

# A Numerical Approach for Static and Dynamic Analysis of Deformable Journal Bearings

D. Benasciutti, M. Gallina, M. Gh. Munteanu, F. Flumian

**Abstract**—This paper presents a numerical approach for the static and dynamic analysis of hydrodynamic radial journal bearings. In the first part, the effect of shaft and housing deformability on pressure distribution within oil film is investigated. An iterative algorithm that couples Reynolds equation with a plane finite elements (FE) structural model is solved. Viscosity-to-pressure dependency (Vogel-Barus equation) is also included. The deformed lubrication gap and the overall stress state are obtained. Numerical results are presented with reference to a typical journal bearing configuration at two different inlet oil temperatures. Obtained results show the great influence of bearing components structural deformation on oil pressure distribution, compared with results for ideally rigid components. In the second part, a numerical approach based on perturbation method is used to compute stiffness and damping matrices, which characterize the journal bearing dynamic behavior.

**Keywords**—Journal bearing, finite elements, deformation, dynamic analysis

## I. INTRODUCTION

JOURNAL bearings are machine elements in which the applied force is entirely supported by an oil film pressure. They are used in many different engineering applications, for example as supports of rotating shafts. They are considered superior to roll-bearings because of their higher load-bearing capacity, higher operating angular speed, lower cost and easier manufacturing. Furthermore, a proper design can assure very large service lives. The early studies on the fluid-dynamic behavior of journal bearings based on the numerical solution of Reynolds equation date back to the fifties, thanks to the work of Raimondi and Boyd (R&B) [1], [2]. They summarized results in useful dimensionless charts ready for design, which are nowadays accepted also in code standards [3].

Raimondi and Boyd analysis is based on some simplifying assumptions, as the hypothesis of constant viscosity of oil film, independency of viscosity on pressure and finally the postulation of perfectly rigid components (shaft and bushing). Such assumptions, however, can be somewhat oversimplified, considering for example that deformation of journal bearing components under imposed oil film pressure is expected to produce a change in lubrication gap and thus a modification in the resultant pressure distribution. Moreover, also the assumption of constant viscosity and its independence from pressure should be critically reviewed, as it is experimentally known how viscosity depends, other than temperature, also on pressure, as summarized by many constitutive models [4].

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It would be then of interest to investigate in more detail the correlation existing between all the above-mentioned aspects and journal bearing performance and design.

In light of the above considerations, the present paper aims to present a general numerical approach to study the static and dynamic behavior of hydrodynamic radial journal bearings, by including in the analysis the effect of the aforementioned aspects.

In the first part, attention will focus on computation of pressure distribution as a function of temperature variation within lubrication gap, viscosity-to-pressure sensitivity (according to the Vogel-Barus constitutive model [4]) and components flexibility. An iterative algorithm using a finite difference scheme will be developed to solve the Reynolds equation, based on the deformed lubrication gap calculated by a coupled structural finite elements (FE) analysis. The numerical approach will compute the pressure distribution and the local stress field including shaft and bushing structural deformation. Results will clearly emphasize the strong influence of component flexibility on journal bearing performance, with a significant reduction of peak pressure caused by components deformation.

In the second part of the paper, the dynamic behavior of journal bearing will be also investigated. A numerical procedure implementing the so-called "perturbation approach" will be developed to compute the stiffness and damping matrices characterizing the dynamic behavior of hydrodynamic journal bearings. Numerical examples considering a typical journal bearing configuration will be presented.

## II. JOURNAL BEARINGS: BASIC CONCEPTS

A typical configuration of radial journal bearing under a vertical load (see Fig. 1) consists on a shaft rotating inside a fixed support (choke), where it is usually fitted a bush. The nominal radial clearance between shaft (diameter  $d=2r$ ) and choke (diameter  $D=2R$ ) is  $c=R-r$ .

The steady-state response of a journal bearing is governed by the fundamental equation of lubrication theory (Reynolds equation) [5]

$$\frac{1}{r^2} \frac{\partial}{\partial \theta} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = \frac{6U}{r} \frac{dh}{d\theta} \quad (1)$$

where  $h(\theta) = c - e \cos \theta$  is the oil film thickness as a function of angular coordinate  $\theta$ , symbol  $e$  is the eccentricity,  $U = \omega r$  is the tangential velocity of shaft,  $\omega$  is its angular velocity,  $p(\theta)$  is the resultant oil pressure distribution,  $\mu$  is the oil dynamic viscosity. The numerical solution of Reynolds equation gives the pressure distribution  $p(\theta)$  within the lubrication gap and the system operating parameters (eccentricity, minimum lubrication gap, force resultant components, etc.).

