

# Discovery of Fuzzy Censored Production Rules from Large Set of Discovered Fuzzy if then Rules

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**Abstract**—Censored Production Rule is an extension of standard production rule, which is concerned with problems of reasoning with incomplete information, subject to resource constraints and problem of reasoning efficiently with exceptions. A CPR has a form: IF A (Condition) THEN B (Action) UNLESS C (Sensor), Where C is the exception condition. Fuzzy CPR are obtained by augmenting ordinary fuzzy production rule “If X is A then Y is B with an exception condition and are written in the form “If X is A then Y is B Unless Z is C. Such rules are employed in situation in which the fuzzy conditional statement “If X is A then Y is B” holds frequently and the exception condition “Z is C” holds rarely. Thus “If X is A then Y is B” part of the fuzzy CPR express important information while the unless part acts only as a switch that changes the polarity of “Y is B” to “Y is not B” when the assertion “Z is C” holds. The proposed approach is an attempt to discover fuzzy censored production rules from set of discovered fuzzy if then rules in the form:

$A(X) \Rightarrow B(Y) \parallel C(Z)$ .

**Keywords**—Uncertainty Quantification, Fuzzy if then rules, Fuzzy Censored Production Rules, Learning algorithm.

## I. INTRODUCTION

KNOWLEDGE acquisition is one of the main problems in developing knowledgebase system. Inductive learning technique tries to get the knowledge of a system from a set of examples. In this paper we are interested in discovering knowledge with quantification of uncertainty. We are using a learning algorithm [1] that will discover set of fuzzy if then rules. Our aim in this paper is that we want to capture exception in the data and we also want to quantify uncertainty through fuzzy logic. For measuring exception we are using Censored Production Rule which is an extension of standard production rule, which is concerned with problems of reasoning with incomplete information, subject to resource constraints and problem of reasoning efficiently with exceptions [2]. A CPR has a form: IF A (Condition) THEN B (Action) UNLESS C (Sensor), Where C is the exception condition. Such rules are employed in situations in which the

conditional statement IF A THEN B holds frequently and the assertion C holds rarely. By using a rule of this type we are free to ignore the censor (exception) condition. As time permits, the censor condition C is evaluated establishing the conclusion B with higher certainty if C does not hold otherwise if C holds then the conclusion is  $\sim B$ . For quantification of uncertainty of data we are using concept of Fuzzy CPR which is written in the form “If X is A then Y is B Unless Z is C. Such rules are employed in situation in which the fuzzy conditional statement “If X is A then Y is B” holds frequently and the exception condition “Z is C” holds rarely [3] [4]. The proposed work is related to discovery of fuzzy CPR from set of discovered fuzzy if then rules.

## II. UNCERTAINTY QUANTIFICATION WITH FUZZY LOGIC

Uncertainty pertains to information that is not definitely fixed, not precisely determined, not dependable or that is vague or indistinct [10]. Uncertainty must be quantified in order to use it systematically in decision-making processes. Uncertainty Quantification is the quantitative characterization and use of uncertainty in information applications. Helton, 1994; Kaplan and Garrick, 1981, understands uncertainty should be divided into two categories for purpose of quantification:

(1) Variability: Which can be quantified in principle using classical probability theory. It is also known as Aleatory Uncertainty.

(2) Lack of knowledge: Which requires more than classical probability theory for its quantification. It is also known as Epistemic Uncertainty.

There are three current options for accomplishing the task of Uncertainty quantification:

- Second Order probability
- Bayesian methods
- Generalized probability theories

Propagation of Generalized probability is currently a research problem for complex technical decision problems. Following methods can be used for the quantification of uncertainty.

- Certainty Factor
- Dempster-Shafer Theory
- Fuzzy Logic

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### Fuzzy Logic

Fuzzy logic is well defined reasoning system that is based on the use of fuzzy sets rather than on the binary values. A fuzzy set is a class of elements with loosely defined boundaries. Formally, a fuzzy set  $F$  in a universe of discourse  $U$  is defined by a membership function,  $\mu_f : U \rightarrow [0, 1]$ . The membership function expresses the degree of membership of elements in the fuzzy subset. For example  $\mu = 0$  indicates no membership,  $\mu = 1$  indicates full membership and  $0 < \mu < 1$  indicates partial membership of the fuzzy set.

fuzzy IF...THEN rule may be expressed as:  $Y$  is  $B$  if  $X$  is  $A$ , in which the antecedent, consequent or both are fuzzy rather than crisp. Where  $X$  and  $Y$  are variables whose domains are  $U$  and  $V$  respectively,  $A$  and  $B$  are fuzzy predicates or relations in  $U$  and  $V$ , which play the role of elastic constraints on  $X$  and  $Y$ . Thus IF...THEN rules in a fuzzy system are nothing but relations between fuzzy sets.

