

# Three-phases Model of the Induction Machine Taking Account the Stator Faults

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**Abstract**—In this work we present the modelling of the induction machine, taking into consideration the stator defects of the induction machine. It is based on the theory of electromagnetic coupling of electrical circuits. In fact, for the modelling of stationary defects such as short circuit between turns in the same phase, we introduce only in the matrix the coefficients of resistance and inductance of stator and in the mutual inductance stator-rotor. These coefficients take account the number of turns in short-circuit deducted from the total number of turns in the same phase; in this way we obtain the number of useful turns. In addition, all these faults involved, will be used for the creation of the database that will be used to develop an automated system failures of the induction machine.

**Keywords**—Asynchronous machine, Indicatory Values Stator faults, Multi-turns Model, Three-phases Model.

## I. INTRODUCTION

THE online diagnosis rotor and stator faults in the induction machines in order to reach a predictive maintenance have prompted the researchers to develop various techniques. The work in their majority is based on the signature (harmonic analysis) indicatory values such as current, torque using the theory of rotating fields and electrical circuits.

Moreover, the analysis methods of stator defects using the structural parameters of knowledge model to detect and locate defects. The key point to ensure the effectiveness of these methods is to choose a model of knowledge [1, 2]. Indeed, the type of defect that has to be detected will be based on the model used [1, 2]. Initial work relating parameter estimation began with relatively simple methods (model of Park for example) [3, 4]. The next step is therefore necessarily to pass to a more knowledge model of machine, while retaining the ability to identify the desired parameter. These models can be the three phases model [5], which do not on the assumption of a machine magnetically balanced, or models with 'n' phases (multi-turns model) [6, 7, 8], which can reflect the operating of the machine over a broad frequency range. On the other hand, it should be noted the need for a priori knowledge of the specific model of the induction machine. In addition, the aging and depending on the environment, the representative model of the operating of the machine change. In addition, the diversity of defects (stator, rotor and supply defects) of the induction machine, they can not be obtained by the same

mathematical model. Indeed, every failure should be modelled separately by its own model [1, 2, 3, 4].

## II. MODELLING INDUCTION MACHINE IN CASE OF STATOR DEFECTS

To modelling the defects the short-circuits between coils, we will present another method for modelling of the induction machine, taking into account the changing parameters such as resistors and inductors ie, the matrix is stator resistance and the matrix of stator inductors are variables [5, 8, 9]. This model requires a precise and rigorous study of faults signatures of the induction machine.

In addition, the conventional modelling of a three-phase induction machine to the stator and the rotor wound (if the machine is cage, we can consider the winding-phase equivalent), based on the classics assumptions. Under these assumptions, the machine can be modelled by the following equations:

$$[U_s] = [R_s][I_s] + [P\phi_s] \quad (1)$$

$$[0] = [R_r][I_r] + [P\phi_r] \quad (2)$$

$$[\phi_s] = ([M_{ss}] + [L_{sf}])[I_s] + [M_{sr}][I_r] \quad (3)$$

$$[\phi_r] = [M_{rs}] + ([M_{rr}] + [L_{rf}])[I_r] \quad (4)$$

Where

- Prefers to the differential operator

$$\text{- the variables } [U_s] = \begin{bmatrix} u_{sa} \\ u_{sb} \\ u_{sc} \end{bmatrix}, [I_s] = \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix}, [\phi_s] = \begin{bmatrix} \phi_{sa} \\ \phi_{sb} \\ \phi_{sc} \end{bmatrix}$$

represent the voltages, the currents, and the stator flux.

$$\text{The variables } [I_r] = \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix}, [\phi_r] = \begin{bmatrix} \phi_{ra} \\ \phi_{rb} \\ \phi_{rc} \end{bmatrix} \text{ present the currents}$$

and the rotor flux.

Either  $N_s$  is the number of turns in healthy regime of the induction machine. A short-circuit stator lead to a decrease in the number of turns of each stator phase.

