The Effect of Maximum Strain on Fatigue Life Prediction for Natural Rubber Material

Chang S. Woo, Hyun S. Park, and Wan D. Kim

Abstract—Fatigue life prediction and evaluation are the key technologies to assure the safety and reliability of automotive rubber components. The objective of this study is to develop the fatigue analysis process for vulcanized rubber components, which is applicable to predict fatigue life at initial product design step. Fatigue life prediction methodology of vulcanized natural rubber was proposed by incorporating the finite element analysis and fatigue damage parameter of maximum strain appearing at the critical location determined from fatigue test. In order to develop an appropriate fatigue damage parameter of the rubber material, a series of displacement controlled fatigue test was conducted using three-dimensional dumbbell specimen with different levels of mean displacement. It was shown that the maximum strain was a proper damage parameter, taking the mean displacement effects into account. Nonlinear finite element analyses of three-dimensional dumbbell specimens were performed based on a hyper-elastic material model determined from the uni-axial tension, equi-biaxial tension and planar test. Fatigue analysis procedure employed in this study could be used approximately for the fatigue design.

Keywords—Rubber, Material test, Finite element analysis, Strain, Fatigue test, Fatigue life prediction.

I. INTRODUCTION

THE fatigue life prediction on the rubber components was increasing according to the extension of warranty period of the automotive components. A design of rubber components against fatigue failure is one of the critical issues to prevent the failures during the operation. Therefore, fatigue life prediction and evaluation are the key technologies to assure the safety and reliability of mechanical rubber components [1, 2, 3, 4].

Fatigue life evaluation of rubber components has hitherto relied mainly on a real load test, road simulator test or bench fatigue test. Although above methods have advantages in accuracy of fatigue life, but cannot be used before the first prototype is made and the fatigue test should be always conducted whenever material or geometry changes are made

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[5]. In order to predict the fatigue life of the rubber components at the design stage, a simple procedure of life prediction is suggested in Fig. 1. We used the hourglass shape specimen to evaluate the fatigue life of rubber materials [6]. Fatigue test of natural rubber materials with different hardness and various amplitude conditions were performed [7].

A procedure to predict the fatigue life of rubber material based on the maximum strain method was proposed.
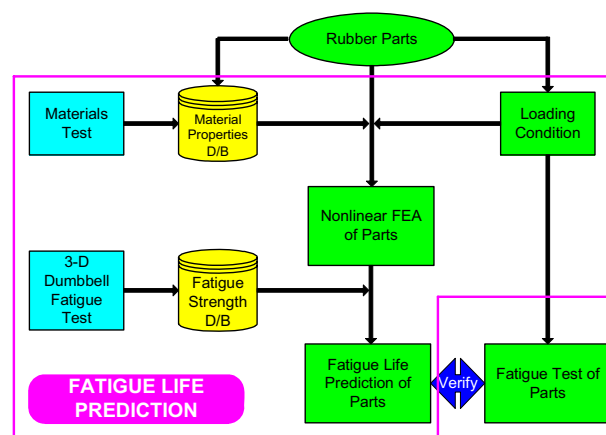

Fig. 1 Procedure to fatigue lifetime prediction system

II. EXPERIMENT

A. Rubber Material

The material used in this study is a carbon-filled vulcanized natural rubber, which have the hardness of the International Rubber Hardness Degree 50, 55, 60, 65(NR50, NR55, NR60, NR65). Compound recipes, including applied cure conditions, are summarized in Table I.

TABLE I
RECIPES FOR NATURAL RUBBER COMPOUNDS

Ingredient	NR50	NR55	NR60	NR65
SMRCV 60	100	100	100	100
C/B FEF	22	27	40	40
C/B SRF	15	18	20	32
S/A	1	1	1	1
ZnO	5	5	5	5

B. Material Test

The rubber material property, which is essential in finite element analysis, is expressed with the coefficient values of strain energy function and these values are determined by fitting stress-strain data obtained from the material tests under various load conditions into the stress-strain curve induced from strain energy function. And it is determined to minimize the differences between the test values and calculated values. Therefore, we analyzed the property of the material and determined the nonlinear material coefficient, which is necessary in finite element analysis, by conducting uniaxial, equi-biaxial tension and pure shear test [8].