Example: Volume is low if pressure is high. In this case  $Y$  = volume,  $X$  = pressure,  
 $B$  = low and  $A$  = high.

According to Driankov and Hellendoorn, a fuzzy conditional statement "IF  $X$  is  $A$  THEN  $Y$  is  $B$ " (of  $A(x) \Rightarrow B(y)$  for short) where "IF  $X$  is  $A$ " is represented by the membership function  $\mu_A(x) : \text{domain}(X) \rightarrow [0, 1]$  which restricts the possible values of the variable  $X$  to a fuzzy set of values having (each to a certain degree) the property  $A$ . In a similar way "Y is  $B$ " is represented by the membership function  $\mu_B(y) : \text{domain}(Y) \rightarrow [0, 1]$ . Then the meaning of the rule  $A(x) \Rightarrow B(y)$  is represented by a fuzzy relation defined on  $\text{domain}(X) \times \text{domain}(Y)$ , i.e.  $\mu_R(x,y) : \text{domain}(X) \times \text{domain}(Y) \rightarrow [0, 1]$

Here the Godels type of implication is chosen:

$$\mu_R(x,y) = 1, \quad \text{if } \mu_A(x) \leq \mu_B(y) \\ = \mu_B(y), \text{ otherwise} \quad (1)$$

then the condition  $B(y)$  is obtained via the operation "composition" between  $\mu_R(x,y)$  and  $\mu_A(x)$  i.e.  $\mu_B(y) = \mu_A(x) \circ \mu_R(x,y)$

$$= \max \{ \min(\mu_A(x), \mu_R(x,y)) \} \quad (2)$$

### III. CENSORED PRODUCTION RULES

As an extension of standard production rule, Michalski & Winston [1986] proposed variable precision logic (VPL), which is concerned with problems of reasoning with incomplete information, subject to resource constraints and problem of reasoning efficiently with exceptions. VLP offers mechanism for handling trade-off between the precision of inferences and computational efficiency of deriving them. Specificity and certainty are the two aspects of precision. Certainty refers to the degree of belief in a statement, whereas specificity refers to the degree of detail of a description. According to Michalski & Winston, a system that gives more specific answers given more time (or resources in general) is called a "Variable Specificity System". A system that gives more certain answers given more time is called a "Variable Certainty System". There can be various combinations of the two systems, reflecting that specificity and certainty are

inversely related; we can gain specificity at the expense of certainty or vice versa.

Michalski & Winston have suggested the Censored Production Rules (CPR) as an underlying representational and computational mechanism to enable logic based systems to exhibit variable precision in which certainty varies while specificity stays constant.

A CPR has a form: IF  $A$  (Condition) THEN  $B$  (Action) UNLESS  $C$  (Censor)

Where  $C$  is the exception condition. Such rules are employed in situations in which the conditional statement IF  $A$  THEN  $B$  holds frequently and the assertion  $C$  holds rarely. By using a rule of this type we are free to ignore the censor (exception) condition when the resources needed to establish its presence are tight or simply no information is available as to whether it holds or does not hold. As time permits, the censor condition  $C$  is evaluated establishing the conclusion  $B$  with higher certainty if  $C$  does not hold otherwise if  $C$  holds then the conclusion is  $\sim B$ . Example: An example of censored production rule is

IF Sunday THEN Toni works in the Yard UNLESS Weather is bad. This rule has an interpretation that if it is Sunday and the weather is good then Toni will work in the Yard. But if it is Sunday and weather is bad then Toni will not work in the Yard.

A CPR may have more than one exception denoting censor. It can be described by following example:

We consider the assertion that birds fly

$\forall x$  is-bird  $(x) \Rightarrow$  flies  $(x)$

This general assertion enables us to express that any newly observed bird flies. But not all birds fly (for example penguins, ostriches, emus, kiwis and domestic turkeys do not fly). To include this information we write censored production rule in the following manner:

$\forall x$  is-bird  $(x) \Rightarrow$  flies  $(x) \perp$

is- penguin(x)  
 $\vee$  is-ostrich(x)  
 $\vee$  is-emu (x)  
 $\vee$  is-kiwi (x)  
 $\vee$  is-domestic-  
 $\vee$  is-dead (x)

Thus the exceptions are disjunctively linked together as one censor condition. Where  $\perp$  notation is used for UNLESS operator and  $\vee$  notation is used for OR operator.

### IV. FUZZY IF-THEN RULES

It associates a condition described using linguistic variables and fuzzy sets to a conclusion. There are two types of fuzzy if then rules.