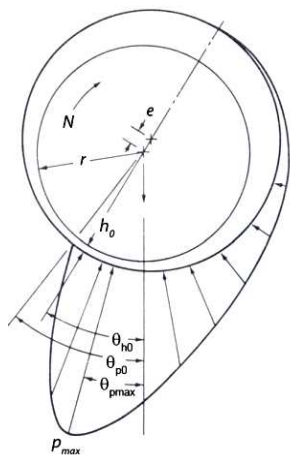


Fig. 1 Sketch of a hydrodynamic journal bearing

Due to the relative velocity between shaft and support, the oil generates a pressure  $p(\theta)$  over the attitude angle  $\beta$ , where  $p_{max}$  is the peak pressure that occurs at angle  $\theta_{pmax}$ . The system moves in a new equilibrium configuration, where the eccentricity  $e$  characterizes the position of shaft axis with respect to the fixed support axis, along direction defined by angle  $\theta_{h0}$  (which also identifies the direction of minimum oil thickness  $h_0$ ).

Several design charts are available in literature [1], [2] which provide journal bearing operation parameters as a function of Sommerfeld number  $S=(r/c)^2 (\mu N/p_m)$ , defined in terms of shaft radius  $r$  and rotational speed  $N$ , while  $p_m=F/(LD)$  is the average (specific) pressure defined as the ratio of the applied radial force  $F$  and the nominal projected area ( $L$  is the length of journal bearing). Such charts were determined by R&B through numerical solution of Reynolds equation under the hypothesis of constant temperature (and thus viscosity) of lubrication film and also under the assumption of perfectly rigid components (shaft and support).

An improvement of the analysis can be obtained by including in Reynolds equation a more sophisticated constitutive model for the viscosity. For example, a coupled temperature-pressure dependency can be summarized by the experimentally determined Vogel-Barus equation  $\mu=\mu_0 \exp(\alpha p)$ , in which  $\mu_0$  is a pressure-independent viscosity term (only function of temperature) and  $\alpha$  is a sensitive parameter related to oil film pressure (typical values are  $\alpha=0.01 \div 0.02 \text{ MPa}^{-1}$ ). In accordance to this constitutive model, an increase in dynamic

viscosity occurs for high pressures, with a solid-like behavior for very high pressures.

This effect, well-known in elasto-hydrodynamic studies (e.g. lubricated contacts), has not been actually investigated in the field of journal bearings.

A further improvement in journal bearing analysis can be obtained by including in the solution of Reynolds equation the deformed shape of lubrication gap caused by deformation of shaft and support under imposed oil pressure  $p(\theta)$ .

TABLE I  
GEOMETRICAL DIMENSIONS USED IN NUMERICAL SIMULATIONS

$d$ mm	$D$ mm	$L$ mm	$F$ kN	$N$ rpm	$p_m$ MPa
500	500.5	300	3600	65	24

This paper will present a general numerical approach to compute the pressure distribution by also including the above mentioned effects. A typical journal bearing configuration (see Table I), operating at two different inlet oil temperatures ( $T_{in}=40$  and  $70 \text{ }^\circ\text{C}$ ), will be investigated. A viscosity-temperature curve typical of an oil ISO VG 680 will be used in all simulations [4].

### III. STEADY-STATE ANALYSIS

In numerical simulations a plane model for the journal bearing is adopted. In the first part of this paper the hypotheses used are rigid components and viscosity function of both temperature and pressure according to the Vogel-Barus model. In the second part, pressure effect will not be considered, while shaft and support deformation will be included into the analysis.

#### A. Temperature and Pressure Effect (Rigid Components)

Reynolds equation (1) is solved by using the finite difference method based on central difference scheme. The unknown function in (1) is the pressure  $p(\theta)$  that, upon integration, gives the resultant applied load  $F$ , which is a given input.

It is worth noting that the problem is actually not linear for several reasons. Although the pressure  $p(\theta)$  is the unknown function, equation (1) does not explicitly depends on load  $F$  (i.e. the resultant of pressure), but on eccentricity through the lubrication gap  $h(\theta)=c-e\cos(\theta)$ . Several iterations (Newton-Raphson method was used) are then required to first impose the input force  $F$  (as resultant of pressure) and to find the appropriate pressure distribution  $p(\theta)$  that solves (1).

