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Pressure Induced Isenthalpic Oscillations with Condensation and Evaporation in Saturated Two-Phase Fluids

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***Abstract*—**Saturated two-phase fluid flows are often subject to pressure induced oscillations. Due to compressibility the vapor bubbles act as a spring with an asymmetric non-linear characteristic. The volume of the vapor bubbles increases or decreases differently if the pressure fluctuations are compressing or expanding; consequently, compressing pressure fluctuations in a two-phase pipe flow cause less displacement in the direction of the pipe flow than expanding pressure fluctuations. The displacement depends on the ratio of liquid to vapor, the ratio of pressure fluctuations over average pressure and on the exciting frequency of the pressure fluctuations.

In addition, pressure fluctuations in saturated vapor bubbles cause condensation and evaporation within the bubbles and change periodically the ratio between liquid to vapor, and influence the dynamical parameters for the oscillation. The oscillations are conforming to an isenthalpic process at constant enthalpy with no heat transfer and no exchange of work.

The paper describes the governing non-linear equation for two-phase fluid oscillations with condensation and evaporation, and presents steady state approximate solutions for free and for pressure induced oscillations. Resonance criteria and stability are discussed.

***Keywords*—**condensation, evaporation, non-linear oscillations, pressure induced, two-phase flow

I. INTRODUCTION

A saturated two-phase fluid is a mixture of liquid and vapor. The volumetric ratio R between the vapor and the liquid portion is defined as

$$R = \frac{V_v}{V_v + V_L}. \tag{1}$$

Where R=0 for pure liquid and R=1.0 for pure vapor. In standard books for Thermodynamics [1], the ratio R is called the quality of the fluid with the symbol x instead of R. V_V is the vapor volume and V_L is the liquid volume. With very good accuracy, the liquid portion can be assumed to be incompressible when compared to the compressible vapor portion. The vapor portion follows approximately the ideal gas equation of state with the vapor volume V_V inversely proportional to the pressure p at constant temperatures. Pressure changes Δp and volumetric changes are related by (2)

$$\frac{p + \Delta p}{p} = \frac{V_V}{V_V + \Delta V_V} \tag{2}$$

Equation (2) may also be written in the form:

$$\Delta p = -p \frac{\Delta V_V}{V_V + \Delta V_V}. \tag{3}$$

Within a controlled volume of a two-phase fluid defined as the sum of V_V and V_L equals one. The volumetric vapor content is equal to

$$V_v = R. \tag{4}$$

The amount of condensing and evaporating fluid depends on the pressure change, and with r as the rate of change the momentary vapor portion V_V as a function of the pressure is

$$V_v = R + r \cdot \Delta p. \tag{5}$$

Fig. 1 shows the asymmetric oscillations in a two-phase pipe flow. For a given absolute pressure difference Δp the absolute displacement $\Delta X(t)$ is smaller for compression than for expansion.

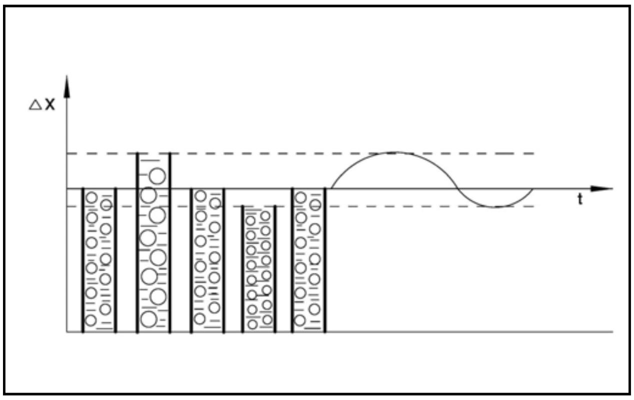

Fig. 1 Asymmetric displacement of vapor bubbles for expansion and compression

Fig. 2 demonstrates the condensation and evaporation

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within the vapor bubble for isenthalpic compression and expansion for saturated two-phase fluids, if the slope of the curves (R=constant) is positive [1]. The slope is positive towards the liquid side of the saturation dome. A pressure increase causes a partial condensation, and a pressure decrease causes a partial evaporation of the vapor bubble.

