

# Fuzzy Control of a Quarter-Car Suspension System

M. M. M. Salem, and Ayman A. Aly

**Abstract**—An active suspension system has been proposed to improve the ride comfort. A quarter-car 2 degree-of-freedom (DOF) system is designed and constructed on the basis of the concept of a four-wheel independent suspension to simulate the actions of an active vehicle suspension system. The purpose of a suspension system is to support the vehicle body and increase ride comfort. The aim of the work described in the paper was to illustrate the application of fuzzy logic technique to the control of a continuously damping automotive suspension system. The ride comfort is improved by means of the reduction of the body acceleration caused by the car body when road disturbances from smooth road and real road roughness.

The paper describes also the model and controller used in the study and discusses the vehicle response results obtained from a range of road input simulations. In the conclusion, a comparison of active suspension fuzzy control and Proportional Integration derivative (PID) control is shown using MATLAB simulations.

**Keywords**—Fuzzy logic control, ride comfort, vehicle dynamics, active suspension system, quarter-car model.

## I. INTRODUCTION

TODAY, a rebellious race is taking place among the automotive industry so as to produce highly developed models. One of the performance requirements is advanced suspension systems which prevent the road disturbances to affect the passenger comfort while increasing riding capabilities and performing a smooth drive. While the purpose of the suspension system is to provide a smooth ride in the car and to help maintain control of the vehicle over rough terrain or in case of sudden stops, increasing ride comfort results in larger suspension stroke and smaller damping in the wheel-hop mode [1]. Many control methods have been proposed to overcome these suspension problems. Many active suspension control approaches such as Linear Quadratic Gaussian (LQG) control, adaptive control, and non-linear control are developed and proposed so as to manage the occurring problems [2-4]. During the last decades fuzzy logic has implemented very fast hence the first paper in fuzzy set theory, which is now considered to be the influential paper of the subject, was written by Zadeh [5], who is considered the founding father of the field. Then in 1975, Mamdani, developed Zadeh's work and demonstrated the viability of Fuzzy Logic Control (FLC) for a small model steam engine. Replacement of the spring-damper suspensions of automobiles by active systems has the potential of improving safety and comfort under nominal

conditions. But perhaps more important, it allows continuous adaptation to different road surface quality and driving situations. For the design of active suspension we know how to build a model and how to define the objective of the control in order to reach a compromise between contradictory requirements like ride comfort and road holding by changing the force between the wheel and chassis masses. In the recent past, it has been reported on this problem successively, about the base of optimization techniques, adaptive control and even, H-infinity robust methods.

The use of active suspension on road vehicles has been considered for many years [6, 7, 8, 9, 10]. A large number of different arrangements from semi-active to fully active schemes have been investigated [11, 12, 13, 14]. There has also been interest in characterizing the degrees of freedom and constraints involved in active suspension design. Constraints on the achievable response have been investigated from "invariant points", transfer-function and energy/passivity point of view in [15, 16, 17, 18, 19]. In [18], a complete set of constraints was derived on the road and load disturbance response transfer functions and results on the choice of sensors needed to achieve these degrees of freedom independently were obtained for the quarter-car model. The generalization of these results to half- and full-car models was then presented in [20]. In [21] it was shown that the road and load disturbance responses cannot be adjusted independently for any passive suspension applied to a quarter-car model.

In this study, an automatic suspension system for a quarter car is considered and a fuzzy logic controller is designed when the vehicle is experiencing any road disturbance (i.e. pot holes, cracks, and uneven pavement), the vehicle body should not have large oscillations, and the oscillations should dissipate quickly. The road disturbance is simulated by a step input as a soft road test and rough road as a simulated to real way and the distance between the body mass and simulation mass is output of the system.

## II. QUARTER-CAR MODEL

In this paper, a quarter car model with two degrees of freedom is considered. This model uses a unit to create the control force between body mass and wheel mass.

