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An Implicit Representation of Spherical Product for Increasing the Shape Variety of Super-quadrics in Implicit Surface Modeling

Pi-Chung Hsu

***Abstract*—**Super-quadrics can represent a set of implicit surfaces, which can be used furthermore as primitive surfaces to construct a complex object via Boolean set operations in implicit surface modeling. In fact, super-quadrics were developed to create a parametric surface by performing spherical product on two parametric curves and some of the resulting parametric surfaces were also represented as implicit surfaces. However, because not every parametric curve can be redefined implicitly, this causes only implicit super-elliptic and super-hyperbolic curves are applied to perform spherical product and so only implicit super-ellipsoids and hyperboloids are developed in super-quadrics. To create implicit surfaces with more diverse shapes than super-quadrics, this paper proposes an implicit representation of spherical product, which performs spherical product on two implicit curves like super-quadrics do. By means of the implicit representation, many new implicit curves such as polygonal, star-shaped and rose-shaped curves can be used to develop new implicit surfaces with a greater variety of shapes than super-quadrics, such as polyhedrons, hyper-ellipsoids, super-hyperboloids and hyper-toroids containing star-shaped and rose-shaped major and minor circles. Besides, the newly developed implicit surfaces can also be used to define new primitive implicit surfaces for constructing a more complex implicit surface in implicit surface modeling.

***Keywords*—**Implicit surfaces, Soft objects, Super-quadrics.

I. Introduction

SINCE super-quadrics were developed by A. Bar [1], they have been widely viewed as a powerful mathematical model to represent an implicit or parametric surface. They have been successfully applied in implicit surface (solid) modeling [2], [3], [4] or computer-aided design. Especially, in soft object model [3] or in constructive geometry [4], implicitly defined super-quadrics can be used as defining functions to define primitive implicit surfaces (solid) or as field functions to define primitive soft objects, and the resulting primitive implicit surfaces (solids) can further be deformed [5] and be used to construct a complex surface via Boolean set operations [6] and [7]. Because defining (field) functions control the shapes of primitive implicit surfaces to be constructed and so they play a very important role in implicit surface modeling. In addition to super-quadrics, a lot of defining functions were developed, such as super-ellipsoids [3], skeletal primitives [8], star solid [9], generalized distance functions [10], implicit sweep objects [11], hyper-quadrics [12] and ratio-quadrics which offer faster calculation of super-quadrics [13].

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Super-quadrics originally were developed to create a parametric surface by performing spherical product on two 2D parametric curves. They generate a shape by modulating and scaling one of the parametric curves with every point on another one. In addition, by choosing parametric curves that can be redefined implicitly, some of the resulting parametric surfaces can be redefined as an implicit surface [1]. However, because not every parametric curve is allowed to be represented implicitly, this causes that in super-quadrics only parametric super-ellipses and super-hyperbolas were used to perform spherical product and consequently only implicitly defined super-ellipsoids and super-hyperboloids had been developed. To increase the variety of the shapes of implicitly defined super-quadrics and expand their application in implicit surface modeling, this paper proposes spherical product functions, which perform spherical product on two 2D implicit curves (a contour and a profile curves) like super-quadrics do. Precisely, the proposed function generates an implicit surface by modulating and scaling the contour curve with every point on the profile curve, and hence it can be viewed as an implicit form of the spherical product of super-quadrics [1]. The major advantage over super-quadrics is that new implicit curves, such as polygonal, rose-shaped and star-shaped iso-curves, with more diverse shapes than super-ellipses and super-hyperbolas adopted by super-quadri can be appled into a spherical product function to perform spherical product for creating a new implicit surface. As a result, new implicit surfaces like polyhedrons, hyper-toroids, super-hypertoroids and hyper-boloids which contiain polygoal, rose-shaped and star-shaped contours and profiles can be developed. In addition, all these new surfaces can also be applied as new primitive implicit surfaces to construit a more complex implicit surface by boolean set operations in constructive solid geometry or in soft object model.

The remainder of this paper is organized as follows. Super-quadrics and implicit surface modeling are introduced in Section II. Spherical product function is presented in Section III. Section IV presents ray-linear functions for defining a spherical product function. Section V gives demonstration of the resulting implicit surfaces. Translated and rotated profile curves are presented in Section VI. Conclusion is given in Section VII.

II. Super-quadrics and Implicit Surface Modeling

This section introduces super-quadrics and their application in implicit surface modeling.

A. Representation of Parametric Surfaces

A parametric surface represents an object by a parametric formula, such as $P(\alpha,\beta):[0, 1]^2\rightarrow R^3$,

$$P(\alpha,\beta) =[X(\alpha,\beta), Y(\alpha,\beta), Z(\alpha,\beta)],$$

where α and β are parameters. The coordinate of every point on the surface can be obtained by calculating $P(\alpha,\beta)$ directly.

