

SDVAR Algorithm for Detecting Fraud in Telecommunications

Fatimah Almah Saaid, Darfiana Nur and Robert King, *University of Newcastle, Australia*

Abstract—This paper presents a procedure for estimating VAR using Sequential Discounting VAR (SDVAR) algorithm for online model learning to detect fraudulent acts using the telecommunications call detailed records (CDR). The volatility of the VAR is observed allowing for non-linearity, outliers and change points based on the works of [1]. This paper extends their procedure from univariate to multivariate time series. A simulation and a case study for detecting telecommunications fraud using CDR illustrate the use of the algorithm in the bivariate setting.

Keywords—Telecommunications Fraud, SDVAR Algorithm, Multivariate time series, Vector Autoregressive, Change points.

I. INTRODUCTION

TELECOMMUNICATION companies are facing fraudulent acts from time to time which greatly affecting the industries' revenue. Although the exact loss figures due to fraud may not be made known to the public but the problems has become global to most telecommunications companies. One of the many ways to detect voice fraud is via the call detailed records (CDR). CDR is a massive amounts of call histories generated in real-time basis where it is among the largest real-time [2]. Part of a larger research project to detect fraudulent acts using the telecommunications CDR is to locate the change points which could lead to detecting suspicious (fraudulent) calls. The aim of this paper is to detect change points from the CDRs (as indicative of fraudulent acts) by incorporating unified detection scheme introduced by [1] where the learning model algorithm is extended to a multivariate time series. The algorithm, called Sequential Discounting for Vector Autoregressive (SDVAR), is proposed to detect fraud as soon as it occurs.

The remainder of this paper is organized as follows. The following section reviews some previous works in change points detection. Section III provides a basic concept of vector autoregressive models. Section IV discusses the method designed for the study. The discussion on the results from the simulation and case studies are presented in Section V. The last section gives some discussions and conclusions.

II. CHANGE POINTS

Change points detection has been used in diverse fields. [3] proposed a geometric method for estimating linear state-

F.A. Saaid is with the School of Mathematical and Physical Sciences, University of Newcastle, Callaghan 2308, NSW, Australia, e-mail: (Fatimah.Saaid@uon.edu.au).

D. Nur is with the School of Mathematical and Physical Sciences, University of Newcastle, Callaghan 2308, NSW, Australia, e-mail (Darfiana.Nur@newcastle.edu.au)

R. King is with the School of Mathematical and Physical Sciences, University of Newcastle, Callaghan 2308, NSW, Australia, e-mail (Robert.King@newcastle.edu.au)

space models for identifying change points in time-series data. Whilst Bayesian change points is applied by [4] to detect regions of genetic alteration in cancer research. It has also been used in detecting change points of the number of annual tropical cyclone [5]. In signal processing, change point based on singular spectrum analysis was applied by [6]. In recent study, [7] introduced the combination of wavelet denoising and sequential approach to detect change points on mobile phone based on the CDR. Network faulty monitoring is studied by [1]. They introduced a two-learning stage to detect outliers and change points in a unifying framework, *ChangeFinder*. The scheme is applied by employing autoregressive process where the model is learned using Sequential Discounting for Autoregressive (SDAR) algorithm, also being used by [8]. Adaptive to non-stationary time series is the key advantage of the algorithm. In this paper we study the call behaviour from the CDR by developing *growth profiles* for unique subscribers. The profiles are considered as the *referenced profiles* for normal callers where deviation (change) from these normal behaviours would lead to the identification of suspicious call (act of fraud). To the authors' best knowledge, the detection scheme proposed by [1] has not been studied in the context of multivariate time series. Due to the needs of using more than one variate to describe the dynamic behaviour of a time series especially when the aim is for detecting change points from the CDR, such issue is warranted.

