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Creating Streamribbons Based on Mass Conservative Streamlines

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Abstract**—**Streamribbon is used to visualize the rotation of the fluid flow. The rotation of flow is useful in fluid mechanics, engineering and geophysics. This paper introduces the construction technique of streamribbon using the streamline which is generated based on the law of mass conservation. The accuracy of constructed streamribbons is shown through two examples.

Keywords**—**Mass conservation, streamline, streamtube, streamribbon.

I. Introduction

STREAMLINE, streamtube and streamribbon are three of the most fundamental features for visualizing steady flow fields. Streamlines are paths of massless particles that are released in a steady flow. Streamtube and streamribbons show the expansion and rotation of the flow.

The path swept by a deformable line segment is a streamribbon. Thus a streamribbon can reveal the translation, the angular rotation, and the rate of shear deformation of the flow. In [1] streamribbons are used in identifying causal relationships between fluid motion and local flow variables such as temperature, static pressure, turbulent kinetic energy, and vorticity.

This paper follows the streamribbon construction in [2, 3] where one streamline and constant-length vectors normal to the local velocity are used to form a streamribbon. In this paper, we use the streamlines constructed in [4]. The process of creating a streamribbon is: (1) generating a streamline; (2) generating a ribbon by passing though all constant-length vectors along the streamline.

The accuracy of created streamribbons is dependent on both the accuracies of the streamline and the constant-length vectors. Li [4] introduced an adaptive streamline tracking method for three-dimensional CFD velocity fields based on the law of mass conservation. This method suits to the CFD velocity fields that more data is available. The advantages of the method introduced in [4] include that the accuracy of the tracked streamlines for a given mesh can be controlled by the threshold number T, i.e., the larger the threshold number T, the more accurate the tracked streamlines are, and the tracking process is stopped when not enough data can be provided. The overall accuracy of tracked streamline depends on the initial mesh and the threshold number T. When CFD velocity fields are given as numerical solutions of mathematical models, more data of the velocity fields are possible to calculate.

The streamtube construction followed the methods in [2, 3] has been introduced in [6] where the streamline for constructing streamtube is generated by the approach in [4]. Such a streamtube is created by connecting the circular cross flow sections along a streamline. The radius of a cross flow section is determined by the local cross flow expansion rate.

The streamribbon construction method in this paper is based on the adaptive streamline tracking method in [4] and the formula for the rotation angle given in [2]. The construction is for incompressible flows but the method can also be used to compressible steady flows by replacing the CFD velocity fields with CFD momentum fields.

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II. Streamline construction

Assume that a CFD velocity field is given in a hexahedral mesh and the further data of the velocity field is available. We subdivide a hexahedron into five tetrahedra (refer Fig. 2 in [4]) when we apply the conditions given in [4] to the hexahedron.

Assume that $\mathbf{V}_l = \mathbf{A}\mathbf{X}+\mathbf{B}$ is the linear interpolation of a given CFD velocity field at the four vertices of a tetrahedron, where

$$\mathbf{A}=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad \mathbf{B}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

are constant matrix and vertical vector respectively. $\mathbf{V}_l$ is unique if the four vertices of the tetrahedron are not on a plane [5]. Assume that f is a scalar function and $f\mathbf{V}_l$ satisfies the law of mass conservation (continuity equation) on the tetrahedron. We can find the expressions of f for different Jacobean forms of matrix $\mathbf{A}$. More data will be required only when $f\mathbf{V}_l$ does not satisfy the law of mass conservation in a hexahedron. We subdivide the hexahedron where $f\mathbf{V}_l$ does not satisfy the law of mass conservation into eight smaller hexahedra. In [4, 5], we find eight conditions of f for which $f\mathbf{V}_l$ does not satisfy the law of mass conservation, and then use these conditions for mesh

refinement. The expressions of f for the eight cases are listed in Table 1 in [4].

A. Conditions for seeking more data in a hexahedron

The conditions (CSFD) for seeking more data of the velocity fields in a hexahedron are: for a hexahedron, subdividing it into five tetrahedra, and then calculating the Jacobean forms of **A** in the linear interpolation of values of the velocity field and the coordinates at the vertices of each of the five tetrahedra respectively, if there exist at least one of the five expressions of f corresponding to the particular Jacobean of **A** in Table 1 in [4] equaling zero or infinity, after taking C as non-zero constant, at some points on the corresponding tetrahedra, further data are needed to be found inside the hexahedron. The locations of the more data are the vertices of the eight smaller hexahedra by connecting the mid points of opposite sides on each of the six faces (refer Fig. 3 in [4]).

B. Algorithm

The following algorithm is used for the adaptive streamline tracking method in [4]. *i* is the index indicating how many times of refinement have been performed. The threshold number *T* represents maximum times of refinements wanted.