B. Representation of Implicit Surfaces

An implicit surface defines an object by using defining functions $f_i(x,y,z):R^3\rightarrow R_+$, i=1,2,…, as a point set

$$\{(x,y,z)\in R^3 \mid f_i(x,y,z)=1\},$$

where R_+ stands for the close set $[0, \infty]$ in R. In the following, an implicit surface is denoted as $f_i(x,y,z)=1$ for short.

In addition, an object can also be represented as an implicitly defined solid by

$$\{(x,y,z)\in R^3 \mid f_i(x,y,z)\leq 1\}.$$

Thus, it can be used as a primitive to construct a more complex object via union or intersection operations. To be a defining function for a solid, $f_i(x,y,z)$ needs to be an inside-outside function, which satisfies the following:

1) if $f_i(x_0,y_0,z_0)=1$, (x_0,y_0,z_0) is on the surface.
2) if $f_i(x_0,y_0,z_0)<1$, (x_0,y_0,z_0) lies inside the surface.
3) if $f_i(x_0,y_0,z_0)>1$, (x_0,y_0,z_0) lies outside the surface.

Two famous families of defining functions are listed as follows:

1) Super-ellipsoids:

$$f(x,y,z)=(|x/a_1|^n+|y/a_2|^n+|z/a_3|^n)^{1/n}, \qquad (1)$$

2) Super-quadrics:

$$f(x,y,z)=((x/a_1)^{2/n_1}+(y/a_2)^{2/n_1})^{n_1/n_2}+(z/a_3)^{2/n_2})^{n_2/2}, \qquad (2)$$

C. Super-quadrics

As in [1], super-quadrics were obtained through spherical product. A spherical product, denoted as $\underline{m}(\beta)\otimes\underline{h}(\alpha)$, on two 2D parametric curves, $\underline{h}(\alpha)$ and $\underline{m}(\beta)$, produces a parametric surface by the position function:

$$\underline{m}(\beta)\otimes\underline{h}(\alpha)=[h_x(\alpha)m_x(\alpha), h_y(\alpha)m_x(\alpha), m_x(\beta)], \qquad (3)$$

where $\underline{h}(\alpha)=[h_x(\alpha), h_y(\alpha)]$, $\underline{m}(\beta)=[m_x(\alpha), m_x(\beta)]$, and $\alpha_1\leq\alpha\leq\alpha_2$ and $\beta_1\leq\beta\leq\beta_2$ are parameters.

Geometrically, parametric surface $\underline{m}(\beta)\otimes\underline{h}(\alpha)$ is viewed as horizontal curve $\underline{h}(\alpha)$ vertically modulated and scalled by vertical curve $m(\beta)$. For example, when super-ellipses are used as $\underline{h}(\alpha)$ and $\underline{m}(\beta)$, i.e. $\underline{h}(\alpha)=[a_1\cos^{n1}\alpha, a_2\sin^{n1}\alpha]$ where $\alpha\in[-\pi, \pi]$,is modulated and scaled by $\underline{m}(\beta)=[\cos^{n2}\beta, a_3\sin^{n2}\beta]$ where $\beta\in[-\pi,\pi]$, then parametrically defined super-ellipsoids are given by

$$\underline{h}(\alpha)\otimes\underline{m}(\beta)=[a_1\cos^{n1}\alpha\cos^{n2}\beta, a_2\sin^{n1}\alpha\cos^{n2}\beta, a_3\sin^{n2}\beta], \qquad (4)$$

In Eq. (4), n_1 is a squareness parameter of the shape in east-west direction and n_2 in north-south direction. When $n_1 < 1$, the shape is square, $n_1 \approx 1$, round, $n_1 \approx 2$, flat-beveled, and $n_1 > 2$, pinched. Parameter n_2 has the same effect as n_1. Fig. 1 shows some shapes of super-ellipsoids with $n_1 = n_2$.

In [1], because super-ellipses can be represented implicitly, Eq. (4) can also be given implicitly by a defining function $f_s(x,y,z)$ and written by:

$$f(x,y,z)=((x/a_1)^{2/n_1}+(y/a_2)^{2/n_1})^{n_1/n_2}+(z/a_3)^{2/n_2}=1$$

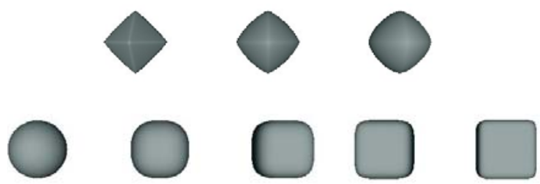

Fig. 1 The shape change of $(x^{2/n1}+y^{2/n1}+z^{2/n1})^{n1/2}=1$ while $n_1=n_2$ varies from 1.9, 15, 1.3, 1, 0.8, 0.57, 0.4, to 0.25 for the surfaces from left to right