1. Fuzzy Mapping Rules
2. fuzzy implication Rules

Structure of fuzzy Rules

If <antecedent> Then <Consequent>.

**Antecedent part:** It is similar to general production rules but difference is that it describe elastic condition (a conclusion that can be satisfied to a degree) while in traditional production rule this part describe rigid condition (the condition that is either satisfied or dissatisfied).

Example: IF the annual income of a person is High then Person is rich.

**Consequent part:** It can be classified into three categories

1. Crisp Consequent  
IF..... THEN  $Y = a$ , where  $a$  is a non fuzzy numeric value or symbolic value
2. Fuzzy Consequent  
IF..... THEN  $Y = A$ , where  $A$  is Fuzzy set.
3. Functional Consequent  
IF  $x_1$  is  $A_1$  AND  $x_2$  is  $A_2$  AND .....  $x_n$  is  $A_n$  THEN  
 $Y = a_0 + \sum a_i \times x_i$   
where  $a_0, a_1, a_2, \dots, a_n$  are constants.

#### V. FUZZY CENSORED PRODUCTION RULES

Fuzzy CPR's are obtained by augmenting ordinary fuzzy production rule "If X is A then Y is B with an exception condition and are written in the form "If X is A then Y is B Unless Z is C. Such rules are employed in situation in which the fuzzy conditional statement "If X is A then Y is B" holds frequently and the exception condition "Z is C" holds rarely. Thus "If X is A then Y is B" part of the fuzzy CPR express important information while the unless part acts only as a switch that changes the polarity of "Y is B" to "Y is not B" when the assertion "Z is C" holds [20]. The fuzzy CPR can be presented as:  $A(X) \Rightarrow B(Y) \parallel C(Z)$ . Here A, B, C are normal fuzzy sets.

#### VI. PROPOSED APPROACH

This section describes a new method to find fuzzy CPR from discovered fuzzy if then rules [22]. Following terms are used:

1. Support of the Rule: Support is a measure related to the relative frequency of the instances covered by a rule.
2. Confidence: It is related to accuracy of the rule.
3. Commonsense Rule: It represents a common phenomenon that comes with high support and high confidence i.e. strong pattern.
4. Reference Rule: It should have low support and low confidence.
5. Exception Rule: It is a pair of Commonsense Rule and Reference Rule. It represents low support but high confidence (similar to the commonsense rule).
6. Rule structure: Rule structure of exceptions can be expressed as [6]:
  - a)  $A \rightarrow X$  General Rule (common sense rule)
  - b)  $B \rightarrow \sim X$  Reference Rule
  - c)  $A, B \rightarrow \sim X$  Exception Rule
 Rule c) can also be expressed as  
 $A \rightarrow X \sqcup B$  where  $\sqcup$  is unless operator.

All things considered our proposed approach is divided in the following three steps:

**Step1:** Mining Fuzzy If Then Rules: In this step we use learning algorithm proposed by A. Gonzalez and R. Perez [2]. The out output of this step will be set of fuzzy if then rules.

**Step2:** Search a pair of two rules

$$A(X) \Rightarrow B(Y) \text{ and } \dots(i)$$

$$C(Z) \Rightarrow \sim B(Y) \dots(ii)$$

**Step3:** looking for exception rule: In this step we use general rule structure described in term no. 6. Fuzzy Exception rule can be formed as :

$$A(X) \wedge C(Z) \Rightarrow \sim B(Y) \dots(iii)$$

Where A, B,  $\sim B$  and C are normal fuzzy sets.

**Step4:** Forming fuzzy CPR: Exception rule can be described by using unless operator. Then the decision will be positive. So rule (iii) can be described as:  $A(X) \Rightarrow B(Y) \parallel C(Z)$  .....(iv)

Rule (iv) is fuzzy CPR where A, B, and C are normal fuzzy sets.

#### VII. EXPERIMENT

Demonstration of proposed approach:

