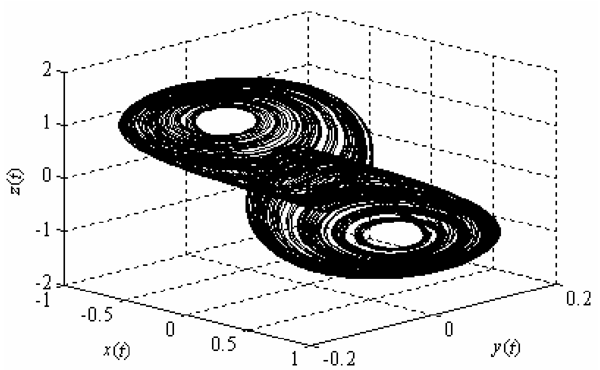

Fig. 1The modified Chua's circuit chaotic attractor

In order to observe synchronization behavior we have two modified Chua's circuits where the drive system with three state variables denoted by the subscript 1 drives the response system having identical equations denoted by the subscript 2. Note that the initial conditions on the drive system are different from that of the response system. The two modified Chua's circuits are described, respectively, by the following equations.

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$$
\begin{aligned}
\dot{x}_1 &= p(y_1 - (2x_1^3 - x_1)/7) \\
\dot{y}_1 &= x_1 - y_1 + z_1 \\
\dot{z}_1 &= -qy_1
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\dot{x}_2 &= p(y_2 - (2x_2^3 - x_2)/7) + u_x(t) \\
\dot{y}_2 &= x_2 - y_2 + z_2 + u_y(t) \\
\dot{z}_2 &= -qy_2 + u_z(t)
\end{aligned} \tag{17}
$$

where $u_x(t)$, $u_y(t)$ and $u_z(t)$ are three control functions that to be determined in order to synchronize two modified Chua's circuits.

Considering $(e_x = x_2 - x_1,\ e_y = y_2 - y_1,\ e_z = z_2 - z_1)$, error dynamics can be written as;

$$
\begin{aligned}
\dot{e}_x &= p(e_y - 2e_x(e_x^2 + 3x_1e_x + 3x_1^2)/7 + e_x/7) + u_x(t) \\
\dot{e}_y &= e_x - e_y + e_z + u_y(t) \\
\dot{e}_z &= -qe_y + u_z(t)
\end{aligned} \tag{18}
$$

To write equation (18) in the form of (3) we introduce the following matrices:

$$
A = \begin{pmatrix} p/7 & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{pmatrix},\ M_{x,y} = \begin{pmatrix} 2p(x_1^3 - x_2^3)/7 \\ 0 \\ 0 \end{pmatrix},\ u = \begin{pmatrix} u_x(t) \\ u_y(t) \\ u_z(t) \end{pmatrix} \tag{19}
$$

and the vector B is selected such that (A, B) is controllable:

$$
B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}
$$

The sliding surface $s(e) = Ce = (6 \quad 4 \quad 2)e$ makes the sliding mode state equation asymptotically stable. Let us choose $k = 5.2$ and $q = 0.1$, large value of k can cause chattering, and appropriate value of q is selected to quicken the time of the reaching sliding mode motion as well as to reduce the system chattering.

From equation (13) we can obtain $v(t)$:

$$
v(t) = \begin{cases} (-21.8857 \quad -24.1143 \quad -7.2)e - 0.05 & s(e) > 0 \\ (-21.8857 \quad -24.1143 \quad -7.2)e + 0.05 & s(e) < 0 \end{cases} \tag{20}
$$

and we can obtain the sliding mode controller from $u(t) = Bv(t) - M_{x,y}$.

IV. Numerical Results

To verify the effectiveness of the proposed synchronization approach, we did some numerical simulations. The initial values of drive system and response system in all simulations are taken $(x_{10} = 0.02,\ y_{10} = 0.05,\ z_{10} = 0.04)$ and