D. Implicit Blends

In constructive geometry of implicit surfaces [3], [4], a more complex implicit surface is generated by constructing k implicit surfaces $f_1(x,y,z)=1,...,k$, smoothly through a blending operator $B_k(x_1,...,x_k):R_+^{\,k}\rightarrow R_+$. This is, a complex object is given by a blend $B_k(f_1(x,y,z),...,f_k(x,y,z))$ and is obtained by calculating the point set

$$\{(x,y,z)\in R^3 \mid B_k(f_1(x,y,z),...,f_k(x,y,z))=1\}$$

Some of the famous blending operators can be found in [4], [5],[6], [7], such as

1) Superellipsoidal intersection blend:

$$B_k(x_1,\ldots,x_k)= (x_1^{\,n}+\ldots+x_k^{\,n})^{1/n},$$

2) Superellipsoidal union blend:

$$B_k(x_1,\ldots, x_k) = (x_1^{-n}+\ldots+ x_k^{-n})^{-1/n}.$$

Fig. 2 demonstrates a dice defined by a difference blend of a cube from twenty-one spheres.

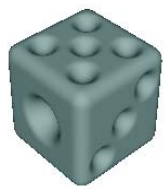

Fig. 2 The difference blend of a cube from twenty-one spheres

III. Implicit Representation of Spherical Product

It can be found from Section II that super-quadrics [1] are faced with two difficulties listed as follows:

1) Not every parametric surface $\underline{m}(\beta)\otimes\underline{h}(\alpha)$ from Eq. (3) can

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be redefined implicitly because not every $\underline{h}(\alpha)$ and $\underline{m}(\beta)$ can be defined implicitly.

2) This above also caused that only super-ellipses and super-hyperbolas were applied as $\underline{h}(\alpha)$ and $\underline{m}(\beta)$ to define a parametric surface $\underline{m}(\beta)\otimes\underline{h}(\alpha)$ from Eq. (3) which has an implicit represntation, too.

Due to these above, only implicitly defined super- ellipsoids and hyperbolids were developed as primitive implicit surfaces for constructing a more complex object in implicit surface modeling. To make super-quadrics have a greater variety of shapes, this section proposes spherical product functions, which is an implicit representation of spherical product and whose iso-surface is generated in the same way as the spherical product in Eq. (3) does but from two implicit curves not parametrc curves.

A. Definition of Spherical Product Functions

Let $h(x,y)$ and $m(x, z)$, called contour and profile functions respectively, map R^2 to R_+. Then a spherical product function, denoted as $m(x, z)\otimes h(x,y)$, is written by

$$m(x, z)\otimes h(x,y)=m(h(x,y), z), \tag{5}$$

Thus, if $h(x,y)=1$, called contour curve, is viewed as a horizontal closed curve, and $m(x, z)=1$, called profile curve, a vertical curve, then the surface $m(x,z)\otimes h(x,y)=1$, called implicit spherical product, has a cross-section like the curve $h(x,y)=1$ and has a profile like the curve $m(x,z)=1$. Namely, every point (x_0,z_0) satisfying the conditions $m(x_0,z_0)=1$ and $x_0\geq 0$ generates a new contour curve $h(x,y)=x_0$ at $z=z_0$, which is like the contour curve $h(x,y)=1$ translated along z-axis by $[0, 0, z_0]$ and scaled by x_0, as shown in Fig. 3.

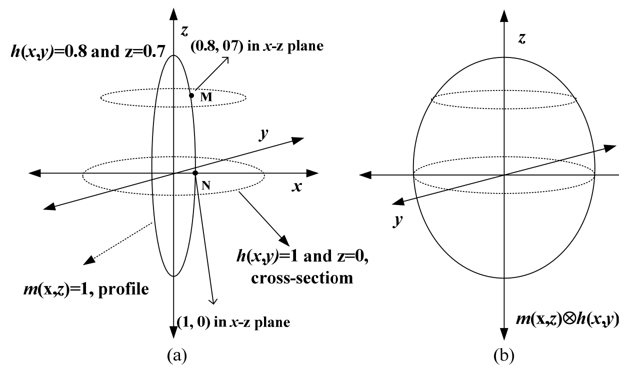

Fig. 3 (a) Dotted curves are cross-sections generated by points $M(0.8,07)$ and $N(1,0)$ on $m(x,z)=1$. (b) The surface $m(x,z)\otimes h(x,y)=1$ has a cross-section like the contour curve $h(x,y)=1$ and a profile like the profile curve $m(x,z)=1$

B. Implcit Representation of Spherical Product

Before explaining why $m(x,z)\otimes h(x,y)$ in Eq. (5) is an implicit form of spherical product, non-negative ray-linear property is defined first:

A function $f(x,y,z):R^3\rightarrow R_+$ is called non-negative ray-linear if $f(ax,ay,az)=af(x,y,z)$ holds for any $(x,y,z)\in R^3$ and $a\in R_+$.