1. Set $i = 0$.
2. Find the hexahedron that contains the seed point and divide the hexahedron into five tetrahedra. If the CSFD conditions are not satisfied in all five tetrahedra, draw the streamline segment that goes through the seed point in the hexahedron; otherwise go to Step 3. Take the intersection of the streamline segment with the boundary of the hexahedron as seed point or end point and go to Step 2. (for a new hexahedron).
3. Subdivide hexahedron into eight equal smaller hexahedra and calculate the values of the velocity field at the vertices of the smaller hexahedra and then let $i = i + 1$ and go to step 4.
4. Take the elements (smaller hexahedra) in the subdivided hexahedron as new elements of the mesh by replacing the initial element and go to step 2 if $i \le T$; Otherwise go to Step 1.

T is the threshold number. The bigger the threshold number *T* (or the more times of subdivisions), the more accurate the tracked streamlines are.

The streamlines constructed by the method reviewed above are very accurate as shown by the examples in [4]. The examples in [4] include closed streamlines and streamlines with asymptotic plane. The accuracy is shown by comparing the tracked streamlines with the exact streamlines.

III. Calculation of Rotation Angle

The surface of the streamribbon is formed in this paper by quadrilateral patches which take constant-length normal vectors as two sides, and the connection of their corresponding points on the streamline as the third side, and the connection of the end points of constant-length normal vectors as the fourth side. The angle of rotating a constant-length vector at a point is governed by the dot product of two vectors [3]:

$$\frac{d\theta}{dt} = \frac{1}{2}\left(\vec{\omega}\cdot\vec{s}\right) \tag{1}$$

where θ is the rotation angle and

$$\vec{\omega} = curl\left(\vec{u}\right),\ \vec{s} = \frac{\vec{u}}{\left\|\vec{u}\right\|}.$$

Since

$$curl\left(\vec{u}\right) = \left\langle \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right\rangle$$

for $\vec{u} = \langle u, v, w \rangle$ in Cartesian coordinates, we evaluate the above curl using the values of velocity fields at two neighboring nodes on the streamline and the difference method for partial derivatives.

We can evaluate θ by taking the previous time step T_0 to the current time step T after approximating $curl\left(\vec{u}\right)$ described above.

$$\begin{aligned}\int_{T_0}^{T}\frac{d\theta}{dt} &= \frac{1}{2}\vec{\omega}\cdot\int_{T_0}^{T}\frac{\vec{u}}{\left\|\vec{u}\right\|}dt \\ &\approx \frac{1}{2}\vec{\omega}\cdot\frac{T-T_0}{2}\left(\frac{\vec{u}(T)}{\left\|\vec{u}(T)\right\|} + \frac{\vec{u}(T_0)}{\left\|\vec{u}(T_0)\right\|}\right) \\ &= \frac{T-T_0}{4}\vec{\omega}\cdot\left(\frac{\vec{u}(T)}{\left\|\vec{u}(T)\right\|} + \frac{\vec{u}(T_0)}{\left\|\vec{u}(T_0)\right\|}\right)\end{aligned} \tag{2}$$

where $\vec{u}(T_0)$ is the velocity field at previous time step and $\vec{u}(T)$ is the velocity field at current time step.

For steady flows, we can write Eq. (2) as

$$\theta(T) = \theta(T_0) + \frac{T-T_0}{4}\vec{\omega}\cdot\left(\frac{\vec{u}(\xi_T)}{\left\|\vec{u}(\xi_T)\right\|} + \frac{\vec{u}(\xi_{T_0})}{\left\|\vec{u}(\xi_{T_0})\right\|}\right)$$

or

$$\theta(h) = \theta(0) + \frac{h}{4}\vec{\omega}\cdot\left(\frac{\vec{u}(\xi_h)}{\left\|\vec{u}(\xi_h)\right\|} + \frac{\vec{u}(\xi_0)}{\left\|\vec{u}(\xi_0)\right\|}\right)$$

where ξ_0 represents the previous node and ξ_h represents the current node, and *h* is the time taken from ξ_0 to ξ_h.

The normal vector $\vec{n_1}$ at the current time step (i.e., at point P_1) is located on the plane which passes through the point P_1

and normal to the velocity field at the current time step (at point P_1). $\theta(h)$ can be interpreted as the angle between normal vectors $\vec{n_0}$ and $\vec{n_1}$ when both have initial points at P_1.