The motion equations of the car body and the wheel are as follows:

$$\begin{aligned} m_b \ddot{z}_b &= f_a - k_1(z_b - z_w) - c_s(\dot{z}_b - \dot{z}_w) \\ m_w \ddot{z}_w &= -f_a + k_1(z_b - z_w) - k_2(z_w - z_r) \end{aligned} \quad (1)$$

with the following constants and variables are shown in Table I:

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TABLE I  
QUARTER CAR PARAMETERS

Parameters	Symbols	Quantities
Body mass	$m_b$	250 kg
Wheel mass	$m_w$	50 kg
Stiffness of the body	$K_1$	16 kN/m
Stiffness of the wheel	$K_2$	160 kN/m
Stiffness of the damper	$C_s$	1.5 kN.s/m

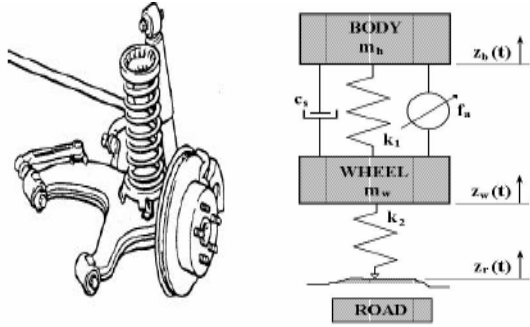


Fig. 1. a Schematic quarter car model b. Quarter-car model

To model the road input let us assume that the vehicle is moving with a constant forward speed. Then the vertical velocity can be taken as a white noise process which is approximately true for most of real roadways.

To transform the motion equations of the quarter car model into a space state model, the following state variables are considered:

$$\underline{X} = [x_1, x_2, x_3, x_4]^T \quad (2)$$

where  $x_1 = z_b - z_w$  is the body displacement,  $x_2 = z_w - z_r$  is the wheel displacement,  $x_3 = \dot{z}_b$  is the absolute velocity of the body, and  $x_4 = \dot{z}_w$  is the absolute velocity of the wheel.

Then the motion equations of the quarter car model for the active suspension can be written in state space form as follows:

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B} \underline{f}_a + \underline{F} \dot{z}_r \quad (3)$$

With

$$\underline{A} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{m_b} & 0 & -\frac{c_s}{m_b} & \frac{c_s}{m_b} \\ \frac{k_1}{m_w} & -\frac{k_2}{m_w} & \frac{c_s}{m_w} & -\frac{c_s}{m_w} \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_b} \\ \frac{1}{m_w} \end{bmatrix}, \quad \underline{F} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

where  $f_a$  is the control force,  $z_r$  is the road input displacement.

### III. FUZZY LOGIC CONTROLLER

The fuzzy logic controller used in the active suspension has three inputs: body acceleration  $\ddot{z}_b$ , body velocity  $\dot{z}_b$ , body deflection velocity  $\dot{z}_b - \dot{z}_w$  and one output: desired actuator force  $f_a$ . The control system itself consists of three stages: fuzzification, fuzzy inference machine and defuzzification.

The fuzzification stage converts real-number (crisp) input

values into fuzzy values while the fuzzy inference machine processes the input data and computes the controller outputs in cope with the rule base and data base. These outputs, which are fuzzy values, are converted into real-numbers by the defuzzification stage.