We definite the coefficients of short-circuit as following:

Coefficient of short-circuit on the 1<sup>st</sup> phase of stator

$$k_{sa} = \frac{N_{cc1}}{N_s}$$

Coefficient of short-circuit on the 2<sup>nd</sup> phase of stator

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$$k_{sb} = \frac{N_{cc2}}{N_s}$$

Coefficient of short-circuit on the 3<sup>rd</sup> phase of stator

$$k_{sc} = \frac{N_{cc3}}{N_s}$$

The number of turns in short-circuit:  $N_{cc}$

The number of useful turns for the three stator phases, is then given by:

$$N_1 = N_s - N_{cc1} = (1 - k_{sa})N_s = f_{sa}N_s$$

$$N_2 = N_s - N_{cc2} = (1 - k_{sb})N_s = f_{sb}N_s$$

$$N_3 = N_s - N_{cc3} = (1 - k_{sc})N_s = f_{sc}N_s$$

The matrixes  $[R_s]$ ,  $[L_{sf}]$ ,  $[M_{ss}]$ ,  $[M_{sr}]$  et  $[M_{rs}]$  depend of three coefficients  $f_{sa}$ ,  $f_{sb}$ ,  $f_{sc}$ . The inductors are given by the following terms:

$$[L_{sf}] = \begin{bmatrix} f_{sa}^2 L_{sf} & 0 & 0 \\ 0 & f_{sb}^2 L_{sf} & 0 \\ 0 & 0 & f_{sc}^2 L_{sf} \end{bmatrix} \quad (5)$$

$$[M_{ss}] = M_s \begin{bmatrix} f_{sa}^2 & -f_{sa}f_{sb}/2 & -f_{sa}f_{sc}/2 \\ -f_{sa}f_{sb}/2 & f_{sb}^2 & -f_{sc}f_{sb}/2 \\ -f_{sa}f_{sc}/2 & -f_{sc}f_{sb}/2 & f_{sc}^2 \end{bmatrix} \quad (6)$$

$$[M_{sr}] = M \begin{bmatrix} f_{sa} \cos \theta & f_{sa} \cos(\theta + \frac{2}{3}\pi) & f_{sa} \cos(\theta - \frac{2}{3}\pi) \\ f_{sb} \cos(\theta - \frac{2}{3}\pi) & f_{sb} \cos \theta & f_{sb} \cos(\theta + \frac{2}{3}\pi) \\ f_{sc} \cos(\theta + \frac{2}{3}\pi) & f_{sc} \cos(\theta - \frac{2}{3}\pi) & f_{sc} \cos \theta \end{bmatrix} \quad (7)$$

with  $[M_{sr}] = [M_{rs}]^T$

The resistance of each stator phase is proportional to the number of useful turns. We write:

The matrix of stator résistances  $[R_s]$  is given by

$$[R_s] = R_s \begin{bmatrix} f_{sa} & 0 & 0 \\ 0 & f_{sb} & 0 \\ 0 & 0 & f_{sc} \end{bmatrix} \quad (8)$$

1-When the stator windings are all identical, the machine is balanced, and three factors are equal, then there is the equivalent classical three-phase model.

2 - When the motor is running, the coefficient matrix is not constant; they vary depending on the angle, angular position between the rotor and the stator. This makes the model phase equivalent hardly usable both in control and the monitoring, in the approach that follows a mathematical transformation is

applied to the equations of the previous model in order to make the entire calculable online [5].

### III. THE TRANSFORMATION MATRIX

The magnetic field created by the current in the rotor has the same pulse as that created by the current in stator. Thus, the magnetic field created by a fictitious stator current. The relationship between the current and the fictitious rotor current is given by a mathematical transformation. Using this transformation, all variables rotor (flux and currents) can be changed into new variables with the same pulsation that the variables stator. Thus, all parameters of the model will be independent of the angular position, the conversion is given by the following matrix [5].

$$[T] = \begin{bmatrix} \cos \theta + \frac{1}{2} & \cos(\theta + \frac{2}{3}\pi) + \frac{1}{2} & \cos(\theta - \frac{2}{3}\pi) + \frac{1}{2} \\ \cos(\theta - \frac{2}{3}\pi) + \frac{1}{2} & \cos \theta + \frac{1}{2} & \cos(\theta + \frac{2}{3}\pi) + \frac{1}{2} \\ \cos(\theta + \frac{2}{3}\pi) + \frac{1}{2} & \cos(\theta - \frac{2}{3}\pi) + \frac{1}{2} & \cos \theta + \frac{1}{2} \end{bmatrix} \quad (9)$$