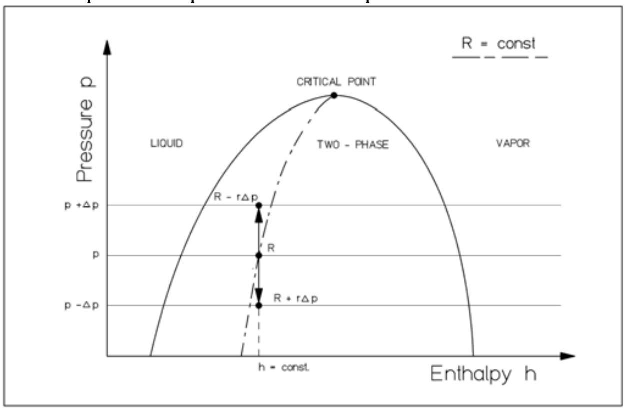

Fig. 2 Isenthalpic oscillations with condensation for increasing pressure

Fig. 3 demonstrates the condensation and evaporation for a negative slope of the *R*=constant curves. The slope is negative towards the vapor side of the saturation dome, and a pressure increase causes a partial evaporation, and a pressure decrease causes a partial condensation of the vapor bubble.

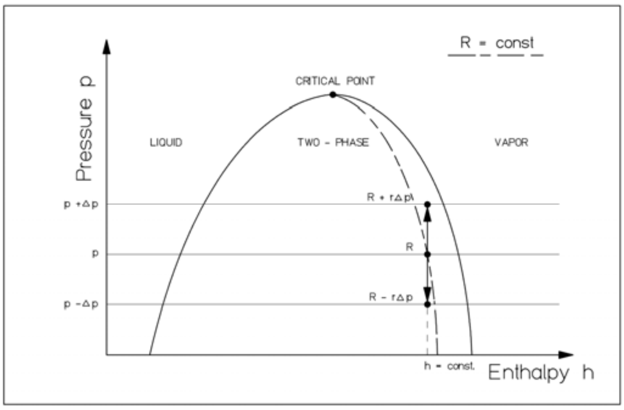

Fig. 3 Isenthalpic oscillations with evaporation for increasing pressure

Any evaporation affects the spring characteristic of the oscillating system to be softer and the mass to be smaller, and any condensation affects the spring characteristic to be stiffer and the mass to be larger. These changes of the system characteristics during a periodic oscillation result in complicated non-linear periodic oscillations.

II. Free Isenthalpic Oscillations in Saturated Fluids

Pressure changes Δp act as a force in the axial direction X of the pipe. The compressible vapor volume and the mass of the liquid volume form an oscillating system. The vapor volume acts as an asymmetric spring and generates axial forces non-linear to the displacement X as a function of the time t.

$$\Delta p \approx -X \approx -\Delta V_V. \tag{6}$$

The density of the vapor is negligibly small compared to the density ρ of the liquid, and the oscillating mass m within the controlled volume $V_V + V_L = 1$ depends only on the volume V_L of the liquid.

$$m = \rho V_L = \rho(1 - V_V) = \rho(1 - R + r\Delta p) = \rho(1 - R - r\,X). \tag{7}$$

Equation (8) is a non-linear differential equation that models the free oscillation in saturated fluids with condensation and evaporation:

$$\rho(1 - R - r\,X(t))\frac{d^2X(t)}{dt^2} + p\frac{X(t)}{R + r\,X(t) + X(t)} = 0. \tag{8}$$