Fig. 2(a) shows the uni-axial tension test by using non contacting strain measurement (laser extensometer). Rubber materials are deformed, their network structure lose their stiffness due to modification and reformation, and damping properties change. Mullins suggested this is due to the stress-strain response, called the Mullins effect [9]. This is more prevalent in carbon black-filled rubbers. In other words, the stiffness of rubber depends on its history and strain range. In addition, the stress-strain curve in initial stage is not repeated anywhere and the curve stabilize after receiving approximately five repetitive loads within the same strain range. The stress-strain curve exhibits yet another change when the rubber material is subjected to a larger strain than the previous one. Lastly, the rubber material possesses properties in which fixed permanent deformation occurs when the material returns to the initial strain value; the strain is not equal to 0 even if the stress is equal 0. Fig. 2(b) shows the Mullins effect in uni-axial tension test.
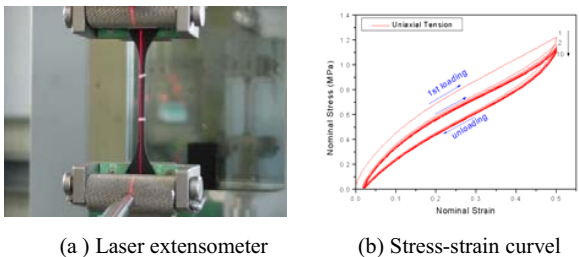
(a) Laser extensometer   (b) Stress-strain curvel

Fig. 2 Uni-axial tension test and Mullins effect

In uni-axial compression test, it is very difficult to obtain the pure compressed stress-strain relationship because of the frictions on the grip and the contact plane of rubber test specimen. Also, there is some bubbling phenomenon in the middle part of test sample due to this friction. Therefore, it is hard to say that the property values of materials obtained from uni-axial compression test are accurate. For equi-biaxial tension tests, we prepared round shaped test specimen shown in Fig. 3, with 16 grips placed on the outer edges of test specimen in order to apply evenly distributed loads in the direction of the circumference. Finite element analysis of the test specimen also proved that stress and strain distribution in the specimen. However, there are stress concentrations at the clamp edges of

the specimen where failure will initiate. FEA of the specimen is required to determine the appropriate geometry of the clamping point in Fig. 3(a). The equi-biaxial strain state may be achieved by radial stretching a circular disc in Fig. 3(b). Once again, a non-contacting strain measuring device must be used such that strain is measured away from the clamp edges in Fig. 3(c)
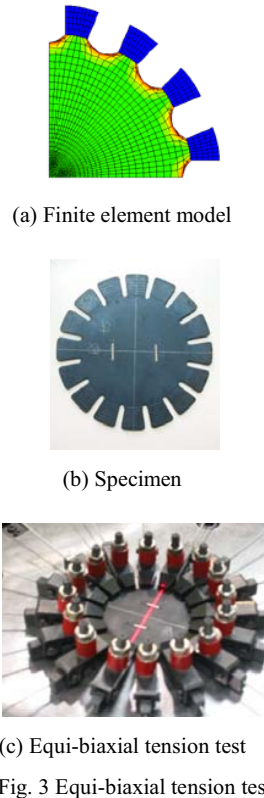

(a) Finite element model

(b) Specimen

(c) Equi-biaxial tension test

Fig. 3 Equi-biaxial tension test

A shear strain state is a more important mode of deformation for engineering applications than tension. There are pure shear and simple shear in shear deformation mode. The quad lap simple shear test piece is standardized. But, pure shear test is not yet standardized. There are two difficulties in the simple shear test. The first difficulty is making the specimen. This may require either bonded to the rigid supports during vulcanization or molded blocks are adhered with a high modulus adhesive. Secondly, the low shear strain range is limited because the rigid plates are bent on straining. Alternatively, pure shear test can be developed high strain range than simple shear test. If the material is incompressible and the width of the specimen is longer than the height, a pure shear state exists in the specimen at 45 degrees angle to the stretching direction. Aspect ratio of the specimen is most significant in pure shear test because the specimen is perfectly constrained in the horizontal direction.