It is called ray-linear for short in this paper. Based on the ray-linear property it can be derived that if $m(x,z)$ and $h(x,y)$ both are ray-linear, then an implicit spherical product surface $m(x,z)\otimes h(x,y)=1$ from Eq. (5) generates a shape in the same way as the spherical product does in Eq. (3). It is explained below. The implicit surface $m(x,z)\otimes h(x,y)=1$ is composed of all the contour curves $h(x,y)=m_x$ with $z=z_0$ where (m_x, z_0) satisfies the condition $m(m_x,z_0)=1$. From the ray-linear property of $h(x,y)$, every contour curve $h(x,y)=m_x$ is equivalent to the curve $h(x,y)/m_x=1$, i.e. $h(x/m_x,y/m_x)=1$. It follows that if the parametric formula of $h(x,y)=1$ is $[h_x(\alpha), h_y(\alpha)]$, then every contour curve $h(x,y)=m_x$ with $z=z_0$ can be given by $[h_x(\alpha)m_x, h_y(\alpha)m_x, z_0]$. This explains why the shape $m(x,z)\otimes h(x,y)=1$ is generated in the same way as the spherical product of super-quadrics in Eq. (3).

IV. Ray-linear Contour and Profile function

As stated in Section III, since 2D ray-linear contour and profile functions enable a spherical product function to have an iso-surface like spherical product of super-quadrics dose, this section develops new ray-linear contour and profile functions, created by constructing lines and super-hyperbolas via an intersection operation, for defining an implicit spherical product function.

A. Ray-linear Lineal Functions

A ray-linear function possessing linear iso-curves in 2D space is proposed and denoted by:

$$f_p(x, y)=|\ \underline{v} \bullet [x, y]\ | \ / \ d_v, \tag{6}$$

where $\underline{v}$ is the unit normal vector of the line $f_p(x, y)=1$ and d_v is the shortest distance from the origin to the line. $f_p(x, y)=1$ is a pair of parallel lines, and it is easy to show that $f_p(x, y)$ is ray-linear.

B. Ray-linear Super-hyperbolic Functions

Let v and u be unit vectors in 2D space, $\underline{v}\bullet \underline{u}=0$, d_v, d_u, and $m>0$. Then, a ray-linear function with super-hyperbolic iso-curves is proposed and defined by

$$f_h(x,y)=\begin{cases} \left(f_v(x,y)^m - f_u(x,y)^m \right)^{\frac{1}{m}} \\ 0 \qquad if \quad \left(f_v(x,y)^m < f_u(x,y)^m \right. \end{cases}, \tag{7}$$

where $f_v(x, y)=|\ \underline{v} \bullet [x, y]\ |/d_v$ and $f_u(x, y)=|\ \underline{u} \bullet [x, y]\ |/d_u$.

It is easy to show that $f_h(x,y)$ is ray-linear. As shown in Fig. 4, the shape of $f_h(x,y)=1$ is a pair of super-hyperbolic and symmetrical curves bounded in specified regions; vectors $\underline{v}$ and $\underline{u}$ both control the orientation of the curves; parameter d_v determines the shortest distance from the origin to the curve; and parameter m controls the squareness of the curve. For example,

1) When $m\approx 1$, $f_h(x,y)=1$ degenerates toward two folded lines passing through points f, e, and g and points f', e', and g', respectively, the red dotted lines in Fig. 4.
2) When $m>1$, $f_h(x,y)=1$ is super-hyperbolic, the solid curve.
3) When $m\approx\infty$, $f_h(x,y)=1$ is two curves that approach the

square dotted lines and pass through points a, c, d, and b and points a', c', d', and b', respectively.

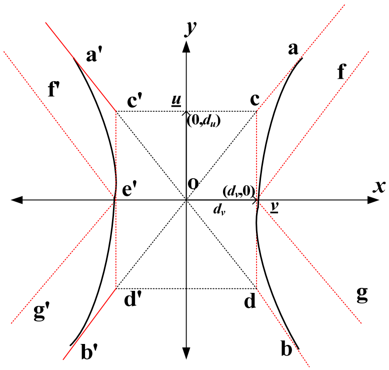

Fig. 4 Super-hyperbolic curves: solid lines bounded between red dotted curves

C. Generalized Elliptic and Hyperbolic Functions

Before introducing the generalized elliptic and hyperbolic functions, Theorem 1 is proposed first:

Theorem 1: If $f_i(x,y)$:$R^2 \rightarrow R_+$, i=1,...,k, and blending operator $B_k(x_1,...,x_k)$:$R_+^k \rightarrow R_+$ all are ray-linear, then the blend $B_k(f_1(x, y),..., f_k(x, y))$ is ray-linear, too.