A possible choice of the membership functions for the four mentioned variables of the active suspension system represented by a fuzzy set is as follows:

for body deflection velocity  $\dot{z}_b - \dot{z}_w$

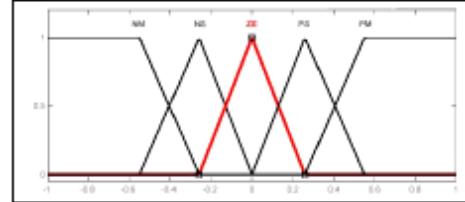


Fig. 2 Membership function for body deflection velocity

for body velocity  $\dot{z}_b$

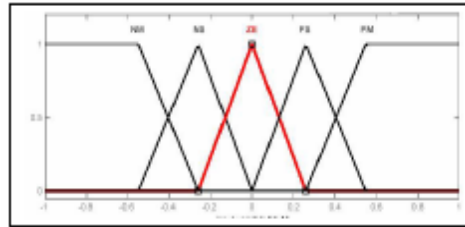


Fig. 3 Membership function for body velocity

for body acceleration  $\ddot{z}_b$

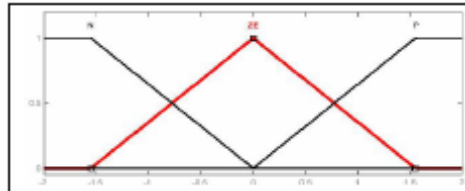


Fig. 4 Membership function for body acceleration

for desired actuator force  $f_a$

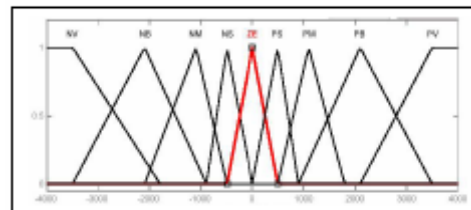


Fig. 5 Membership function for desired actuator force

The abbreviations used correspond to:

NV is Negative Very Big, NB is Negative Big, NM is Negative Medium, NS is Negative Small, ZE is Zero, PS is Positive Small, PM is Positive Medium, PB is Positive Big and PV is Positive Very Big.

The rule base used in the active suspension system showed in Table II with fuzzy terms derived by the designer's knowledge and experience. The table consists of two parts, the left part has zero body acceleration so the control action was chosen to minimize the relative and the absolute body velocities only. The second part, the body acceleration has positive or negative values so important to modify the control action to minimize it also, which will lead to minimize the suspension working space (SWS) and the dynamic tyre load (DTL).

TABLE II  
RULE BASE

$\dot{z}_b - \dot{z}_w$	$\dot{z}_b$	$\ddot{z}_b$	$\ddot{z}_w$	$\dot{z}_b - \dot{z}_w$	$\dot{z}_b$	$\ddot{z}_b$	$\ddot{z}_w$
PM	PM	ZE	ZE	PM	PM	P or N	NS
PS	PM	ZE	NS	PS	PM	P or N	NM
ZE	PM	ZE	NM	ZE	PM	P or N	NB
NS	PM	ZE	NM	NS	PM	P or N	NB
NM	PM	ZE	NB	NM	PM	P or N	NV
PM	PS	ZE	ZE	PM	PS	P or N	NS
PS	PS	ZE	NS	PS	PS	P or N	NM
ZE	PS	ZE	NS	ZE	PS	P or N	NM
NS	PS	ZE	NM	NS	PS	P or N	NB
NM	PS	ZE	NM	NM	PS	P or N	NB
PM	ZE	ZE	PS	PM	ZE	P or N	PM
PS	ZE	ZE	ZE	PS	ZE	P or N	PS
ZE	ZE	ZE	ZE	ZE	ZE	P or A	ZE
NS	ZE	ZE	ZE	NS	ZE	P or N	NS
NM	ZE	ZE	NS	NM	ZE	P or N	NM
PM	NS	ZE	PM	PM	NS	P or N	PB
PS	NS	ZE	PM	PS	NS	P or N	PB
ZE	NS	ZE	PS	ZE	NS	P or N	PM
NS	NS	ZE	PS	NS	NS	P or N	PM
NM	NS	ZE	ZE	NM	NS	P or N	PS
PM	NM	ZE	PB	PM	NM	P or N	PV
PS	NM	ZE	PM	PS	NM	P or N	PB
ZE	NM	ZE	PM	ZE	NM	P or N	PB
NS	NM	ZE	PS	NS	NM	P or N	PM
NM	NM	ZE	ZE	NM	NM	P or N	PS