Fig. 4(a) shows the deformed shapes by the finite element analysis at 100 percent stretching for the aspect ratio of 10:1. Stress-strain curves obtained from the tests are shown Fig. 4(b), compared to those predicted by the finite element analysis. Even though there exist some differences in the stress-strain

responses between the experiment and the analysis, fairly good correlations are observed. A better agreement can be seen for the aspect ratio of 10:1, compared to 5:1. The differences are attributed to the specimen slippage from the clamp edges, leading to the inadequate states of pure shear strain. Therefore, it is necessary to design a gripping device to prevent specimen slippage, in order to improve the test accuracy.
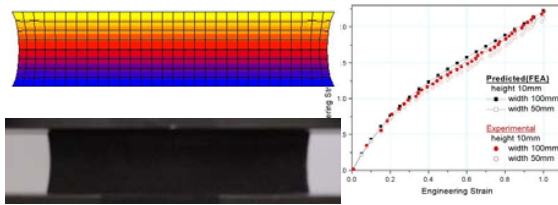

(a) Deformed shape of specimen   *(b) Stress-strain curve*

Fig. 4 Pure shear test

Fig. 5 shows the stress-strain curves obtained from the uni-axial tension, equi-biaxial tension and pure shear tests in which we applied five repetitive loads in each of the vertical and horizontal direction with 25%, 50% and 100% of the strain range for natural rubber. According to Fig. 5, the stress-strain curves during the second repetition showed a greater decline than in the first repetition. The stress-strain curve gradually decreased as the number of repetitions increased, and ultimately stabilized to a fixed stress-strain value. In order to predict the behavior of the rubber components using the finite element analysis, the rubber material constants must be determined from the stabilized cyclic stress-strain curve. The stress-strain curve varies significantly depending on the cyclic strain levels. A 5th loading cycle was selected as the stabilized stress-strain relationship in this study. But this stabilized relation should be shifted to pass through the origin of the curve, to satisfy the hyper-elastic nature of rubber. Fig. 6 shows the stress-strain relation of rubber material for various physical tests. The shift of curve meant that the gage length and initial cross sectional area were changed as shown in Eq. (1).

$$\varepsilon = \frac{\varepsilon' - \varepsilon_p}{1 + \varepsilon_p} \qquad \sigma = \sigma'(1 + \varepsilon_p) \qquad (1)$$

We performed the curve fitting with uni-axial tension, equi-biaxial tension and pure shear test data. Ogden 3-terms fits that uses progressively more information as the basis for the curve fitting. Table II contains the values of rubber material coefficient calculated in each case.
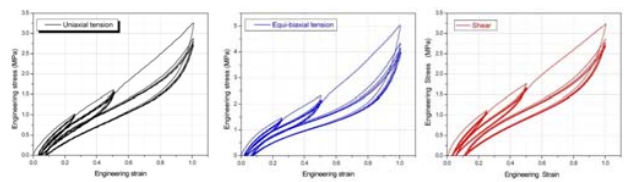
(a) Uni-axial   (b) Equi-biaxial   (c) Pure shear

Fig. 5 Stress-strain curve at various test type
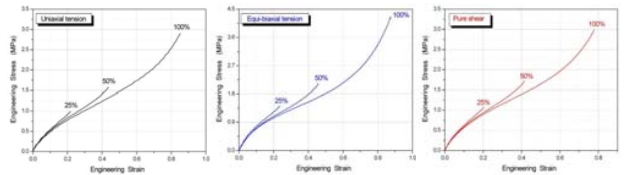
(a) Uni-axial   (b) Equi-biaxial   (c) Pure shear