For very small displacements, $X(t)$, the oscillating system can be considered as a linear harmonic oscillator,

$$\rho(1 - R)\frac{d^2X(t)}{dt^2} + p\frac{X(t)}{R} = 0, \tag{9}$$

with the natural frequency,

$$f_N = \sqrt{\frac{p}{\rho R(1-R)}}, \tag{10}$$

and,

$$\frac{d^2X(t)}{dt^2} + f_N^2\,X(t) = 0 \tag{11}$$

The non-linear differential equation can be written now in term of the natural frequency and transforms into:

$$(1 - R - r\,X(t))\frac{d^2X(t)}{dt^2} + f_N^2\frac{X(t)}{R + (1+r)X(t)} = 0. \tag{12}$$

Steady state approximate solutions for free isenthalpic oscillations are included in the solutions for the pressure induced oscillations.

III. Forced Pressure Induced Oscillations in Saturated Fluids

In the case of forced oscillations with harmonic excitation the non-linear differential equations transforms into:

$$\left(1 - R - r\,X(t)\right)\frac{d^2X(t)}{dt^2} + f_N^2\frac{R(1-R)X(t)}{R + (1+r)X(t)} = Gf^2\cos(ft). \tag{13}$$

The exciting force depends on the exciting amplitude G and the square of the exciting frequency f. This highly non-linear differential equation cannot be solved by analytical methods and numerical or approximation methods have to be applied. The method described by A. Kimmel [2] of using Fourier coefficients for the periodic error is applied to generate an approximate analytical solution for the steady state case of pressure induced isenthalpic oscillations in saturated two-phase fluids.

If *X(t)* is an approximate solution, then *E(t)* is the function of the error for this approximate solution *X(t)*:

$$E(t) = \big(1 - R - r\,X(t)\big)\frac{d^2X(t)}{dt^2} + f_N^2\frac{R(1-R)X(t)}{R+(1+r)X(t)} - Gf^2\cos(ft). \quad (14)$$

The periodic functions of the error *E(t)* can be expanded in a Fourier series and the Fourier coefficients a_k and b_k of the periodic error are determined by (15) and (16).

$$a_k = \int_0^{2\pi} E(t)\cos(kft)\,d(ft) \quad (15)$$

$$b_k = \int_0^{2\pi} E(t)\sin(kft)\,d(ft) \quad (16)$$

By setting certain a_k and b_k equal to zero, the error for the *k-th* term of the Fourier series *E(t)* becomes zero. The approximate solution *X(t)* is then correct for the *k-th* term of the function *E(t)*.

IV. Approximate Solution

For the given differential equation with forced oscillation through harmonic excitation the approximate solution *X(t)* corresponds to the exciting periodic function but with a constant term *S* and the amplitude *A*.

$$X(t) = S + A\cos(ft) \quad (17)$$

This is part of the Fourier series with $\frac{1}{2}a_0=S$ and $a_1=A$ and all other Fourier coefficients are equal to zero. By substituting *X(t)* into the differential equation the Fourier coefficients a_0 and a_1 of the error, *E(t)* can be determined by integration:

$$E(t) = -f^2(G + A(1 - R - r\,(S + A\cos(ft))) \cdot Cos(ft) + f_N^2\frac{R(1-R)(S+A\cos(ft))}{R+(1+r)(S+A\cos(ft))}. \quad (18)$$

Integration of (18) leads to functions (19) and (20) for a_0 and a_1 respectively.

$$a_0 = a_0(A, S, f, f_N, R, r, G) = 0 \quad (19)$$

$$a_1 = a_1(A, S, f, f_N, R, r, G) = 0 \quad (20)$$

Setting both (19) and (20) equal to zero produces the approximate solutions for *A* and *S* as a function of the exciting frequency *f* and the exciting force *G* determined by the vapor to liquid ratio *R* and the condensation and evaporation parameter *r*. The coefficient S can be determined from $a_0(A,S,f,fN,R,r,G)=0$ with (21).

$$S = -\frac{R}{1+r} + \sqrt{A^2 + \frac{4(1-R)^2R^4}{(1+r)^2(A^2f_S^2r(1+r)+2(1-R)R)^2}} \quad (21)$$