**Step1:** Mining fuzzy If Then rules by using learning algorithm [2]. The output will be:

$$\begin{aligned} A_1(X) &\Rightarrow B_1(Y) \\ C_2(Z) &\Rightarrow \sim B_2(Y) \\ A_2(X) &\Rightarrow B_2(Y) \\ C_1(Z) &\Rightarrow \sim B_1(Y) \\ A_3(X) &\Rightarrow B_3(Y) \\ C_3(Z) &\Rightarrow \sim B_3(Y) \\ A_4(X) &\Rightarrow B_4(Y) \\ C_5(Z) &\Rightarrow \sim B_5(Y) \\ A_5(X) &\Rightarrow B_5(Y) \\ C_4(Z) &\Rightarrow \sim B_4(Y) \\ A_6(X) &\Rightarrow B_6(Y) \\ C_7(Z) &\Rightarrow \sim B_7(Y) \\ A_7(X) &\Rightarrow B_7(Y) \\ C_6(Z) &\Rightarrow \sim B_6(Y) \\ A_8(X) &\Rightarrow B_8(Y) \\ C_8(Z) &\Rightarrow \sim B_8(Y) \\ A_9(X) &\Rightarrow B_9(Y) \\ C_{10}(Z) &\Rightarrow \sim B_{10}(Y) \\ A_{10}(X) &\Rightarrow B_{10}(Y) \\ C_9(Z) &\Rightarrow \sim B_9(Y) \end{aligned}$$

**Step2:** Searching a pair of two rules in the form:

$$A(X) \Rightarrow B(Y) \text{ and } \dots(i)$$

$$C(Z) \Rightarrow \sim B(Y) \dots(ii)$$

The output of this step will be:

$$\begin{aligned} A_1(X) &\Rightarrow B_1(Y) \\ C_1(Z) &\Rightarrow \sim B_1(Y) \\ \} \\ A_2(X) &\Rightarrow B_2(Y) \\ C_2(Z) &\Rightarrow \sim B_2(Y) \\ \} \end{aligned}$$

$$\begin{array}{l}
 A_3(X) \Rightarrow B_3(Y) \\
 C_3(Z) \Rightarrow \sim B_3(Y) \\
 \left. \begin{array}{l} \\ \end{array} \right\} \\
 A_4(X) \Rightarrow B_4(Y) \\
 C_4(Z) \Rightarrow \sim B_4(Y) \\
 \left. \begin{array}{l} \\ \end{array} \right\} \\
 \dots\dots\dots \\
 \dots\dots\dots \\
 A_{10}(X) \Rightarrow B_{10}(Y) \\
 C_{10}(Z) \Rightarrow \sim B_{10}(Y) \\
 \left. \begin{array}{l} \\ \end{array} \right\}
 \end{array}$$

**Step3:** Applying Rule structure on above rules and forming fuzzy exception rules:

$$\begin{array}{l}
 A_1(X) \wedge C_1(Z) \Rightarrow \sim B_1(Y) \\
 A_2(X) \wedge C_2(Z) \Rightarrow \sim B_2(Y) \\
 A_3(X) \wedge C_3(Z) \Rightarrow \sim B_3(Y) \\
 A_4(X) \wedge C_4(Z) \Rightarrow \sim B_4(Y) \\
 \dots\dots\dots \\
 \dots\dots\dots \\
 A_{10}(X) \wedge C_{10}(Z) \Rightarrow \sim B_{10}(Y)
 \end{array}$$

**Step4:** Generating Fuzzy CPR's

$$\begin{array}{l}
 A_1(X) \Rightarrow B_1(Y) \parallel C_1(Z) \\
 A_2(X) \Rightarrow B_2(Y) \parallel C_2(Z) \\
 A_3(X) \Rightarrow B_3(Y) \parallel C_3(Z) \\
 A_4(X) \Rightarrow B_4(Y) \parallel C_4(Z) \\
 \dots\dots\dots \\
 \dots\dots\dots \\
 A_{10}(X) \Rightarrow B_{10}(Y) \parallel C_{10}(Z)
 \end{array}$$

#### Example:

Using proposed approach following knowledgebase could be discovered:

1. Height-is-tall  $\Rightarrow$  X wears-large-shoes  $\parallel$  feet-are-small
2. X is-rich  $\Rightarrow$  X wears-expensive-leather-shoes  $\parallel$  shortage-of-money
3. X is-poor  $\Rightarrow$  X wears-inexpensive-leather-shoes  $\parallel$  gifted-shoes-are-expensive
4. X likes-synthetic-item  $\Rightarrow$  X wears-purely-synthetic-fiber-shoes  $\parallel$  X is-allergic-to-synthetic-items.

#### VIII. CONCLUSION

In this paper idea of discovering fuzzy censored production rules have been presented. We have proposed an approach for the discovery of quantified rules with exceptions in the form of censored production rules (CPR) from the large set of discovered if then rules. We have used fuzzy logic to capture the exception in the data and to produce the discovered rules in the form:

$A(X) \Rightarrow B(Y) \parallel C(Z)$ . The performance of the proposed Algorithm is demonstrated through examples. Discovered quantified CPRs would facilitate quantitative reasoning and learning. One of the most important extension of the present work is to develop schemes for discovering fuzzy Hierarchical Production Rules.

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