It is easy to prove this theorem. Based on Theorem 1, one can integrate super-ellipsoidal intersection blend: $B_k(x_1,\ldots,x_k) =(x_1^n+\ldots+x_k^n)^{1/n}$, with Eqs. (6)-(7) together, and then develop a new ray-linear blending operation (function) by

$$B_k(f_1,\ldots,f_k) =(f_1(x, y)^n+\ldots+f_k(x, y)^n)^{1/n}, \quad (8)$$

where $f_i(x, y)$, i=1,...,k, is $f_h(x, y)$ or $f_p(x, y)$ from Eqs. (6)-(7).

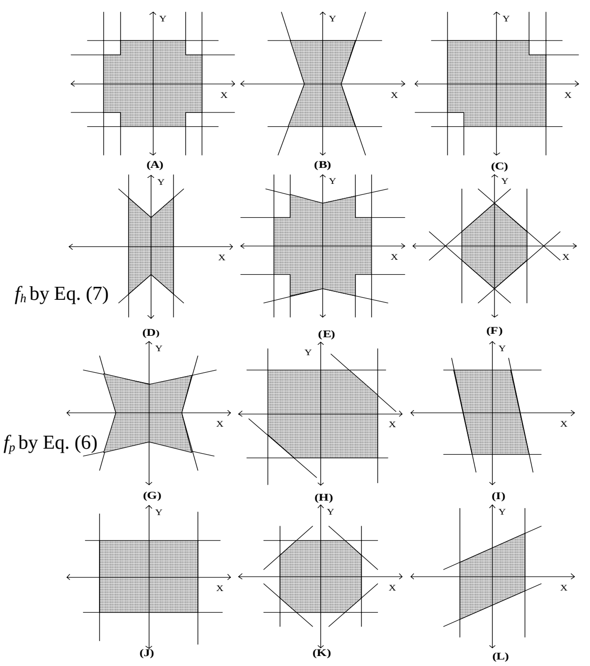

Fig. 5 Contour and profile curves, $h(x,y)$=1 and $m(x,z)$=1 by Eq. (8), where parallel lines are defined by f_p in Eq. (6) and folded lines by f_h in Eq. (7).

Some of the contour curves $h(x,y)$=1 or profile curves $m(x,z)$=1 defined by Eq. (8) are displayed in Fig. 5. They can be used to define a new implicit spherical product surface.

V. Demonstration of Implicit Spherical Product Surfaces

Some of the implicit spherical product surfaces defined from Eq. (8) are demonstrated in this section.

A. Generalized Elliptic Contour and Profile Functions

When $f_p(x, y)$ in Eq. (6) acts as $f_i(x, y)$ in Eq. (8) and namely contour function $h(x,y)$ is an intersection of four pairs of parallel lines, an eight-sided curve shown in Fig. 6,

$$h(x,y)= (f_{p1}(x, y)^{n1}+ f_{p2}(x, y)^{n1}+f_{p3}(x, y)^{n1}+f_{p4}(x, y)^{n1})^{1/n1},$$

where f_{p1}=|x/30|, f_{p2}=|y/30|, $f_{p3}=|x/\sqrt{2}+y/\sqrt{2}|/30$ and $f_{p4}=|-x/\sqrt{2}+y/\sqrt{2}|/30$, and profile function $m(x,z)$ is also an intersection of four pairs of parallel lines,

$$m(x,z)= (f_{p1}(x,z)^{n2}+ f_{p2}(x,z)^{n2}+f_{p3}(x,z)^{n2} +f_{p4}(x,z)^{n2})^{1/n2},$$

where f_{p1}=|x|, f_{p2}=|z/30|, $f_{p3}=|x/\sqrt{2}+z/(30\sqrt{2})|$ and $f_{p4}=|-x/\sqrt{2}+z/(30\sqrt{2})|$, then surfaces $m(x,z)\otimes h(x,y)$=1, where n_2 of $m(x,z)$ is set 100 for a contracted polygonal profile and n_1 of $h(x,y)$ is set from 100,10, to 6 for a changing polygonal contour, are shown in Fig. 7 from left to right, and the first one is a polyhedron.

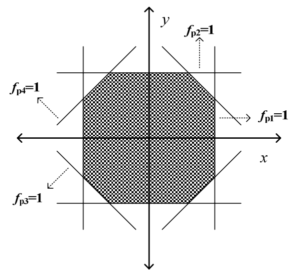

Fig. 6 The curves of f_{p1}=|x/30|, f_{p2}=|y/30|, $f_{p3}=|x/\sqrt{2}+y/\sqrt{2}|/30$, and $f_{p4}=|-x/\sqrt{2}+y/\sqrt{2}|/30$

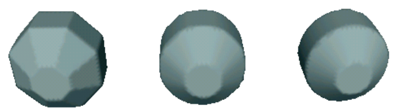

Fig. 7 The surfaces $m(x,z)\otimes h(x,y)$=1 defined by using an intersection blend of four pairs of parallel lines as $h(x, y)$ and $m(x, z)$ both

B. Generalized Hyperbolic Contour and Profile Functions

When $f_h(x,y)$ in Eq. (7) acts as $f_i(x,y)$ in Eq. (8), i.e. contour function $h(x,y)$ is a polygonal curve in Fig. 6 and profile functions $m(x,z)$ are those shapes in Figs. 5(A)-(I), then the shapes of $m(x, z)\otimes h(x,y)$=1 are listed in Fig. 8.