By substituting *S* into the function $a_1(A,S,f,fN,R,r,G)=0$ the solution for *A* and f_S is found as an implicit function.

$$\frac{R}{1+r} + \frac{Af_S^2((A+G)(1+r)-A(1+2r)R)}{2(A^2f_S^2r(1+r)+(1-R)R} - \sqrt{A^2 + \frac{4(1-R)^2R^4}{(1+r)^2(A^2f_S^2r(1+r)+2(1-R)R)^2}} = 0 \quad (22)$$

The scaled frequency f_S is the ratio of exciting frequency *f* over natural frequency f_N.

$$f_S^2 = f^2/f_N^2 \quad (23)$$

Solutions of the functions a_0 and a_1 are the stable steady state solutions for pressure induced isenthalpic oscillations with condensation and evaporation in saturated two-phase fluids.

V. Approximate Solution Applied to Free and Forced Oscillations

Free oscillations in saturated fluids without exciting forces, with *G*=0, reduces the approximate solution to:

$$\frac{R}{1+r} + \frac{A^2f_S^2(1+r-R-2rR)}{2(A^2f_S^2r(1+r)+(1-R)R} - \sqrt{A^2 + \frac{4(1-R)^2R^4}{(1+r)^2(A^2f_S^2r(1+r)+2(1-R)R)^2}} = 0 \quad (24)$$

and for two-phase fluids without condensation and evaporation, like air in water, with r=0 the equation reduces further to:

$$A = \pm\frac{2R}{f_S^2}\sqrt{1-f_S^2}. \quad (25)$$

The constant term *S* simplifies to:

$$S = -R + \sqrt{A^2 + R^2} \quad (26)$$

Fig. 4 shows the amplitude response *A* as a function of the frequency f_S for a free oscillation with *R*=0.5 and without quality change *r*=0. The displacement *X(t)* for pressure reduction is maximal *S+A* and the maximal displacement for pressure increase is *S-A*. Stable oscillations exist only for frequencies less or equal to the natural frequency.

Fig. 5 shows the amplitude response for free oscillations with *R*=0.5 and a small positive quality change *r*=0.03. There exist two stable solutions for a scaled frequency of less than 1, and the so called jump phenomenon occurs randomly between the two stable solutions, as indicated with the vertical line. For high frequency the amplitude approaches asymptotically a maximum value for the amplitude. The maximum value for the amplitude can be calculated with (27).