We can show easily that the matrix orthogonal i, e.

$$[T]^{-1} = [T]^T \quad (10)$$

Using the same steps as those of the transformed PARK we get the new three phases model which represent the model of the induction machine in the presence of failures to the stator. Considering the equation (3) using the matrice T as:

$$\begin{aligned} [\phi_s] &= [M_s][I_s] + [M_{sr}][I_r] \\ &= [M_s][I_s] + [M_{sr}][T^{-1}][T][I_r] \end{aligned} \quad (11)$$

It allows to write:

$$[\phi_s] = [M_s][I_s] + [M_{sr}^s][I_r^s] \quad (12)$$

$$\begin{aligned} \text{Where: } [M_{sr}^s] &= [M_{sr}][T^{-1}] \\ [I_r^s] &= [T][I_r] \end{aligned} \quad (13)$$

With this manner the mutual inductors matrix become independent of position angle ( $\theta$ ):

$$[M_{sr}^s] = \begin{bmatrix} f_{sa}M & -f_{sa}M/2 & -f_{sc}M/2 \\ -f_{sb}M/2 & f_{sb}M & -f_{sc}M/2 \\ -f_{sc}M/2 & -f_{sc}M/2 & f_{sc}M \end{bmatrix}$$

The global model by considering all factors of defects between coils becomes:

Flux equations:

$$\begin{aligned}\frac{d\phi_{ra}}{dt} &= \delta \left( f_{sa} i_{sa} - \frac{f_{sb}}{2} i_{sb} - \frac{f_{sc}}{2} i_{sc} \right) - \frac{R_r A}{C} \phi_{ra} - \left( \frac{R_r B}{C} + \frac{\sqrt{3}}{3} w r \right) \phi_{rb} - \left( \frac{R_r B}{C} - \frac{\sqrt{3}}{3} w r \right) \phi_{rc} \\ \frac{d\phi_{rb}}{dt} &= \delta \left( -\frac{f_{sa}}{2} i_{sa} + f_{sb} i_{sb} - \frac{f_{sc}}{2} i_{sc} \right) - \left( \frac{R_r B}{C} - \frac{\sqrt{3}}{3} w r \right) \phi_{ra} - \frac{R_r A}{C} \phi_{rb} - \left( \frac{R_r B}{C} + \frac{\sqrt{3}}{3} w r \right) \phi_{rc} \\ \frac{d\phi_{rc}}{dt} &= \delta \left( -\frac{f_{sa}}{2} i_{sa} - \frac{f_{sb}}{2} i_{sb} + f_{sc} i_{sc} \right) - \left( \frac{R_r B}{C} + \frac{\sqrt{3}}{3} w r \right) \phi_{ra} - \left( \frac{R_r B}{C} - \frac{\sqrt{3}}{3} w r \right) \phi_{rb} - \frac{R_r A}{C} \phi_{rc}\end{aligned}$$

Where

$$A = (l_{rf} + M_r)^2 - \frac{M_r^2}{4} ; \quad B = \frac{M_r l_{rf}}{2} + \frac{3M_r^2}{4} ;$$

$$C = l_{rf}^3 + 3l_{rf}^2 M_r + \frac{9}{4} M_r^2 l_{rf}$$

Stator current equations:

$$\begin{aligned}\frac{di_{sa}}{dt} &= V_{SA} + K_{A1} i_{sa} + K_{A2} i_{sb} + K_{A3} i_{sc} \\ &+ K f_{sa} f_{sb}^2 f_{sc}^2 \left( G \phi_{ra} + \left( \frac{\sqrt{3}}{2} w r - \frac{G}{2} \right) \phi_{rb} - \left( \frac{\sqrt{3}}{2} w r + \frac{G}{2} \right) \phi_{rc} \right) \\ \frac{di_{sb}}{dt} &= V_{SB} + K_{B1} i_{sa} + K_{B2} i_{sb} + K_{B3} i_{sc} \\ &+ K f_{sa}^2 f_{sb} f_{sc}^2 \left( -\left( \frac{\sqrt{3}}{2} w r + \frac{G}{2} \right) \phi_{ra} + G \phi_{rb} + \left( \frac{\sqrt{3}}{2} w r - \frac{G}{2} \right) \phi_{rc} \right) \\ \frac{di_{sc}}{dt} &= V_{SC} + K_{C1} i_{sa} + K_{C2} i_{sb} + K_{C3} i_{sc} \\ &+ K f_{sa}^2 f_{sb}^2 f_{sc} \left( \left( \frac{\sqrt{3}}{2} w r - \frac{G}{2} \right) \phi_{ra} - \left( \frac{\sqrt{3}}{2} w r + \frac{G}{2} \right) \phi_{rb} + G \phi_{rc} \right)\end{aligned}$$