In the same case as that in Fig. (8) except the contour $h(x,y)$ is replaced with the four-sided curve in Fig. 5(J) given by:

$$h(x,y)= (f_{p1}(x, y)^{n1}+f_{p2}(x, y)^{n1})^{1/n1},$$

where $f_{p1}=|x|$ and $f_{p2}=|z/30|$, the resulting shapes of $m(x, z)\otimes h(x,y)=1$ are listed in Fig. 9.

Consider the case that $h(x,y)$ is an intersection of four pairs of super-hyperbolas (folded lines) by Eq. (7) and it is like a star shape shown in Fig. 10 and defined by:

$$h(x,y)=(f_{h1}(x,y)^{n_1}+f_{h2}(x,y)^{n_1}+f_{h3}(x,y)^{n_1}+f_{h4}(x,y)^{n_1})^{1/n_1},$$

where f_{h1}: $f_{v1}(x, y)=|x/25|$ and $f_{u1}(x, y)=|y/25|$,
f_{h2}: $f_{v2}(x, y)=|y/25|$ and $f_{u2}(x, y)=|x/25|$,

Fig. 8 The surfaces $m(x,z)\otimes h(x,y)=1$ created using the curves in Figs. 5(A)-(I) as $m(x, z)=1$ and the curve in Fig. 6 as $h(x, y)=1$

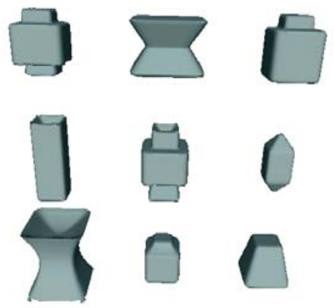

Fig. 9 The surfaces $m(x,z)\otimes h(x,y)=1$ defined using the curves shown in Figs. 5(A)-(I) as $m(x,z)=1$ and the curve in Fig. 5(J) as $h(x,y)=1$

f_{h3} : $f_{v3}(x, y)=|x/\sqrt{2}+y/\sqrt{2}|/25$ and
$f_{u3}(x, y)=|-x/\sqrt{2}+y/\sqrt{2}|/25$,
f_{h4} : $f_{v4}(x, y)=|-x/\sqrt{2}+y/\sqrt{2}|/25$ and
$f_{u4}(x, y)=|x/\sqrt{2}+y/\sqrt{2}|/25$,

and the squareness parameters m of f_{h1}, f_{h2}, f_{h3} and f_{h4} are all set close to 1. Besides, $m(x,z)$ is an intersection of two pairs of parallel lines,

$$m(x,z)=(f_{p1}(x, z)^{n2}+f_{p2}(x, z)^{n2})^{1/n2},$$

where $f_{p1}=|x|$, $f_{p2}=|z/25|$ and n_2 of $m(x,z)$ is set close to 1. Thus, as n_1 of $h(x,y)$ are set from 100, 8, 2, 1.5, 1, to 0.7, causing the contour curve to change from a star, a rose to a concave rose, the surfaces $m(x, z)\otimes h(x, y)=1$ are shown from top left to bottom right in Fig. 11.

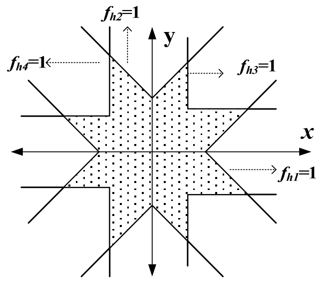

Fig 10 The intersection of four pairs of super-hyperbolas (folded lines), f_{h1}= , f_{h2}=1, f_{h3}=1 and f_{h4}=1 defined using $f_h(x, y)$ in Eq. (7)

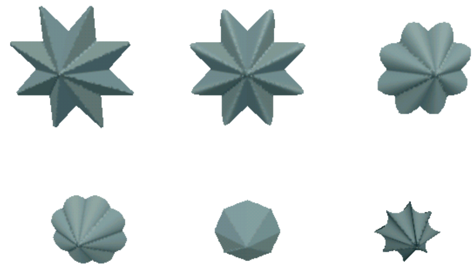

Fig. 11 Star-shaped and rose-shaped surfaceS $m(x,z)\otimes h(x,y)=1$ where the star-shaped curve in Fig. 10 is used to define the contour curve $h(x, y)=1$