Fig. 6 Stress-strain curve at shift curve to zero

TABLE II
OGDEN FUNCTION OF RUBBER MATERIAL

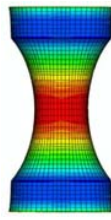| strain	Ogden 3-terms					
	μ_1	α_1	μ_2	α_2	μ_3	α_3
25%	1.1e-4	2.29	0.96	3.71	4.5e-4	3.08
50%	2.0e-5	1.03	2.14	1.3e-6	1.182	2.60
100%	0.028	4.39	5.81	0.03	0.953	2.59

C. Fatigue test

The fatigue test piece has the basic shape of three-dimensional dumbbell specimen with a metal fitting cure bonded to each end. The geometry of the central part of the cylinder was designed to meet the following criteria in relation to fatigue test data for rubber components and strain distribution profile. The test piece should be capable of compression and tensile deformation without developing slackness under cyclic deformation. It should have a smooth strain distribution and the position at which maximum tensile strain develops should be the same for any deformation. Three-dimensional dumbbell specimen has an elliptical cross-section and parting lines are located on the minor axis of specimen to avoid undesirable failure at the surface discontinuities. The basic geometry of the test piece for materials fatigue testing is shown in Fig. 7(a). The finite element analysis is executed to obtain the relationship between the displacement and strain [10,11,12]. It was assumed that the ends of the specimen were constrained by bonding to the fitting and the rubber part was deformed in the longitudinal direction. Finite element model of specimen is shown in Fig. 7(b). The maximum strain was found in the surface at the major axis of the rubber part of specimen as shown in Fig. 7(c). To avoid the mold parting line is acted as a source of crack initiation, the parting line should placed on the minor axis side. Fig. 7(d) shows the strain distribution according to finite element analysis from compression and tension displacement. The maximum Green- Lagrange strain was found to develop at a constant

position in the surface at the centre of the rubber part of the test piece in both compression and tension.
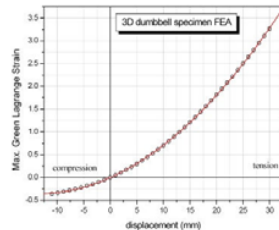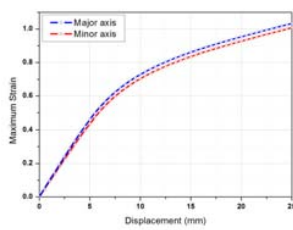

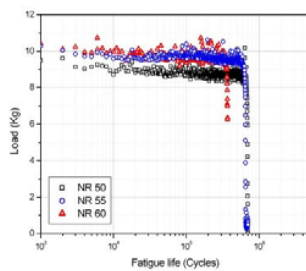(a) Hourglass shape   (b) Finite element model

(c) Strain distribution of specimen (d) Displacement and strain curve

Fig. 7 Three-dimensional dumbbell specimen for fatigue test

Fatigue tests were conducted in an ambient temperature under the stroke-controlled condition with a sine waveform of 5 Hz and the mean displacement is 0, 3, 5, 8, 10mm at the displacement range is -11~21mm. The fatigue failure was defined as a number of cycles at which the maximum load dropped by 20 percent. As increasing the cycles in initial phase, the maximum load decreased little by little. When the crack grew over the critical size, the maximum load decreased suddenly and the final failure reached. Fig. 8 shows the fatigue testing system and fatigue life curve.