Used Coefficients:

$$V_{SA} = d_1 f_{sb}^2 f_{sc}^2 u_{sa} + d_2 f_{sa} f_{sb} f_{sc}^2 u_{sb} + d_3 f_{sa} f_{sb}^2 f_{sc} u_{sc}$$

$$V_{SB} = d_2 f_{sa} f_{sb} f_{sc}^2 u_{sa} + d_1 f_{sa}^2 f_{sc}^2 u_{sb} + d_3 f_{sa}^2 f_{sb} f_{sc} u_{sc}$$

$$V_{SC} = d_3 f_{sa} f_{sb}^2 f_{sc} u_{sa} + d_2 f_{sa}^2 f_{sb} f_{sc} u_{sb} + d_1 f_{sa}^2 f_{sb}^2 u_{sc}$$

The torque equation is given by:

$$C_e = [(i_b \Phi_c - i_c \Phi_b) - (i_a \Phi_c - i_c \Phi_a) + (i_a \Phi_b - i_b \Phi_a)]$$

#### IV. INTERPRÉTATION OF THE SIMULATION RESULTS OF THE STATOR DEFECTS (SHORT-CIRCUIT BETWEEN COILS)

With regard to the defects of short-circuit between coils, we have seen two cases of failures, where  $N_{cci} = 20$  and  $N_{cci} = 40$  the coils in short- circuit, therefore we will have respectively  $N_{cci} = 140$  and  $N_{cci} = 120$  coils that phase will be useful. The figures.1 and 2 represent the changing parameters (current, speed and torque) by considering these defects. Each defect has been applied in the empty running the machine (at  $t = 0.8$  seconds). We notice that the current after the appearance of default, increases considerably compared to the regime established current (healthy machine). This increase is related to the number of coils in short-circuit. This is obviously the simultaneous decrease of resistance and the reactance phase of the stator. We also noticed a slight disturbance at the speed and torque, and then they will be reinstated after a brief period.

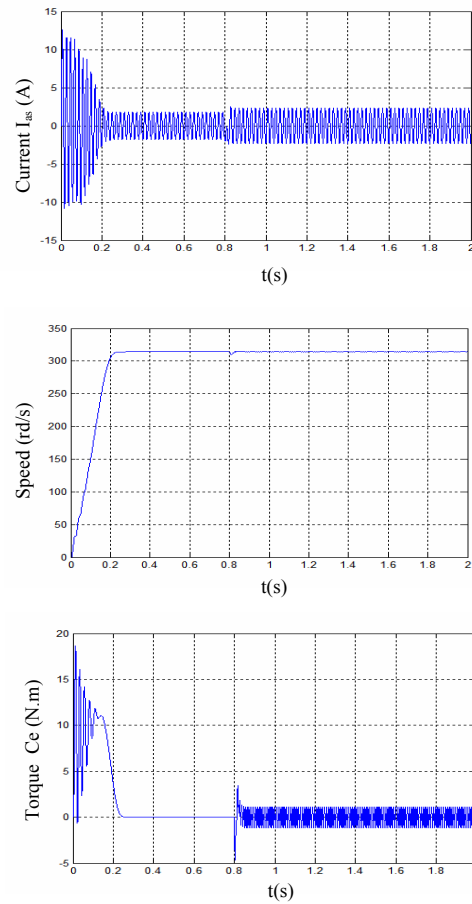
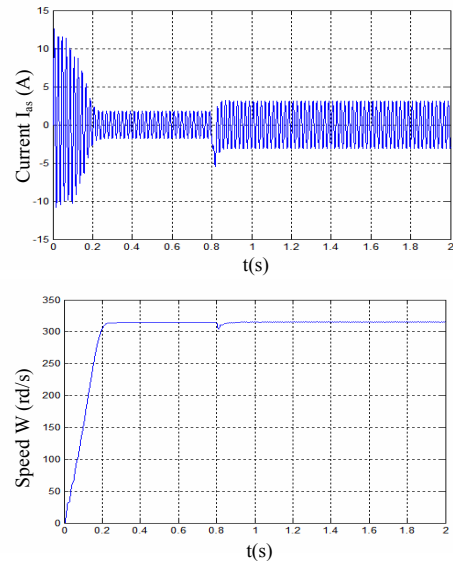