VI. Translated Profile Curve

When a profile curve is translated by [C, 0], $C\geq 0$, and is written as:

$$m(x-C, z)=1,$$

then a new spherical product function that has a shape of hyper-toroids is given by:

$$m(x-C, z)\otimes h(x, y). \qquad (9)$$

As shown in Fig. 12, the shape of $m(x-C, z)\otimes h(x,y)=1$ is like a toroidal surface whose major circle, contour curve $h(x,y)=1$, is translated along z-axis forwards and backwards and controlled by the points possessing x-cooridinate larger than or equat to 0 on the minor circle, the translated profile curve $m(x-C, z)=1$. Geometrically speaking, Eq. (9) can be used to define hyper-toroids containing polygonal, rose-shaped and star-shaped major and minor circles, as shown in Fig. 13. When the polygonal curve in Fig. 6 is used as both the contour and the profile curves, then the surface $m(x-C,z)\otimes h(x,y)=1$ is shown in Fig. 13(A); in the same case as that in Fig. 13(A) but the contour curve is replaced with the star-shaped curve in Fig. 10 where all the squareness parameters m of f_{h1} , f_{h2} , f_{h3} and f_{h4} are set 1.1 and 2 respectively, then the surfaces $m(x-C,z)\otimes h(x,y)=1$ are shown in Figs. 13(B)-(C).

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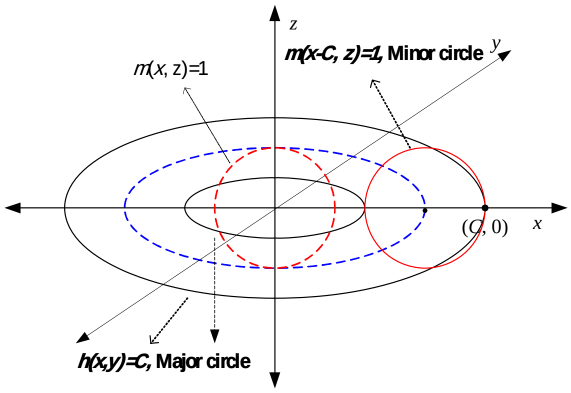

Fig. 12 A translated profile curve, in a red line

Fig. 13 Hyper-toroids created by Eq. (9), where an eight-sided polygon, a star and a rose in Figs. 6 and 10 are used as the contour curves (major circles) respectively for the surfaces from left to right and a translated eight-sided polygonal curve in Fig. 6 as the profile curves (minor circles)

Moreover, when a profile curve is translated and given by:

$$m(|x-C|, z)=1 \tag{10}$$

where $|x-C|$ means the absolute value of $(x-C)$ and $C\geq 0$, then a new spherical product function is given by:

$$m(|x-C|, z)\otimes h(x, y). \tag{11}$$

Because the value of $h(x, y)$ is larger than or equal to 0, the profile curve $m(|x-C|, z)=1$ used to modulate the contour curve $h(x, y)=1$ is composed of two curves: $m(x-C, z)=1$, $x>C$, and $m(-(x-C),z)=1$, $0\leq x<C$, which are symmetrical to line $x=C$. In fact, the curve $m(-(x-C), z)=1$, $0\leq x<C$, is the reflection of the curve $m(x-C, z)=1$, $x>C$, with respect to line $x=C$. For example, when the profile curve $m(x,z)=1$ is as in Fig. 14(A), as C increases, translated profile curves by Eq. (10) may be those on the boundaries of the shaded areas in Figs. 14(B)-(F). It follows from Fig. 14 that Eq. (11) can be used to create hyper-toroids with a minor circle of symmetric shape. For example, when Figs. 14(A)-(F) are used as profile curves and the eight-sided polygon of Fig. 6 as a contour curve, the shapes of spherical product surfaces by Eq. (11) are shown in the surfaces from top left to bottom right in Fig. 15.

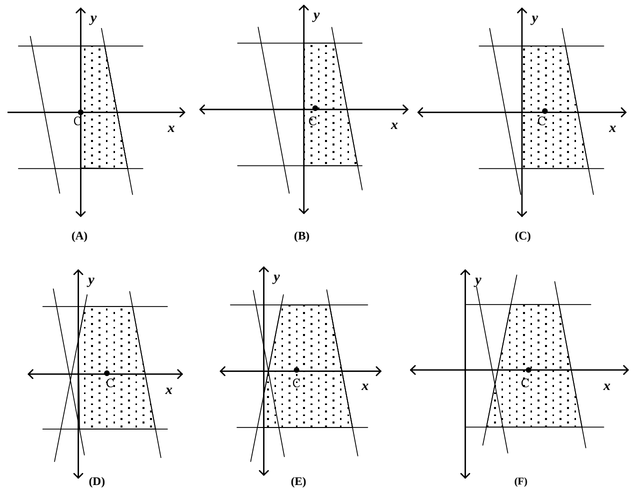

Fig. 14 (A) A profile curve to be translated. (B)-(F) The translated profile curves of Fig. 14(A) by Eq. (10) where the value of C increases for the curves from Fig. 14 (B) to (F)