(a) Fatigue test equipment   (b) Load and fatigue life

Fig. 8 Fatigue test of three-dimensional dumbbell specimen

III. RESULT AND DISCUSSION

A. Fatigue Test Result

Fig. 9 shows the relationship between displacement and fatigue life for the three-dimensional dumbbell specimens with different hardness under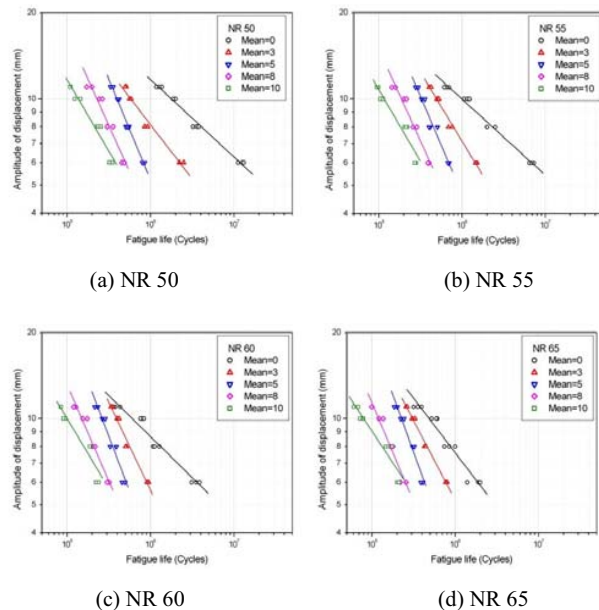 various mean displacements. The fatigue life at the same displacement amplitude decrease as the mean displacement and hardness increases.

(a) NR 50   (b) NR 55

(c) NR 60   (d) NR 65

Fig. 9 Fatigue life of three-dimensional dumbbell specimen

B. Fatigue Damage Parameter

Fatigue process begins with the accumulation of damage at a localized region due to alternating load and displacement, leading to crack nucleation, growth, and final fracture [13, 14]. The crack nucleation life of the component may be defined as the number of cycles required for the appearance of a macro-crack. Therefore, the crack nucleation life of the component can be related to the life of a smooth specimen that is cycled to the same stresses or strains as the material at the critical region of the component.

In this study, the fatigue damage of the natural rubber was evaluated from smooth dumbbell specimens. Fig. 10(a) shows the relationship between the maximum tension displacement and fatigue life of three-dimensional dumbbell specimen. The fatigue life decreased as the maximum tension displacement increases. It is possible to express the fatigue life with the maximum tension displacement fairly good.

In large deformation analysis, the most finite element analysis codes use a strain measure as the Green-Lagrange strain instead of the nominal strain [15]. Fig. 10(b) shows the true strain and the Green-Lagrange strain at the position of major axis from finite element analysis of the three-dimensional dumbbell specimen. The Green- Lagrange strain is higher than the true strain. The maximum displacement in Fig. 10(a) is converted to the maximum true and maximum Green-Lagrange strain in Fig. 10 (c) and (d).

(a) Displacement and fatigue life

(b) Analysis of strain by FEA

(c) Max. true strain and fatigue life   (d) Max.G-L strain and fatigue life
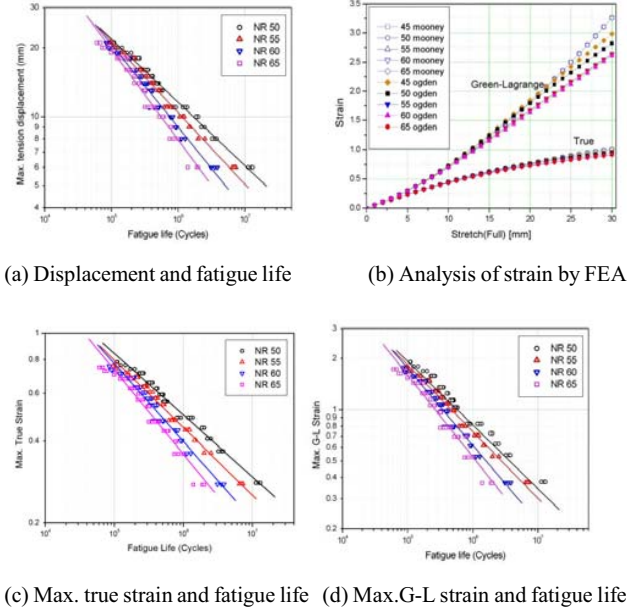
Fig. 10 Fatigue life of natural rubber

C. Fatigue Lifetime Prediction

Fatigue life of the three-dimensional dumbbell specimen represented by the maximum strain parameter is shown in Fig. 10, where the true and Green-Lagrange strain for each dumbbell specimen is calculated from the displacement versus strain curve in Fig. 10(b). It can be seen from Fig. 10(c) and (d) that the fatigue lives with different hardness can be effectively represented by a following function using the maximum true and Green-Lagrange strain, thus taking into account the mean displacement and amplitude.