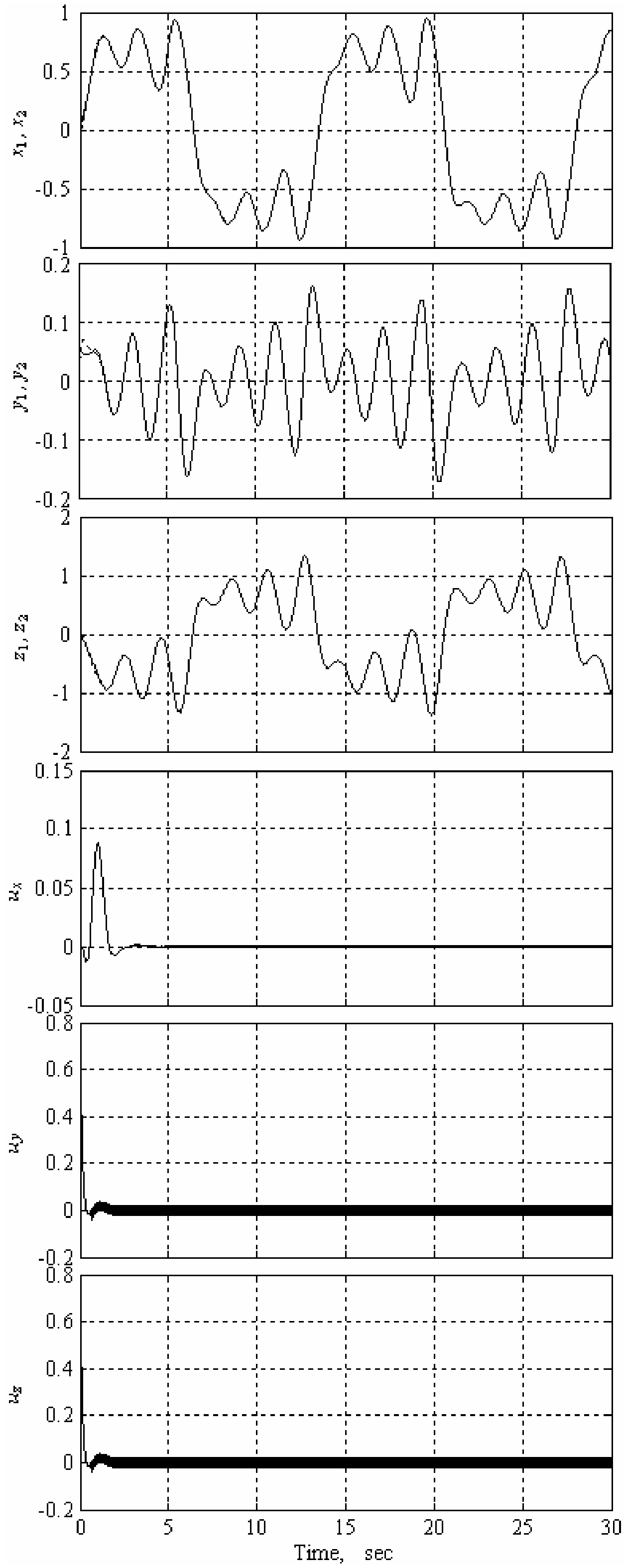

Fig. 2 Synchronized states of modified Chua's circuits and control signals

$(x_{20} = 0.0002 \ \ y_{20} = 0.0005, \ \ z_{20} = 0.0004)$ respectively. The synchronized states of the drive and response systems and the control signals are shown in Fig. 2. Error trajectories are shown in Fig. 3. To verify the robustness of the proposed method in presence of noise, white Gaussian noise with mean 0 and variance 0.01 is added to the drive system states. As it is expected and shown in Fig. 4 the controlled system is stable and the error values are bounded. In order to have better performance in presence of noise, $k = 1.5$ is taken in the sliding surface.

V. Numerical Results

To verify the effectiveness of the proposed synchronization approach, we did some numerical simulations. The initial values of drive system and response system in all simulations are taken $(x_{10} = 0.02, \ \ y_{10} = 0.05, \ \ z_{10} = 0.04)$ and $(x_{20} = 0.0002 \ \ y_{20} = 0.0005, \ \ z_{20} = 0.0004)$ respectively. The synchronized states of the drive and response systems and the control signals are shown in Fig. 2. Error trajectories are shown in Fig. 3. To verify the robustness of the proposed method in presence of noise, white Gaussian noise with mean 0 and variance 0.01 is added to the drive system states. As it is expected and shown in Fig. 4 the controlled system is stable and the error values are bounded. In order to have better performance in presence of noise, $k = 1.5$ is taken in the sliding surface.

VI. Conclusion

In this paper the synchronization of chaos by sliding mode controller has been developed and applied to a coupled modified Chua's circuits. Based on variable structure and sliding mode theorems the control function is derived. Finally the numerical results are presented to verify the proposed method.

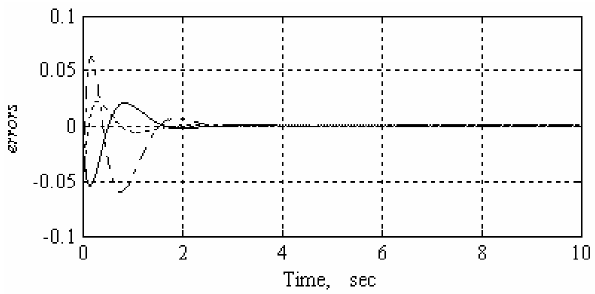

Fig. 3 Error trajectories of synchronized coupled modified Chua's circuits

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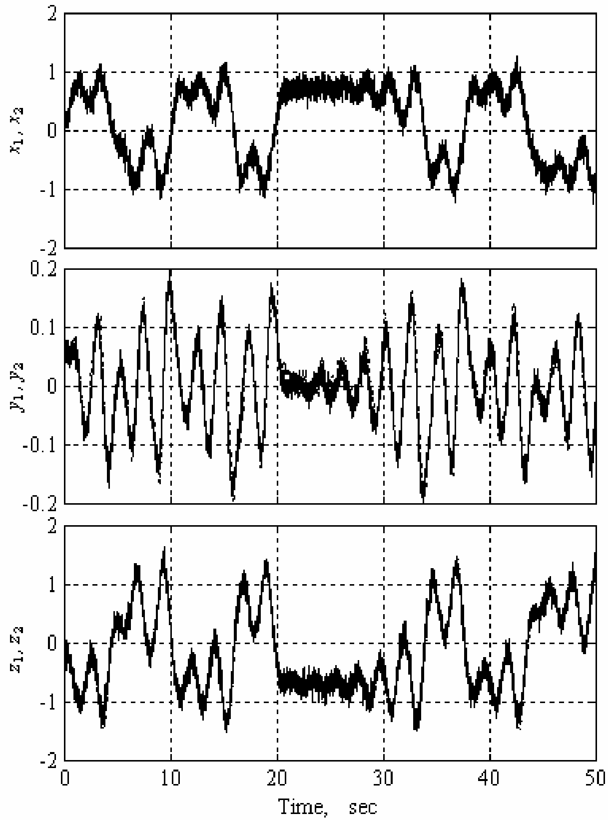

Fig. 4 Synchronization of coupled modified Chua's circuits in presence of noise