III. VECTOR AUTOREGRESSIVE (VAR) MODELS

Autoregressive (AR) model is the most typical time series model to predict the current value from the past values in a same univariate time series. The number of the past values (or lag values) is referred to the order of the model. However, with the increase interest in modelling a series with more than one variable, multivariate time series model is required. In a VAR, or also known as multivariate AR (MAR) model, the value of each variable at each time point is predicted from the values of the same series and those of all other time series, depending on the variables used in the model. Consider the VAR model with p^{th} order where N be the length of m series. Let $\mathbf{x}_t = [x_{1,t}, \dots, x_{m,t}]^T$ denote $(m \times 1)$ vectors of time series variables. Then VAR(p) model is given by:

$$\mathbf{x}_{i,t} = \boldsymbol{\mu} + \boldsymbol{\Phi}_1 \mathbf{x}_{t-1} + \dots + \boldsymbol{\Phi}_p \mathbf{x}_{t-p} + \boldsymbol{\varepsilon}_{i,t}, \quad (1)$$

where $t = 1, \dots, N$, $i = 1, \dots, m$, $\boldsymbol{\Phi}_k$, $k = 1, \dots, p$ is $(m \times m)$ coefficient matrices, $\boldsymbol{\mu}$ is $(m \times 1)$ vector and $\boldsymbol{\varepsilon}_{i,t}$ is the $(m \times 1)$ vectors of i.i.d Gaussian noise with mean 0 and covariance matrix $\boldsymbol{\Sigma}_\varepsilon$. The mean for the i -series is given as $E[x_{i,t}] = \mu_i$.

Hence the mean vector is given by

$$\boldsymbol{\mu} = [\mu_1, \mu_2, \dots, \mu_m]^T. \quad (2)$$

The covariance matrix function, $\Gamma(h)$ of the vector process \mathbf{x}_t , given by [9] is:

$$\begin{aligned} \Gamma(h) &= \text{Cov}\{\mathbf{x}_t, \mathbf{x}_{t+h}\} \\ &= E[(\mathbf{x}_t - \boldsymbol{\mu})(\mathbf{x}_{t+h} - \boldsymbol{\mu})^T] \\ &= \begin{pmatrix} \gamma_{11}(h) & \gamma_{12}(h) & \cdots & \gamma_{1m}(h) \\ \gamma_{21}(h) & \gamma_{22}(h) & \cdots & \gamma_{2m}(h) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{m1}(h) & \gamma_{m2}(h) & \cdots & \gamma_{mm}(h) \end{pmatrix}. \end{aligned} \quad (3)$$

For $i = j$, $\gamma_{ii}(h)$ is the autocovariance function for the i^{th} components of \mathbf{x}_t . For $i \neq j$, $\gamma_{ij}(h)$ is the *cross-covariance* function between $x_{i,t}$ and $x_{j,t}$. The cross-covariance $\gamma_{ij}(h)$ is calculated by

$$\gamma_{ij}(h) = E[(x_{i,t} - \mu_i)(x_{j,t+h} - \mu_j)].$$

By mapping with the SDAR, the SDVAR algorithm is developed to estimate the model parameters using VAR estimations by [9] and [10].

IV. METHOD

The detection flow, as displayed in Figure 1, shows the unified scheme in change detection proposed by [1] called *ChangeFinder*. It presents a two-stage learning scheme of the data in detecting outliers and change points in a single framework. We describe the unified scheme in change detection using the two-stage learning framework but with an extension of employing multivariate autoregressive representative of the time series.

A. Unified Detection Scheme

Unified detection scheme is to detect multiple outliers and change points in a time series. The estimation of parameters in VAR(p) models is done after a series is observed. The SDVAR algorithm in the flow of Figure 1 involves online estimation of a time series data by introducing discounting parameter, r . The value of r is between 0 and 1, where the smaller r indicates higher influence of the past data.

The following describes the flow of the detection process used in this study by combining the SDVAR and SDAR algorithms to learn the model in the first and second detection stage, respectively:

- Step 1. Read the series, where the elements of vector \mathbf{x}_t denote the measurement of length of m at each time $t = 1, 2, \dots, N$.
- Step 2. First stage: Perform the SDVAR algorithm from 1-4 (in section IV-B), to produce the densities for p_{it} .
- Step 3. Calculate the model prediction loss using either logarithm loss function (or quadratic loss function). The scores of \mathbf{x}_t in this paper is using the logarithmic loss scoring technique where the series using the joint probability densities of x_1 and x_2 .