$$(1+r)^2(4A^4r^2-(A+G)^2) \\ +2A(A+G)(1+r)R-A^2R^2 = 0 \quad (27)$$

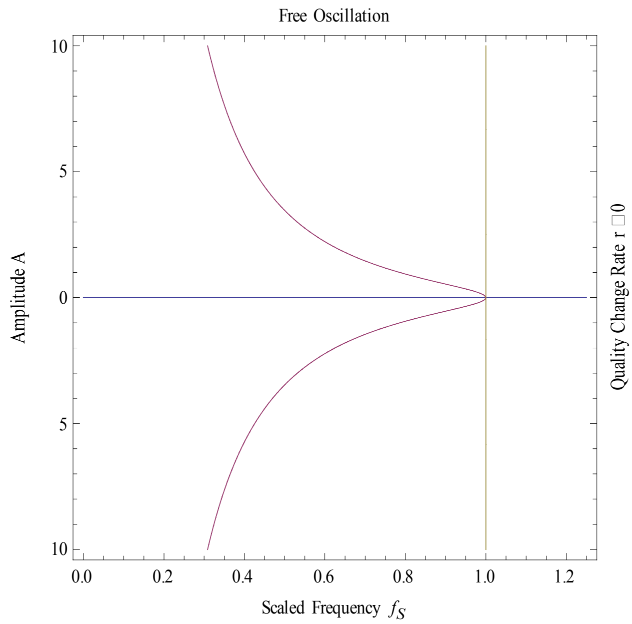

Fig. 4 Amplitude response for free oscillation without quality change

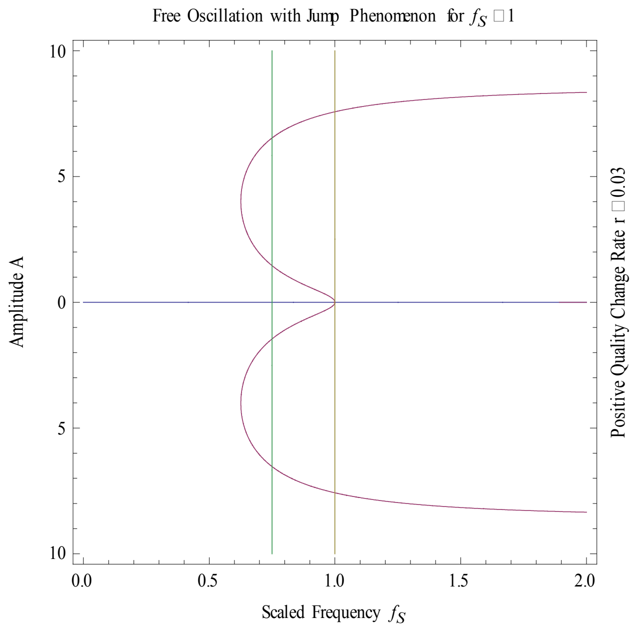

Fig. 5 Free oscillation for positive quality change and jump phenomenon

Fig. 6 shows the amplitude response for free oscillations with R=0.5 and a large positive quality change r=0.3. There is only one stable solution. The asymptotic maximum value for the amplitude follows the same equation as in Fig. 5.

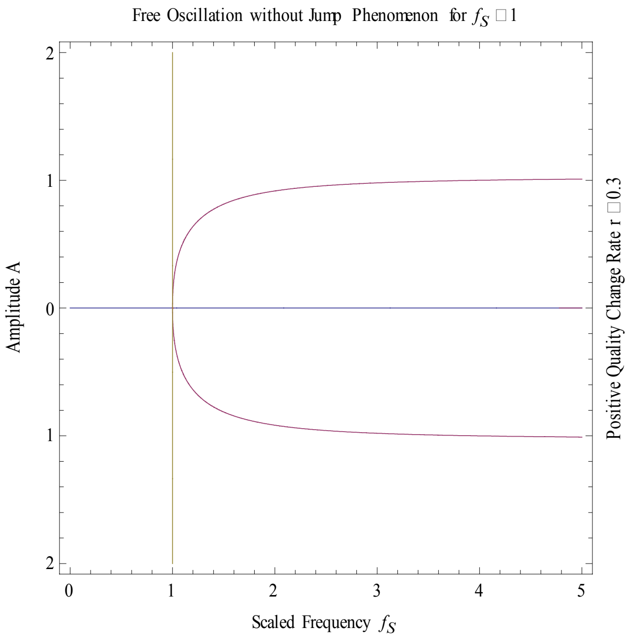

Fig. 6 Free oscillation for positive quality change without jump phenomenon

Equation (28) is applied to determine if two, or only one stable solution exists.

$$(1+r)^2-2(1+r)(1+4r)R \\ +(1+r(8+9r)R^2 = 0 \quad (28)$$

Fig. 7 demonstrates the graph for (28) and jump phenomena exists only within the dark regions.