Fig. 1 Results of simulation of 12.5% turns in short-circuit



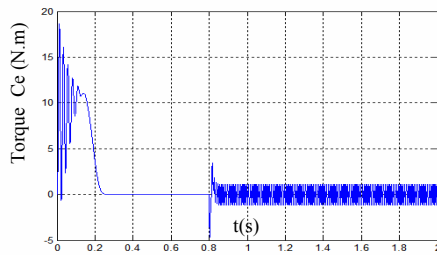


Fig. 2 Results of simulation of 25% turns in short-circuit

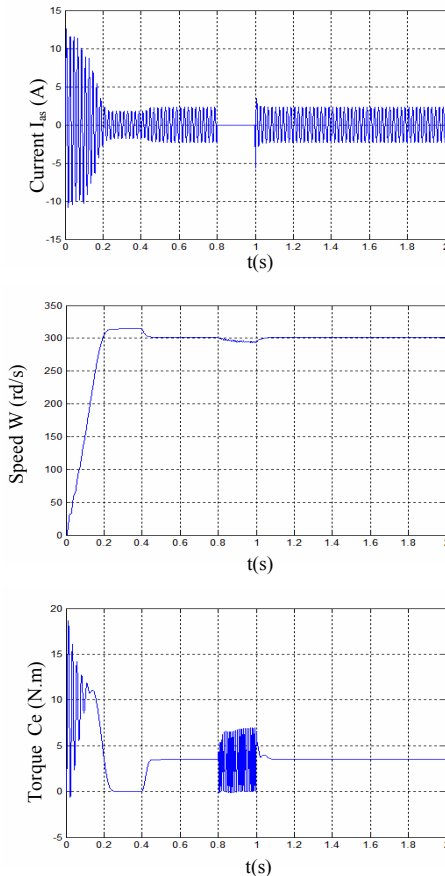


Fig. 3 Results of simulation of one phase cut

## V. CONCLUSION

In this work, in order to facilitate the study and to reveal defects stator, we preceded to the transformation model multi-coils in a three-phase model. In addition, the pre-established model can be developed to identify other defects that may arise on the induction machine, on the one hand, in its rotor (eccentricity of the motor shaft, deformation of the air gap.) on the other hand, in the stator (shorted turns between turns, etc.). For this purpose, we developed a three-phase model equivalent to the model multi-coils.

Furthermore it is possible to use different methods of diagnosis for the detection and localization of the defects of the induction machine (based on the signatures of electrical and mechanical properties obtained from the model used for this study, namely: the sampled values of current stator and the rotational speed as well as the values effective stator currents, voltages and the value of the rotational speed).

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## APPENDIX

$P_n$ : nominal power 1.1 kW  
 $V$ : tension of a stator phase 220 V  
 $p$ : a number of pairs of poles 1  
 $f_s$ : stator frequency of tension 50 Hz  
 $R_s$ : resistance of a stator phase 7.58  $\Omega$   
 $R_R$ : resistance of the rotor cage 6.3  $\Omega$   
 $L_R$ : rotor inductance 0.1612H  
 $L_s$ : stator cyclic inductance 0.5976H  
 $J_m$ : moment of inertia 0.0054 Nms<sup>2</sup>  
 $N_s$ : a number of coils per stator phase: 160  
 $M_{SR}$ : mutual inductance stator nets 26.5mH