TABLE II  
OVERALL COMPARISON OF RESULTS FROM NUMERICAL SIMULATIONS FOR RIGID COMPONENTS

Configurations	$T_{in}$ $^\circ\text{C}$	$T_m$ $^\circ\text{C}$	$T_{out}$ $^\circ\text{C}$	$\mu$ Pa·s	$S$ -	$e$ mm	$p_{max}$ MPa	$\theta_{pmax}$ deg	$h_0$ mm	$\theta_{h0}$ deg
R&B ( $L/D \sim \infty$ , 2D analysis)						0.2352	87.30	15.50	0.0148	-
$T_m$ cost. $\alpha=0$	$\uparrow$	$\uparrow$	$\uparrow$	0.1678	0.00786	0.2335	82.26	15.62	0.0165	26.29
$T_m$ cost. $\alpha=0.01$	40	60	80	$\downarrow$	$\downarrow$	0.2286	83.74	15.03	0.0214	27.03
$T_{in}-T_{out}$ lin. $\alpha=0$	$\downarrow$	$\downarrow$	$\downarrow$	Not defined	Not defined	0.2392	83.17	22.82	0.0108	32.15
$T_{in}-T_{out}$ lin. $\alpha=0.01$						0.2350	80.18	22.43	0.0150	33.76
JB2 R&B ( $L/D \sim \infty$ , 2D analysis)	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	0.2447	205.50	6.60	0.0053	-

$T_m$ cost.	$\alpha = 0$	70	80	90	0.0655	0.00298	0.2440	136.92	10.27	0.0060	16.27
$T_m$ cost.	$\alpha = 0.01$	↓	↓	↓	↓	↓	0.2412	149.54	9.34	0.0088	16.67
$T_{in}-T_{out}$ lin.	$\alpha = 0$				Not	Not	0.2453	151.08	10.85	0.0047	16.18
$T_{in}-T_{out}$ lin.	$\alpha = 0.01$				defined	defined	0.2431	173.29	9.80	0.0069	16.47

Secondly, the force-eccentricity relationship  $F-e$  is highly non-linear, especially for eccentricity values  $e$  approaching the nominal radial clearance  $c$ . Another source of non-linearity is that negative pressure values must be set to zero during the iterative process.

To evaluate the effect of temperature on viscosity, and consequently on pressure distribution, the journal bearing configuration in Table I was studied at two operating conditions (JB1, JB2) characterized by two different inlet temperatures ( $T_{in}=40, 70$  °C). Two hypotheses are then adopted to compute the pressure-independent viscosity term  $\mu_0$  as a function of oil temperature: in the first, using an average constant oil temperature  $T_m$  resulting by a thermal balance inside the oil film (as in R&B approach), in the second using, as a first approximation, a linear temperature variation from inlet value  $T_{in}$  to the outlet value  $T_{out}$  (that has been calculated by previous thermal balance). Note that in both cases the same average oil film temperature  $T_m$  is obtained.

For both temperature distributions within lubrication gap (constant  $T_m$ , linear  $T_{in}-T_{out}$ ), the Vogel-Barus equation has been implemented with two different cases ( $\alpha=0$  and  $\alpha=0.01$ ).

Table II shows an overall comparison of obtained results, while Fig. 2 and Fig. 3 compare the pressure distribution under an imposed vertical load, with a linear temperature variation within oil film and assuming different pressure sensitivity values for viscosity.

The effect of temperature variation of oil film is first analyzed. Referring to JB1 configuration in Table I, a negligible difference is observed between the case of constant and linearly varying temperature, for both  $\alpha=0$  and  $\alpha=0.01$  values. Instead, larger differences (with a 10-12% increase of  $p_{max}$  value) are observed for JB2 configuration, considering both  $\alpha=0$  and  $\alpha=0.01$  values.

This emphasizes how the variation of oil film temperature could have some effect on pressure distribution, at least for high temperature values. Considering the viscosity-temperature strong correlation, this seems to confirm that pressure distribution is more sensitive to a change of small (rather than high) viscosity values within lubrication gap. In any case, the constant temperature assumption used in R&B calculations seems too simplified.