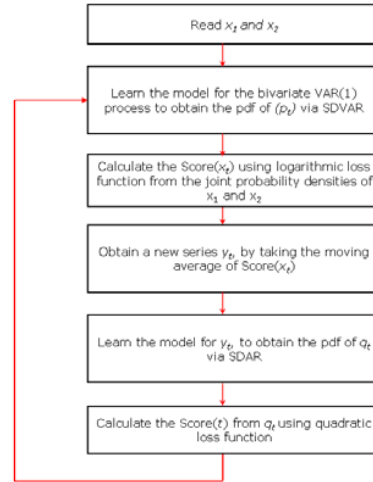


Fig. 1. Unified outliers and change points detection scheme proposed in the study using the framework by [1]

Step 4. Average the scores with arbitrary constant T to isolate the outliers in the time series which yielding a new series, \mathbf{y}_t .

Step 5. Second stage: Learn the model again by using SDAR [1], to produce the densities, q_t .

Step 6. Calculate the model prediction loss via quadratic scoring.

The following section outlines the SDVAR algorithm introduced in this paper to learn the multivariate time series by employing vector autoregressive model.

B. Sequential Discounting VAR (SDVAR) Algorithm

Sequentially Discounting VAR (SDVAR) is introduced in this paper to learn the model with VAR process. Let the discounting parameter $0 < r < 1$ and the parameters are estimated after a set of data is observed.

- 1) Initialize the parameter estimates, $\hat{\boldsymbol{\mu}}_0$, Γ_j , $\hat{\boldsymbol{\Phi}}_j$, $\hat{\boldsymbol{\Sigma}}$ as denoted in equations (1) - (3). The VAR model consists of parameters for each series \mathbf{x}_t where the estimations of these parameters are all represented by matrices.
- 2) Parameter update: For each new data arrives ($t=k+1, k+2, \dots$), read \mathbf{x}_t and update the mean, the variances-covariances (and the cross-covariances) and the model's parameters:

$$\hat{\boldsymbol{\mu}} = (1-r)\hat{\boldsymbol{\mu}}_0 + r * \mathbf{x}_t,$$
 which produces the mean vectors, $\boldsymbol{\mu}_t$, $t=1,2,\dots$. The parameter updates to produce the variance-covariance matrix (Γ_0) as well as variance-cross-covariance matrix ($\Gamma(h)$ for $h > 0$):

$$\Gamma(h) = (1-r) * \Gamma(h) + r * (\mathbf{x}_t - \hat{\boldsymbol{\mu}})(\mathbf{x}_{t+h} - \hat{\boldsymbol{\mu}})^T, h=0,1,\dots,p.$$
- 3) Calculate $\hat{\mathbf{x}}_t$ and $\hat{\boldsymbol{\Sigma}}$ where

$$\hat{\mathbf{x}}_t = \hat{\boldsymbol{\Phi}}_i(\mathbf{x}_{t-i} - \hat{\boldsymbol{\mu}}) + \hat{\boldsymbol{\mu}}$$
 and

$$\hat{\boldsymbol{\Sigma}} = (1-r)\hat{\boldsymbol{\Sigma}} + r(\mathbf{x}_t - \hat{\mathbf{x}}_t)(\mathbf{x}_t - \hat{\mathbf{x}}_t)^T.$$

C. Kullback-Leibler Divergence

The Kullback-Leibler (KL) divergence is a statistical distance measuring between two probability distributions. The measure has been widely applied in control systems, communication and information theory [11]. It is an intuitive understanding from information theory that statistically measure the quantities in bits on how close a density $p = p_i$ is to a candidate density $q = q_i$, for $i = 1, 2, \dots$. The KL divergence for two probability mass functions of $p(x)$ and $q(x)$, $KL(p \parallel q)$ is given by:

$$\begin{aligned} KL(p \parallel q) &= \int p_i(x) [\log p_i(x) - \log q_i(x)] dx \\ &= \int p_i(x) \log \frac{p_i(x)}{q_i(x)} dx \\ &= E_p \log \frac{p_i(x)}{q_i(x)}, \end{aligned} \quad (4)$$