For example, the linguistic control rules of the fuzzy logic controller obtained from the table above used in such case are as follows:

**R17:**

IF ( $\dot{z}_b - \dot{z}_w = \text{PS}$ ) AND ( $\dot{z}_b = \text{NS}$ ) AND ( $\ddot{z}_b = \text{ZE}$ ) THEN ( $f_a = \text{PM}$ )

**R22:**

IF ( $\dot{z}_b - \dot{z}_w = \text{PS}$ ) AND ( $\dot{z}_b = \text{NM}$ ) AND ( $\ddot{z}_b = \text{ZE}$ ) THEN ( $f_a = \text{PM}$ )

Thus the rules of the controller have the general form of:

**Ri:**

IF ( $\dot{z}_b - \dot{z}_w = A_i$ ) AND ( $\dot{z}_b = B_i$ ) AND ( $\ddot{z}_b = C_i$ ) THEN ( $f_a = D_i$ )

where  $A_i$ ,  $B_i$ ,  $C_i$  and  $D_i$  are labels of fuzzy sets representing the linguistic values of  $\dot{z}_b - \dot{z}_w$ ,  $\dot{z}_b$ ,  $\ddot{z}_b$  and  $f_a$  respectively, which are characterised by their membership functions.

The output of the fuzzy controller is a fuzzy set of control. As a process usually requires a non-fuzzy value of control, a method of defuzzification called "centre of gravity method" (COG), is used here [22]:

$$f_a = \frac{\int_{F_a} f * \mu_D(f) . df}{\int_{F_a} \mu_D(f) . df} \quad (4)$$

where  $\mu_D(f)$  is corresponding membership function. The actuator force ( $f_a$ ) is chosen to give  $\pm 6$  kN as a maximum and minimum values, [23].

#### IV. SIMULATION RESULTS

In the simulation results of two types of controllers PID and Fuzzy control are compared for two kinds of road conditions, namely smooth and random road excitations. The quarter car model parameters are shown in Table I and the simulation results are shown in Figs. 6-7 shows comparison between the body acceleration, the suspension working space and dynamic tyre load for smooth road showing good results in ride comfort and handling characteristics for active suspension. It can clear that there is an improvement in ride comfort performance. The proposed fuzzy logic control gives percentage reduction in body acceleration, suspension working space and dynamic tyre load amplitudes less than PID by 65%, 19.35% and 30.43% respectively. Fig. 7 represents the controller signal for both of PID and Fuzzy controllers actions. It is apparent that, the signal amplitude of fuzzy control is less than the half of the PID signal for the same road.

Simulation results of active suspension controlled by PID and Fuzzy control are compared in Figs. 8-9 for real road roughness, Fig. 8 show the comparison between the body acceleration, suspension working space and dynamic tyre load results respectively. It can be noticed that the fuzzy logic control provided good results than PID. The fuzzy logic control gives percentage reduction in body acceleration, suspension working space and dynamic tyre load amplitudes less than the PID by 61.3%, 6.9% and 24.24% respectively and also Fig. 9 assures the results of Fig. 7 which will be effective in designing smaller actuator size when we use fuzzy control as an implemented controller in the suspension system. Fig. 10 shows the road input power spectral density. It provides information on the frequency range at which the majority of the output occurs. It clears the majority of the response occurs at around 1Hz.

#### V. CONCLUSION

The idea of PID and Fuzzy control for controlling active suspension was presented in this paper. Simulation results showed Fuzzy control is very effective and can be used in vehicles that will be manufactured in future. In this paper, the new active suspension control system is proposed to achieve both ride comfort and good handling. The results of the active suspension system based on the fuzzy logic controller also show the improved stability of the one-quarter-car model.