Numerical solutions for constant  $T_m$  and  $\alpha=0$  were also compared with results given by R&B charts, showing a good agreement only for JB1 configuration, while some difference characterizes JB2 configuration. The observed discrepancy can be attributed to the very low Sommerfeld number ( $S=0.00298$ ) characterizing JB2 configuration, which makes difficult using R&B design charts and thus can be source of interpolation errors.

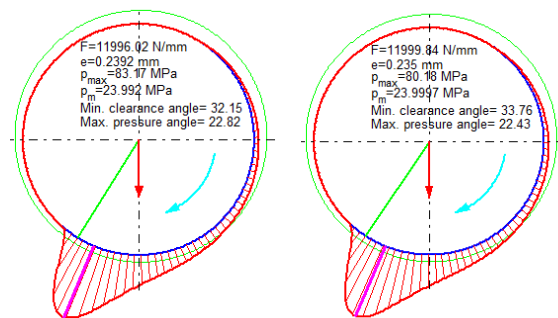


Fig. 2 Results for JB1 configuration (linear temperature variation,  $T_{in}=40^{\circ}\text{C} - T_{out}=80^{\circ}\text{C}$ ), with  $\alpha=0$  (left) and  $\alpha=0.01$  (right)

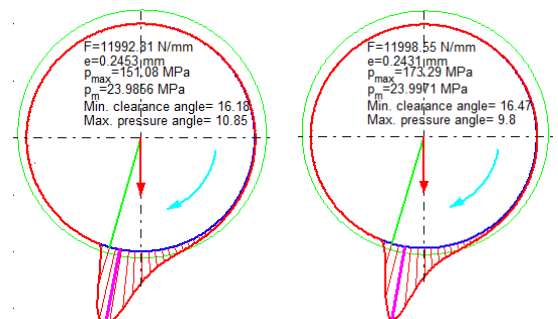


Fig. 3 Results for JB2 configuration (linear temperature variation,  $T_{in}=70^{\circ}\text{C} - T_{out}=90^{\circ}\text{C}$ ), with  $\alpha=0$  (left) and  $\alpha=0.01$  (right)

In general, a non-zero viscosity-to-pressure sensitivity ( $\alpha=0.01$ ) determines a variation in the overall pressure distribution (change of attitude angle  $\beta$ ) and in its maximum value  $p_{max}$ , depending on the general pressure levels attained. Limiting the attention to the case with  $T_{in}-T_{out}$  linear temperature variation, for peak pressures  $p_{max}<100$  MPa (case JB1), the pressure effect is actually negligible, as shown in Fig. 2, with only a small decrease of the maximum pressure of about 3.5%. For larger pressure levels (case JB2), an increment of  $p_{max}$  of about 12% is observed, see Fig. 3. The minimum oil thickness increment ( $h_0=c-e$ ) produced by the pressure effect is relevant in both cases, with a variation respectively of 28% and 32%.

The obtained results can be summarized by saying that, if the influence of pressure on viscosity is taken into consideration, when  $\alpha$  increases the peak pressure  $p_{max}$  increases, while the eccentricity  $e$  decreases. However, the pressure-to-viscosity effect is smaller compared to temperature influence, at least for the maximum pressure peaks encountered in the examples studied. Accordingly, pressure dependence will be intentionally neglected in the rest of this paper.

### B. Effect of Component Deformation (with $\alpha=0$ , $T$ Linear)

In the second part of this work, the pressure distribution will be calculated by considering the real geometry of lubrication gap resulting from component deformation. Pressure values calculated by solving the Reynolds equation (1) are used, as input mechanical loads in a FE model, to compute the geometry of lubrication gap after deformation, which is next used to solve again equation (1) with an iterative analysis scheme.

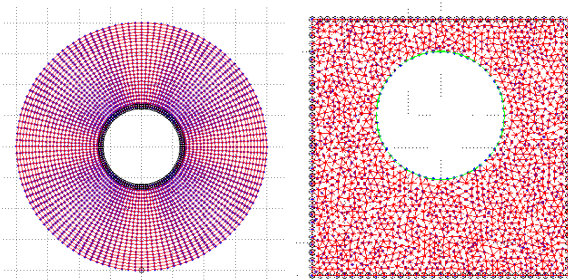


Fig. 4 Finite element model of shaft (left) and support (right)

A fluid-structural coupled numerical procedure was developed in Matlab environment. The first analysis step is the calculation of pressure distribution  $p(\theta)$  and eccentricity  $e$  for the case of not deformable components, by solving Reynolds equation (1). The obtained pressure distribution is next applied as input mechanical load in a plane structural FE model, which gives the relative radial displacements between shaft and support after deformation, and the resulting gap deformation  $g(\theta)$ . A new oil film geometry  $h'(\theta)=c-e\cos(\theta)+g(\theta)$  that incorporates mechanical deformation (thus it differs from the case of perfectly rigid components) can be thus calculated. At second iteration step this updated gap geometry  $h'(\theta)$  is entered in (1) to get a new pressure distribution  $p'(\theta)$  that balances the input force  $F$ . This iterative procedure is repeated until convergence is achieved with respect to an imposed threshold tolerance on the imposed force.