Let $\vec{n_1} = \langle x_1, y_1, z_1 \rangle, \left\|\vec{n_1}\right\| = a;$

$\vec{n_0} = \langle x_0, y_0, z_0 \rangle, \left\|\vec{n_0}\right\| = a,$ where a is the constant length for all normal vectors on the streamribbon. Now we find $\vec{n_1}$ for the current time step when $\vec{n_0}$ is known as the normal at the previous time step. Since $\vec{n_0} \cdot \vec{n_1} = \left\|\vec{n_0}\right\|\left\|\vec{n_1}\right\| \cos\theta(h)$, we have

$$x_0x_1 + y_0y_1 + z_0z_1 = a^2 \cos\left\{\frac{h}{4}\vec{\omega}\cdot\left[\frac{\vec{u}(\xi_h)}{\left\|\vec{u}(\xi_h)\right\|}\right] + \frac{\vec{u}(\xi_0)}{\left\|\vec{u}(\xi_0)\right\|}\right\}$$

and

$$\vec{n_1}\cdot\vec{u}(P_1) = x_1u_1 + y_1u_2 + z_1u_3 = 0$$

Where $\vec{u}(P_1) = \langle u_1, u_2, u_3 \rangle$. Taking $z_1 = 1$ leads to a linear system

$$\begin{aligned} x_0x_1 + y_0y_1 &= a^2\cos\theta - z_0 \\ x_1u_1 + y_1u_2 &= -u_3 \end{aligned} \qquad (3)$$

Solving (3) for x_1 and y_1, and then let

$$\vec{n_1} = a\frac{\langle x_1, y_1, z_1 \rangle}{\left\|\langle x_1, y_1, z_1 \rangle\right\|}. \qquad (4)$$

IV. Streamribbon Construction

Theoretically a streamribbon can be created by drawing constant-length normal vectors at all corresponding points on streamlines if the normal vector at the starting point is given. The streamline constructed using the algorithm in Section 2 consists of a series points. The density of the points of a segment on a streamline is big when they are close to a singular point or asymptote line. The algorithm for generating streamribbon is as follows.

1. Draw the constant-length normal vector at the second point on the streamline after the constant-length normal vector at the first point is chosen using the angular calculated from Eq. (4)
2. Draw a quadrilateral patch using the four points (two on streamlines and two end points of the constant-length normal vectors at the two points on the streamline).
3. Take the last point on the streamline and the constant-length vector in Step 2 as the initial point and vector, and find the next point on the streamline and the corresponding normal vector using Eq. (4).
4. Repeat Steps 2 and 3 until finish.

V. Examples

The following two examples show the adaptively tracked streamline by the method in [4] and the streamribbon constructed based on the streamlines. The tracked streamlines in red in Fig. 1 and Fig. 5 are drawn for the threshold numbers T=15.

Example 1 Saddle-spiral flow

$$\mathbf{V} = \left(xz - y, yz + x, -z^2\right)$$

with seed point (-0.8, 0.8, 1).

Fig. 1 shows the streamribbon in three dimensions. Fig. 2 shows the projection of the streamribbon in Fig. 1 on yz-plane. It is shown that the normal vector at the seed point is perpendicular to yz-plane and then it rotates gradually up to almost perpendicular to xy-plane. Fig. 3 shows the projection of the streamribbon in Fig. 1 on xy-plane. This figure indicates that the variation of rotations are very small from the point of view on z- axis. Fig.4 shows the projection of the streamribbon in Fig. 1 on xz-plane.