The true strain and the Green-Lagrange strain are defined as;

$$\varepsilon_{true} = \ln(\lambda)$$

$$\varepsilon_{G-L} = \frac{1}{2}(\lambda^2 - 1) \tag{2}$$

ε_{true} : true strain, ε_{G-L} : Green-Lagrange strain,

λ : strech ration

By using the fatigue test and finite element analysis, the normalized maximum strain defined as dividing by elongation at break (ε_{EB}) for the maximum true strain (ε_{true}) and Green-Lagrange strain (ε_{G-L}) for each hardness.

Fig. 11 and 12 show relation of normalized maximum strain and fatigue life. It was observed that the maximum strain was a good parameter to account for the hardness, mean displacement, amplitude effects. Fatigue life of natural rubber represented by the maximum strain and elongation at break are shown in (3). The fatigue lives effectively represented by a single function using the maximum true and Green-Lagrange strain and elongation at break for each natural rubber materials.

$$N_f = 4,466 \cdot [\varepsilon_{true} / \varepsilon_{EB(true)}]^{-3.85}$$

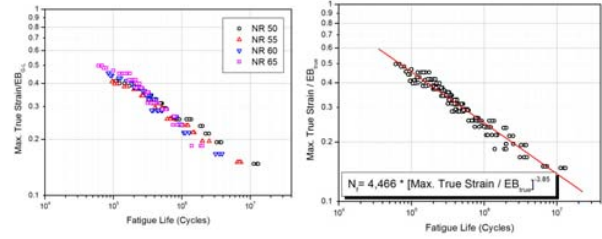$$N_f = 1,096 \cdot [\varepsilon_{G-L} / \varepsilon_{EB(G-L)}]^{-2.22} \tag{3}$$

(a) Max. true strain and fatigue life   (b) Fatigue life prediction curve

Fig. 11 Fatigue life prediction on base maximum true strain

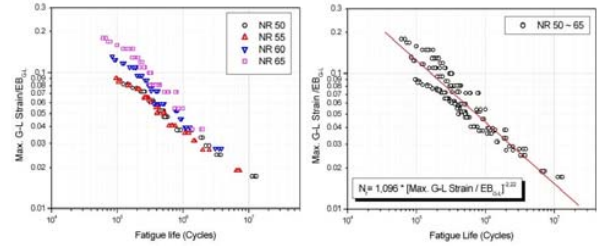(a) Max. G-L strain and fatigue life   (b) Fatigue life prediction curve

Fig. 12 Fatigue life prediction on base maximum G-L strain

Fig. 13 compares the predicted and experimental fatigue lives for the variable amplitude signals for natural rubber in terms of sequences to failure along with reference lines indicating factors of 2 scatter in fatigue life.
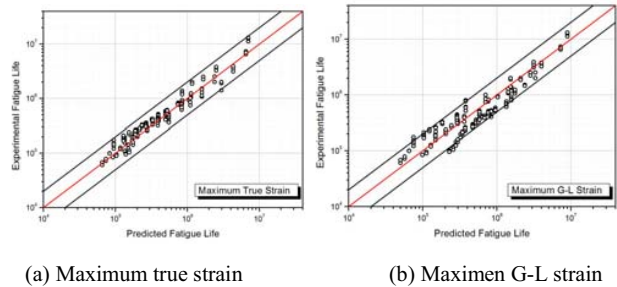

(a) Maximum true strain   (b) Maximen G-L strain

Fig. 13 Comparison of predicted and experimental fatigue life

IV. CONCLUSION

In this paper, to develop the fatigue analysis process for vulcanized rubber materials, which is applicable to predict fatigue life at initial product design step, methodology to extract the material properties for finite element analysis input data from limited minimum test results was proposed. Also, in order to investigate the applicability of commonly used fatigue damage parameters, fatigue tests and corresponding finite element analyses were carried out and optimum fatigue damage parameter was selected. Fatigue life prediction methodology of the rubber components was proposed by incorporating the finite element analysis and fatigue damage parameter.