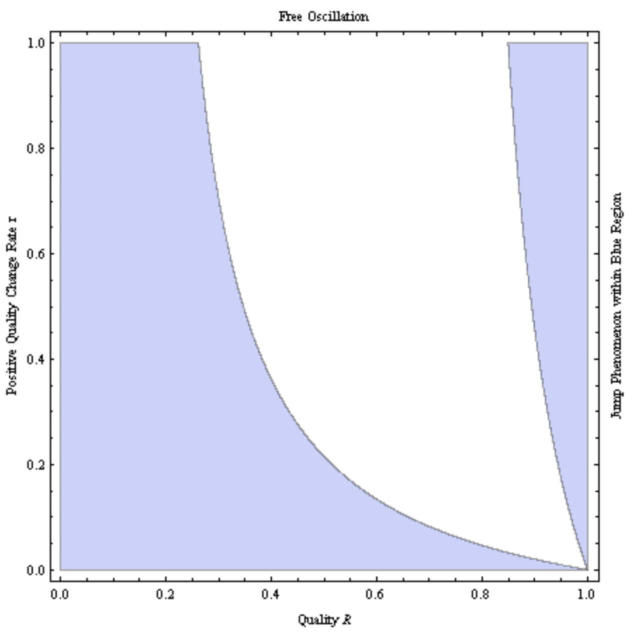

Fig. 7 Jump Phenomenon region for free oscillation and positive quality change

A typical free oscillation with R=0.5 and a small negative quality change r= -0.03 is shown in Fig. 8. There are two stable solutions for negative quality changes.

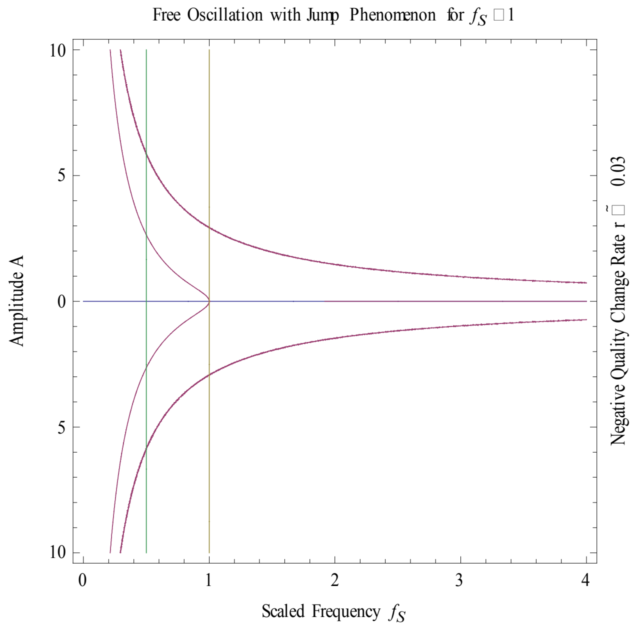

Fig. 8 Free oscillation for negative quality change with jump phenomenon

A typical forced oscillation with G=0.5 and R=0.5, without quality change r=0 is shown in Fig. 9. Jump phenomena occurs between two or three stable solutions, with one of the stable solutions not in phase with the exciting frequency, and a negative amplitude A for a positive exciting force G.

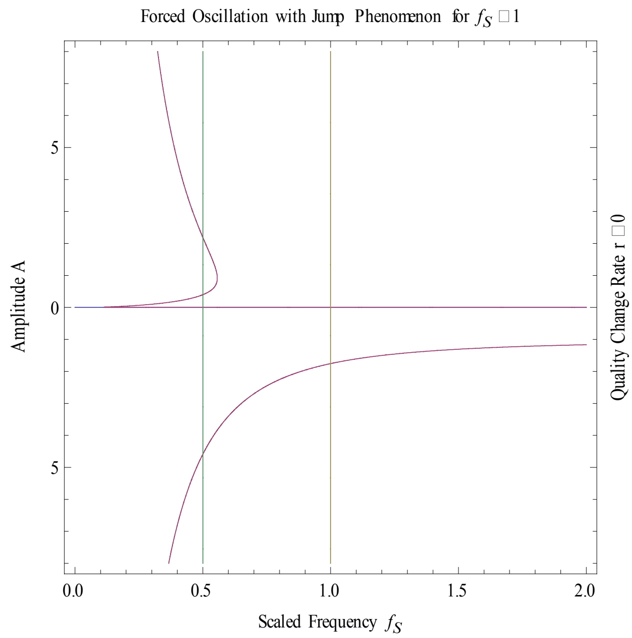

Fig. 9 Forced Oscillation with G=0.5 without quality change

Small positive quality changes, typically r=0.03 results also in three stable solutions for forced oscillations G=0.5, but with two solutions out of phase, as shown in Fig. 10 with R=0.5. All three solutions approach asymptotically certain amplitude values for high frequencies.