The plane FE models of both shaft and support used in the analysis are shown in Fig. 4. The shaft is modeled by a mapped *mesh* with 4-nodes isoparametric linear elements, while the choke is free meshed using 3-nodes CST triangular elements.

Shaft and choke are loaded by the same oil pressure distribution  $p(\theta)$  applied on the outer and inner surfaces, respectively. Analysis assumes small displacements and a plane strain condition. Material has linear elastic behavior, with properties typical of a structural steel.

It is worth noting that the use of a plane FE model for the structural analysis of a journal bearing requires a special attention in modeling mechanical constraints. In fact, in a real journal bearing the applied load  $F$  and the resulting pressure distribution are actually applied along different longitudinal locations along the shaft axis. Instead, in the plane FE model here adopted the external load  $F$  that balances the oil pressure is replaced by an appropriate constrain on shaft geometry.

For this purpose, the shaft has been modeled with a central hole and all nodes on the inner circumference have imposed zero radial displacements; the choke, instead, has all the external edges constrained. This modeling strategy, however, affects the shaft structural stiffness: a large inner radius determines an anomalous increment of shaft stiffness, while a very small inner hole gives rise to very large deformations and abnormally high reaction forces at constrained nodes. A proper sensitivity analysis has been preliminarily carried out, in order to find the optimal radius of inner hole.

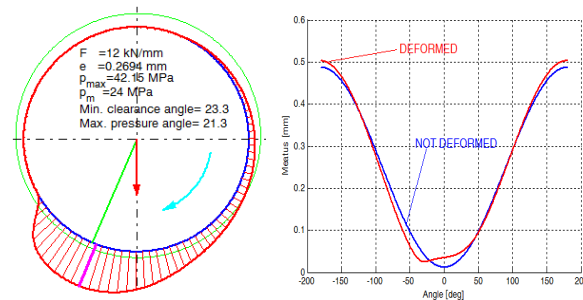


Fig. 5 Pressure distribution for JB1 configuration ( $\alpha=0$ ,  $T_{in}=40^{\circ}\text{C}$  –  $T_{out}=80^{\circ}\text{C}$ ) with deformable components (left). Lubrication gap for rigid and deformable components (right)

The coupled numerical approach was applied to study the JB1 configuration (with  $\alpha=0$  and linear temperature variation in  $T_{in}=40^{\circ}\text{C}$ – $T_{out}=80^{\circ}\text{C}$ ). Fig. 5 (left) shows the result for the case of deformable components. The comparison with the case of rigid components in Fig. 2 (left) clearly emphasizes how component deformation determines a reduction of about 48% (from 83 MPa to 43 MPa) of the maximum peak pressure and, accordingly, an increase in the attitude angle  $\beta$  (under the same applied resultant force  $F$ ). The pressure profile, more uniform than the case of rigid components (R&B solution), seems to support the idea of using the average pressure  $p_m$  as a structural design parameter, as suggested in some design codes [3].

Fig. 5 (right) also compares the geometry of lubrication gap for the case of deformable and rigid components (angles are referred to the position of minimum oil gap  $\theta_{h0}$ ). It is observed that for deformable components the gap is not symmetric and that eccentricity can assume values greater than the nominal clearance, as deformation can increase the gap between shaft and support.