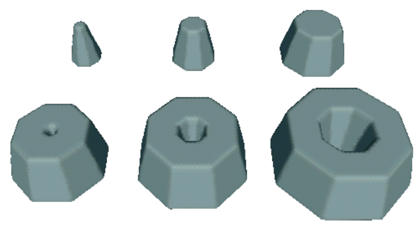
Fig. 15 Hyper-toroids by Eq. (11) with a symmetric shape of minor circle, where Figs. 14(A)-(F) are used as profile curves (minor circles) for the surfaces from top left to bottom right and the eight-sided polygon of Fig. 6 as the contour curve (major circle)

VII. Conclusion

Super-quadrics were developed by performing spherical product on two 2D parametric curves. Because not every parametric curve can be represented implicitly, only super-ellipses and super-hyperbolas are appled to define super-quadrics and so only implicit super-ellipsoids and super-hyperboloads have been developed. To increase the variety of the shapes of implicitly defined super-quadriics and to expand their applicition on implicit surface modeling, this paper has developed an implicit representaion of spherical product of super-quadrics, which performs spherical product like super-quadrics do but on two 2D implicit curves. The implicit representaion of spherical product is written as a spherical product function, which is a composition of a contour and a profile curves and whose iso-surfaces are generated by modulating the contour curve via the profile curve. Based on spherical product functions, this paper has developed implicit generallized ellipses and hyperbolas with polygoanl, star-shaped and rose-shaped curves as contour and profile curves, and consequently a new set of implicit hyper-ellipsoids and hyperboloids with polyhedral, concave, convex, star and rose shapes have been developed, and it is more diverse than super-quadrics.

Besides, translated profiles have been successfully appled into spherical product functions in this paper. As a result, hyper-toroids that have a symmetric shape of minor circle and possess polygonal, rose-shaped and star-shaped major and minor circles have been created, too. Furthermore, all these newly proposed functions can be further used as new field functions in soft object model [3] or as new defining functions in constructive solid geometry [4] to define new primitive implicit surfaces. That is, this paper has offered a greater variety of primitive implicit surfaces with more diverse shapes than super-quadrics for constructing a complex implicit surface via boolean set operations in implicit surface modeling.

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REFERENCES

[1] A. H. Barr, "Superquadrics", *IEEE Computer Graphics and Applications*, Vol. 1, No 1, pp. 11-23, 1981.

[2] J. F. Blinn, "A generalization of algebraic surface drawing", *ACM Transactions on Graphics*, Vol. 1, No 3, pp. 235-256, 1982.

[3] G. Wyvill and B. Wyvill, "Field functions for implicit surfaces", *The Visual Computer*, Vol. 5, pp. 78-52, 1989.

[4] A. Ricii, "A Constructive Geometry for Computer Graphics", *The Computer Journal*, Vol.16, No 2, pp. 157-160, May 1973.

[5] B Wyvill, A Guy, and E Galin, "Extending the CSG tree: warping, blending and boolean operations in an implicit surface modeling systems", in *Proc. of Implicit Surfaces'98*, pp.128-136, 1998.

[6] P.-C. Hsu and C. Lee, "The scale method for blending operations in functionally based constructive geometry", *Computer Graphics Forum*, Vol. 22, No 2, pp. 143-158, 2003.

[7] Q. Li, "Smooth piecewise polynomial blending operations for implicit shaped", *Computer Graphics Forum*, Vol. 26, No 2, pp. 143-158, 2007.

[8] J. Bloomenthal and B. Wyvill, "Interactive techniques for implicit modeling", in *SIGGRAPH Computer Graphics*, Vol.24, No 2, pp. 109-116, 1990.

[9] E. Akleman, "Interactive construction of smoothly blended star solids", in *Graphical Interface'96*, pp. 159-167, May 1996.

[10] E. Akleman and J. Chen, "Generalized distance functions", in *Proc. Shape Modeling International '99*, pp. 72-79, 1999.

[11] B. Crespin, C. Blanc, and C. Schlick, "Implicit sweep objects", in *Eurographics'96*, Vol.15, No 3, pp. 165-175, 1996.

[12] A. Hanson, "Hyperquadrics: smoothly deformable shapes with convex polyhedral bounds", *Computer Vision, Graphics and Images Processing*, Vol. 44, No 1, pp. 191-210, 1988.

[13] C. Blanc and C. Schlick, "Ratioquadrics: an alternative model to superquadrics", *The Visual Computer*, Vol. 12, pp. 420-428, 1996.