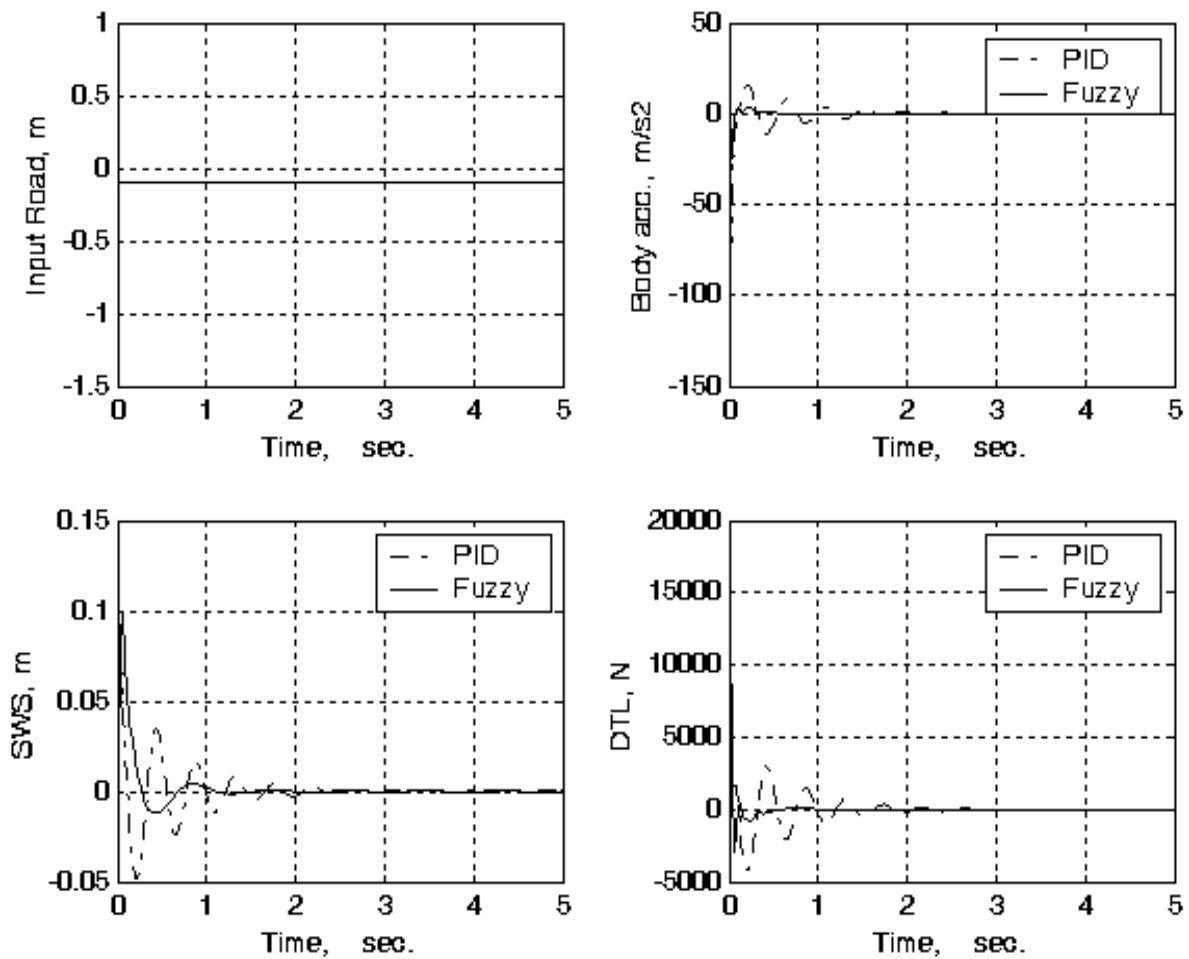


Fig. 6 The effect of PID and Fuzzy control systems on smooth road

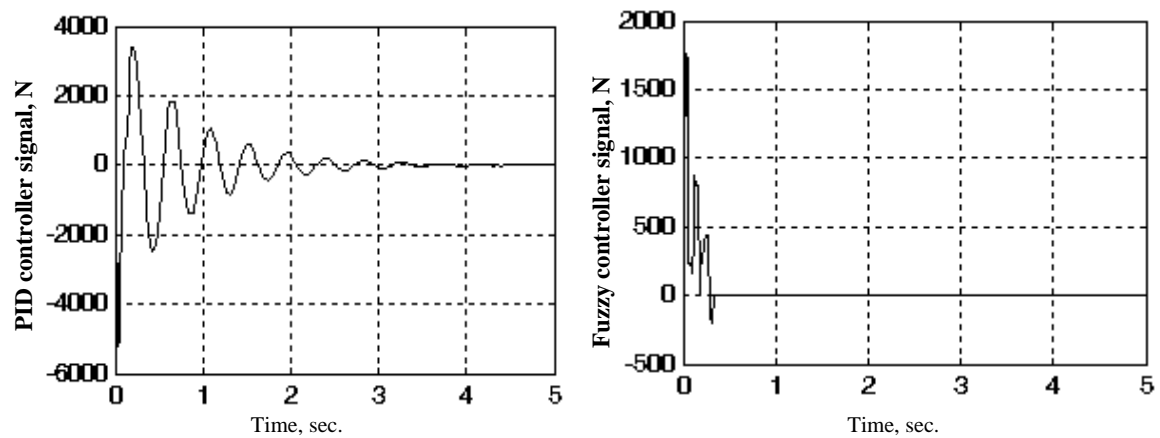


Fig. 7 PID and Fuzzy control signal for smooth road (step response)