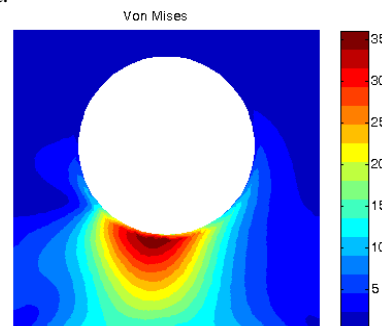


Fig. 6 Von Mises stress distribution in support (MPa units)

For what concerns the calculated mechanical stresses, Fig. 6 show the von Mises stress distribution in the support (choke). The inner surface of support undergoes to stress state that is approximately equibiaxial, where the maximum hoop and radial stresses are comparable to the maximum applied pressure ( $\sigma_\theta \approx \sigma_r = p_{\max} = 43$  MPa), while the axial stress under plane strain is  $\sigma_z = \nu(\sigma_\theta + \sigma_r)$  where  $\nu$  is the Poisson ratio. Compared to the case of perfectly rigid components, this explains the relatively small value ( $\sim 35$  MPa) of von Mises stress calculated in the choke, which is actually comparable with static strength properties of materials usually employed in the bush (for instance, white metal generally used as internal coating has a yield stress of about 50 MPa [6]).

#### IV. DYNAMIC ANALYSIS

The dynamic behavior of rotating shaft supported by journal bearings is strongly influenced by the hydrodynamic forces produced in lubricant film that oppose to shaft movement. Determination of rotor dynamics then requires full characterization of the dynamic response of bearing lubricant film, which is a non-linear function of position and velocity of journal center.

In the dynamic analysis of a rotor-bearing configuration, a simple spring-dashpot model is usually adopted to account for journal bearing contribution [5], [7]. With small displacements increments ( $\delta x, \delta y$ ) and small velocities increments ( $\delta \dot{x}, \delta \dot{y}$ ) in the vicinity of the journal bearing static equilibrium position, a linearized relationship, between the incremental oil-film forces  $\delta F_i$  and journal displacements and velocities increments that cause them, can be written as [5], [7]

$$\begin{pmatrix} \delta F_x \\ \delta F_y \end{pmatrix} = - \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{pmatrix} \delta x \\ \delta y \end{pmatrix} - \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \begin{pmatrix} \delta \dot{x} \\ \delta \dot{y} \end{pmatrix} \quad (2)$$

where  $k_{ij} = (\delta F_i / \delta x_j)$  and  $c_{ij} = (\delta F_i / \delta \dot{x}_j)$  are the linear stiffness and damping coefficients, respectively.

A classical "perturbation method" is followed to compute the increase in oil film forces resulting from a departure (perturbation) from the static equilibrium position. A journal bearing configuration, characterized by given displacement ( $u, v$ ) and velocities ( $\dot{u}, \dot{v}$ ) of journal centre, is first assigned. The reference pressure distribution  $p(\theta)$  and oil film forces  $F_x$  and  $F_y$  are then calculated by Eq. (3), the Reynolds equation in dynamic regime

$$\mu \frac{h^3}{r^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\mu} \frac{\partial p}{\partial \theta} \right) + \frac{3h^2}{r^2} \frac{\partial h}{\partial \theta} \frac{\partial p}{\partial \theta} = \frac{6U\mu}{r} \frac{\partial h}{\partial \theta} + 12 \frac{\partial h}{\partial t} \quad (3)$$

which explicitly depends also on the time derivative of lubrication gap  $h(\theta) = -\dot{u} \cos \theta - \dot{v} \sin \theta$ . Independent displacement and velocity perturbations are next applied and the corresponding force increments calculated. Solution of (3) gives the increased pressure distribution (say  $p + \delta p$ ), and

therefore the increased resultant of oil film forces  $F_x + \delta F_x$  and  $F_y + \delta F_y$ , with respect to the reference equilibrium position, for shaft displacement and velocity increments ( $\delta x, \delta y, \delta \dot{x}, \delta \dot{y}$ ). Thus stiffness and damping coefficients can be thus determined, as for example:

$$[k] \approx \begin{bmatrix} \frac{\delta F_x}{\delta u} & \frac{\delta F_x}{\delta v} \\ \frac{\delta F_y}{\delta u} & \frac{\delta F_y}{\delta v} \end{bmatrix} \quad (4)$$

To include also the contribution of the structure deformation into stiffness and damping matrices, in the above expression the displacement increments are substituted by  $\delta u_o$  and  $\delta v_o$ , total displacement increments of the bearing center. Absolute displacements  $u_o$  and  $v_o$  of the bearing center and their increments  $\delta u_o$  and  $\delta v_o$  are found as explained in Section III point B. A similar approach is used to determine damping matrix.