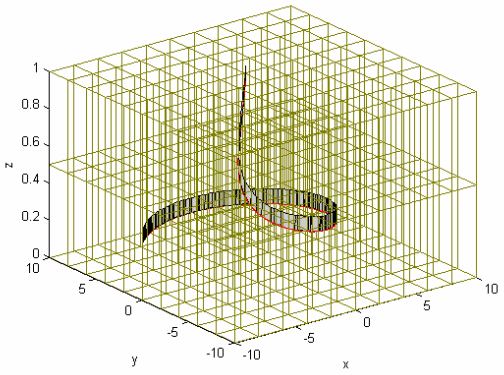

Fig. 1 Streamribbon for Example 1 in 3D

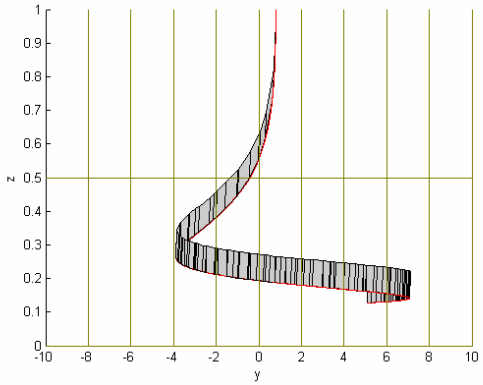

Fig. 2 Projection of the streamtube in Fig. 1 on yz-plane

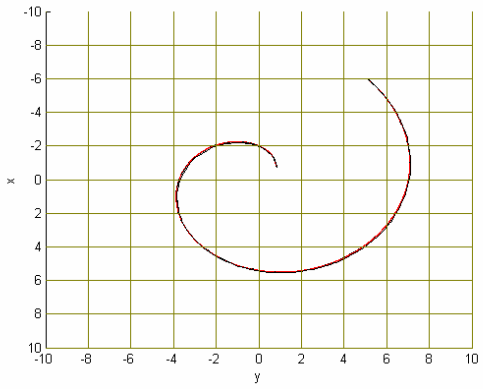

Fig. 3 Projection of the streamtube in Fig. 1 on xy-plane

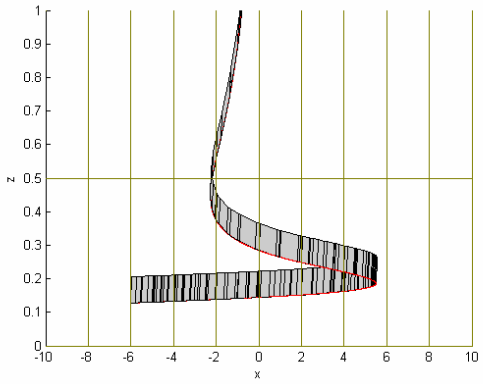

Fig. 4 Projection of the streamtube in Fig. 1 on xz-plane

Example 2 Toroidal flow velocity field

$$\mathbf{V} = \left(\frac{2x(z-1)}{r^2} - \frac{2y}{5r}, \frac{2y(z-1)}{r^2} + \frac{2x}{5r}, -\frac{2(r-9)}{r} \right)$$

with seed point (6.0006, 6.7076, 1), where $r = \sqrt{x^2 + y^2}$.

Fig. 5 shows the streamribbon in three-dimensions. The rotation for this example varies not significantly from Fig. 5. Fig. 6 shows the projections of Fig.5 on yz-plane. This figure shows the rotations clearly on yz-plane. Fig. 7 shows the projection of the streamribbon xy-plane and it again shows the rotation varies not significantly. Fig. 8 shows the projection of the streamribbon xz-plane in which we can see clearly the density of normal vectors.

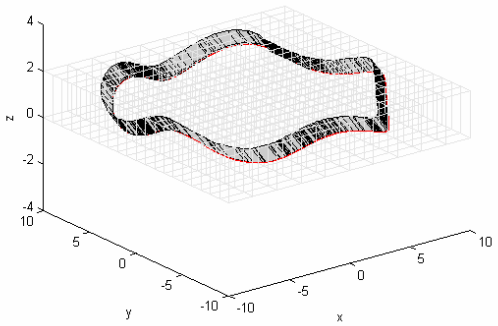

Fig. 5 Streamtube for Example 2 in 3D

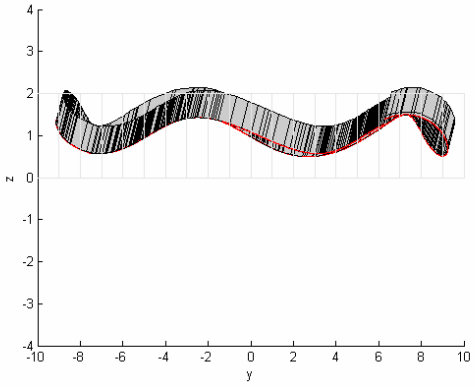

Fig. 6 Projection of the streamtube in Fig. 5 on yz-plane

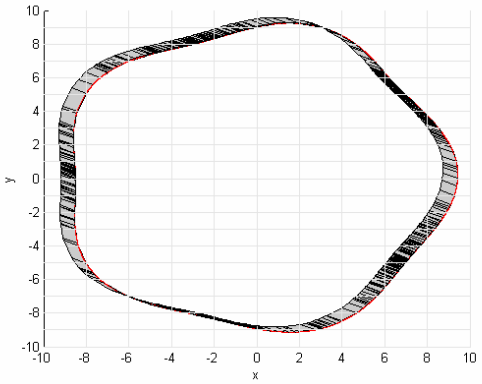

Fig. 8 Projection of the streamtube in Fig. 5 on xy-plane

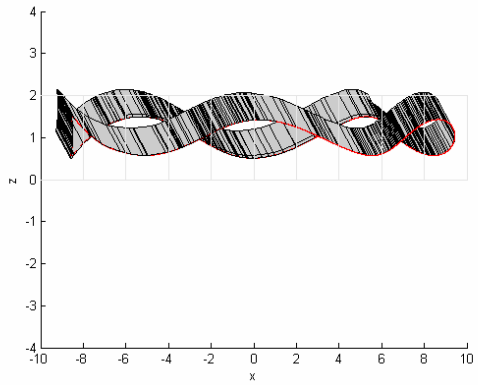

Fig. 9 Projection of the streamtube in Fig. 6 on xz-plane

VI. Conclusion

This paper has introduced a streamribbon construction method based on the streamlines tracked by the mass conservative, adaptive streamline tracking method in [4]. As shown in [4], the streamlines are very accurate. We can conclude that the streamribbons constructed in this paper are also accurate.

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