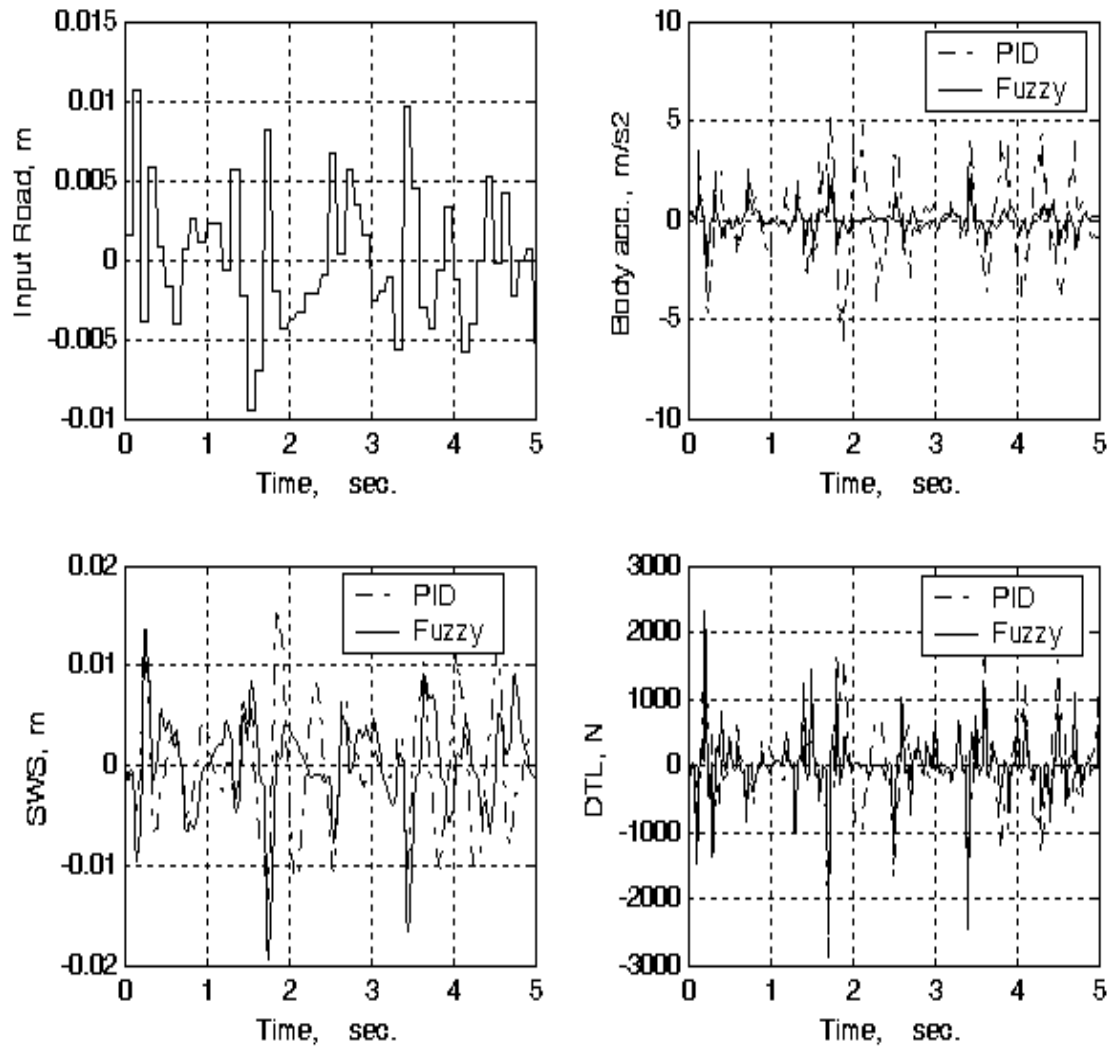


Fig. 8 The effect of PID and Fuzzy on real road roughness

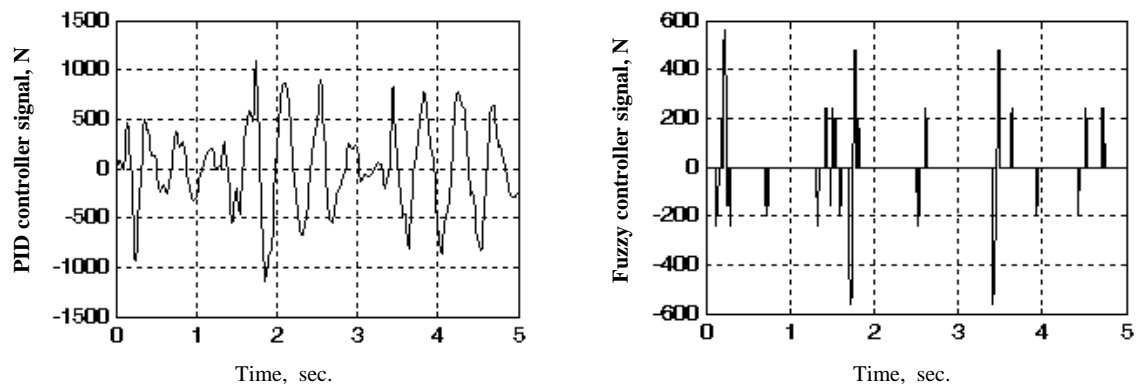


Fig. 9 PID and Fuzzy control signal for real road roughness

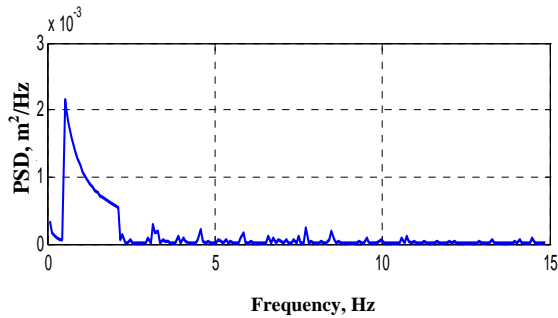


Fig. 10 The Road Power Spectral Density

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