Stiffness  $[k]$  and damping  $[c]$  matrices characterize the dynamic behavior of journal bearing and they enter into the dynamic equilibrium equations of the shaft. It is worth noting that, due to the non-linear nature of the Reynolds equation (3), both matrices explicitly depend on the assigned journal bearing displacement ( $u, v$ ) and velocities ( $\dot{u}, \dot{v}$ ), that is they have to be interpreted as tangent matrices. Therefore, a transient dynamic analysis of a rotor supported by journal bearings is non-linear and  $[k]$ ,  $[c]$  matrices must be calculated at every time integration step. In addition,  $[k]$  and  $[c]$  are in general not symmetric.

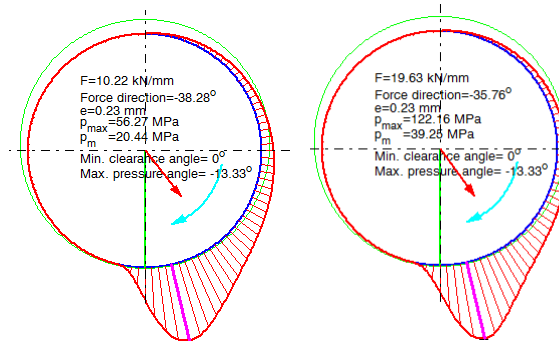


Fig. 7 Pressure distribution calculated for  $u=0, \dot{u}=0, v=0.23$  mm and  $\dot{v}=0$  (left) and  $\dot{v}=0.5$  mm/s (right)

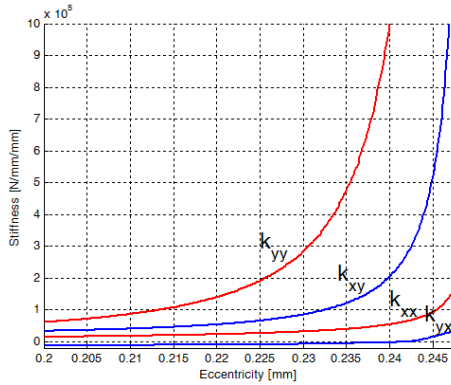


Fig. 8 Stiffness coefficients (units N/mm/mm) for JB1 configuration versus eccentricity (for  $u=0$ ,  $\dot{u}=0$ ,  $\dot{v}=0$ )

A numerical algorithm has been specifically developed to compute stiffness and damping matrices by the perturbation method previously described. The procedure has been applied to characterize different journal bearing configurations.

An example of calculated pressure distribution for JB1 configuration for two different velocities,  $\dot{v}=0$  and  $\dot{v}=0.5$  mm/s, is shown in Fig. 7. The calculated stiffness and damping coefficients are reported in Fig. 8 and Fig. 9: a high non-linear dependence on the eccentricity  $e$  is observed.

#### V. CONCLUSIONS

The present papers developed a numerical procedure for the steady state and dynamic analysis of hydrodynamic radial journal bearing. Influence of temperature and pressure on viscosity and thus on resultant pressure distribution were studied. A mechanical plane finite element model, coupled with solution of Reynolds equation, was also developed to study journal bearing structural behavior and its influence on pressure distribution. Finally, a perturbation approach was implemented to evaluate stiffness and damping coefficients.

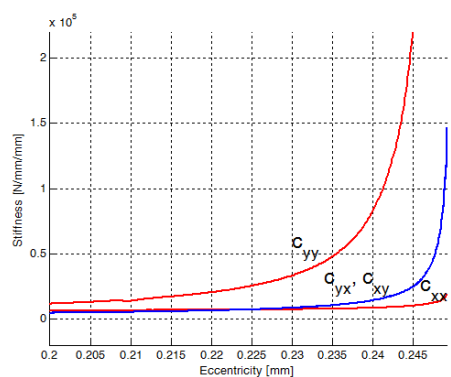


Fig. 9 Damping coefficients (units Ns/mm/mm) for JB1 configuration versus eccentricity (for  $u=0$ ,  $\dot{u}=0$ ,  $\dot{v}=0$ )

The main findings of the work can be summarized as follows:

- temperature increase was shown to give a decrease of

- attitude angle  $\beta$  and an increase in pressure peak;
- an increase of viscosity-to-pressure sensitivity ( $\alpha$  value) gives a general increase of peak pressure, at least for pressure peaks greater than about 100 MPa;
- temperature effect was shown to be generally greater than pressure effect;
- component deformation gives a more uniform pressure distribution, with a reduced peak pressure compared to the case of ideally rigid components;
- stiffness and damping coefficient were calculated and a high non-linear trend with journal bearing eccentricity  $e$  was observed.

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