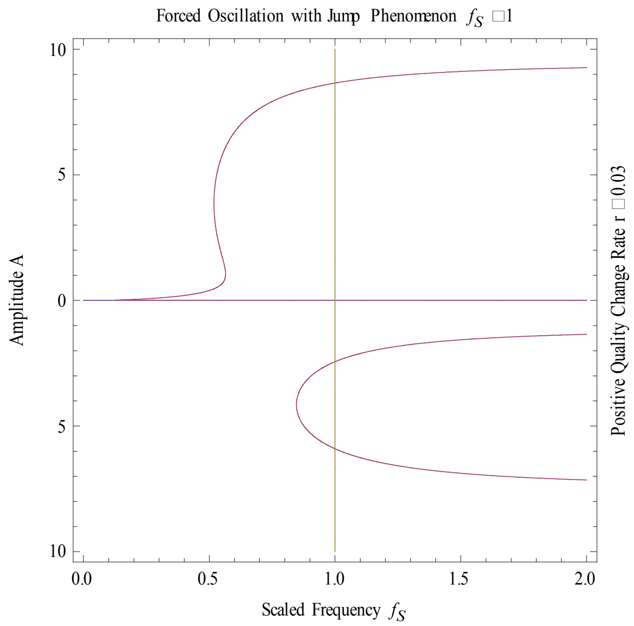

Fig. 10 Forced oscillation with G=0.5 and small positive quality change

Large positive quality changes like r=0.3 in forced oscillations G=0.5 with R=0.5 result in one stable solution approaching asymptotically a certain amplitude value, shown in Fig. 11.

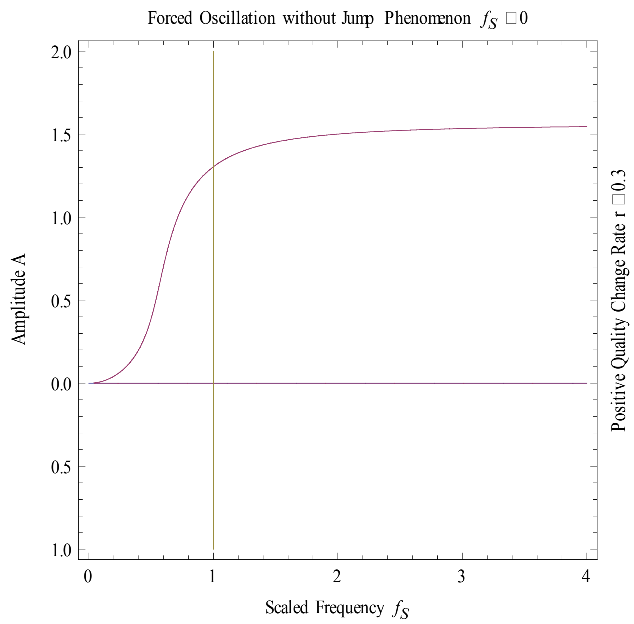

Fig. 11 Forced oscillation with G=0.5 and large positive quality change

Forced oscillations *G*=0.5 with negative quality changes *r*=-0.05 and *R*=0.5 result in a complicated solution pattern for the amplitude with up to five stable solutions, only one of which is in phase with the exciting force. Multiple jump phenomena with and without phase change occur randomly.

VI. Conclusion

Pressure induced isenthalpic oscillations with condensation and evaporation in saturated two-phase fluids have stable solutions for the whole range of the saturated fluid within the saturation dome. The solution pattern is more complicated close to the vapor side of the saturation dome at negative slopes of the constant quality curve. There are no instabilities for free or forced oscillations

References

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