The fatigue life of rubber material was effectively represented by the maximum strain. Predicted fatigue lives of the rubber material were in fairly good agreements with the experimental lives. Therefore, fatigue life estimation procedure employed in this study could be used approximately for the fatigue design of rubber component at the early design stage.

REFERENCES

[1] Lake, "Aspect of Fatigue and Fracture of Rubber", *Progress of Rubber technology*, Vol. 45, 1983, pp.89-143

[2] D. Klenke, A. Beste, "Ensurance of the fatigue Life of Metal -Rubber Components*", Gummi Kunsftoffe*, Vol. 40, 1987, pp. 1067- 1071

[3] S. Thong-om, W. Payakcho, J. Grasaesom and B. Marungsri, "Study of Ageing Deterioration of Silicone Rubber Housing Material for Outdoor Polymer Insulators", *World Academy of Science, Engineering and Technology,* Issue 80, August 2011, pp. 533-539.

[4] Ekis N. Herliyana, Kunio Tsunoda, Yusuf S. Hadi, Arinana, and Dewi A. Natalia, "Pleurotus Ostreatus for Furability Test of Rubber and Sengon Woods using Indonesian National Standard and Japanese Standard Methods", *World Academy of Science, Engineering and Technology,* Issue 74, 2013, pp. 52-56.

[5] Thomas, "Factors Influencing the Strength of Rubbers", *Rubber Chemistry and technology*, Vol. 48, 2001, pp. 902-912

[6] H. Yamaguchi, M. Nakagawa, " Fatigue Test Technique for Rubber materials of Vibration Insulator**", *International Polymer Science and Technology*, vol. 20, 1993, p 64-69

[7] S. Moon, C. Woo & W. Kim, "Study on the Determination of Fatigue Damage Parameter for Rubber Component under Multi-axial Loading", *Elastomers and Composites*, vol. 47, 2012, pp. 194-200.

[8] Day,J., Miller,K., "Equibiaxial Stretching of Elastometric Sheets, An Analytical Verification of an Experiment technique", ABAQUS Users Conference procedings, Vol. 35, No. 4, pp. 205-219, 2000.

[9] Mullins, L., "Softening of Rubber by Deformation", Rubber Chemistry and Technology, Vol. 42, pp. 339-362, 1969.

[10] S. Lee, S. Yeom, C. Han & C. Woo, "A Study on Finite Element Analysis and Aging Test for Automotive Grommet", Elastomers and Composites, vol. 47, 2012, pp. 201-209.

[11] S. Atieh, M. Kalantari, R. Ahmadi, J. Dargahi, M. Packirisamy, and M. H. Zadeh, "FEM Analysis of the Interaction between a Piezoresistive Tactile Sensor and Biological Tissues", *World Academy of Science, Engineering and Technology,* Issue 54, 2011, pp. 106-110.

[12] M. Foroutan, H. Dalayeli, and M. Sadeghian, "Simulation of Large Deformations of Rubbers by the RKPM Method", *World Academy of Science, Engineering and Technology,* Issue 2, 2007, pp. 178-182.

[13] H. Oh, "The fatigue life model of rubber bushing", *Rubber Chem. & Technology*, vol. 53, 1980, pp. 1226-1238.

[14] T. Alshuth, T. & Abraham F., Parameter Dependence and Prediction of Fatigue life of Elastomers products, *Rubber Chem. & Technology*, vol. 75, 2002, pp. 635-642.

[15] MARC Users manual, MSC software Inc., 2000.

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