where E_p is the expectation with respect to the p distribution. Eq.(4) is in the form of minimizing the divergence to obtain the moment matching, i.e. the expectation. In this paper, we define the divergence of two stochastic process at time t_0 , where t_0 indicates a change point occurs. Let p_t be the stochastic process of $p_t = p(x_t | x_{t-1}, \dots, x_1)$ at time $t < t_0$, and $q_t = p(x_t | x_{t-1}, \dots, x_1)$ at time $t > t_0$. At time $t = t_0$ is considered as a change point if p_t is different from q_t , as measured using Eq.(4). The KL divergence for two Gaussian distributions is given as

$$\frac{1}{2\sigma^2}(\mu_1 - \mu_2)^2 + \frac{1}{2} \left[\frac{\sigma_1^2}{\sigma_2^2} - 1 - \log \left(\frac{\sigma_1^2}{\sigma_2^2} \right) \right]. \quad (5)$$

Following the works of [1], the detection of multiple change points in a time series is done by considering a sliding window indicating an approximately stationary stochastic process. The sudden occurrence of changing point in that particular sliding window can be measured using KL divergence where they defined two main cases:

1) Jumping Mean

Consider two i.i.d Gaussian densities of p and q , with identical variances. The mean for p is μ_1 and 2 is μ_2 . Therefore Eq. (5) becomes

$$\begin{aligned} KL(p \parallel q) &= \frac{1}{2\sigma^2}(\mu_1 - \mu_2)^2 + \frac{1}{2} [0] \\ &= \frac{1}{2\sigma^2}(\mu_1 - \mu_2)^2. \end{aligned} \quad (6)$$

2) Jumping Variance

Two i.i.d Gaussian densities of p and q , with both means are 0 and the variances are σ_1^2 and σ_2^2 , respectively. The divergence is calculated using,

$$\begin{aligned} KL(p \parallel q) &= \frac{1}{2\sigma^2}(0)^2 + \frac{1}{2} \left[\frac{\sigma_1^2}{\sigma_2^2} - 1 - \log \left(\frac{\sigma_1^2}{\sigma_2^2} \right) \right] \\ &= \frac{1}{2} \left[\frac{\sigma_1^2}{\sigma_2^2} - 1 - \log \left(\frac{\sigma_1^2}{\sigma_2^2} \right) \right]. \end{aligned} \quad (7)$$

D. Scoring

In this study, the VAR is stochastic process used to predict the next observation and after the prediction is made, a loss will be incurred. Among common methods for calculating the prediction loss (or scores) are via logarithmic loss function and quadratic loss function.

1) Logarithmic Loss Function:

$$Score(x_t) = -\log p_{t-1}(x_t | x^{t-1}), \quad (8)$$

where the score for x_t is based on the data generated from the probability density of p_{t-1} (where $p_{t-1} = x_t | x_1 \dots x_{t-1}$).

2) Quadratic Loss Function:

$$Score(x_t) = (x_t - \hat{x}_t)^2, \quad (9)$$

where \hat{x}_t is the prediction of x_t given x^{t-1} based on the learned model p_{t-1} , and $\hat{x}_t = \int x p_{t-1}(x_t | x^{t-1}) dx$.

Eq.(8) refers to the predicted probability of the likelihood of the observation and the conventional least squares error criterion is as given in Eq.(9).

V. RESULTS

A. Simulation Study

Following the examples demonstrated by [1], three simulation studies are run to illustrate the use of the SDVAR algorithm to learn the three stochastic models in the context of multivariate autoregressive process: jumping mean, jumping mean with varying variance and jumping variance. This study focuses on the VAR(1) process with bivariate system. Hence 2-variate of 1st order VAR (two variables of x_1 and x_2) with Gaussian white noise are simulated using R version 2.13. The scoring rule used is logarithmic loss function for the first detection stage whilst quadratic loss function for the second. Moreover, using the joint probability density function of x_1 and x_2 ([12], [13]), SDAR is used to learn the new series in the second stage.

1) *Jumping mean*: Using *mAr.sim* function of *mAr* package in R, as given in Eq. (10) and Eq. (11) respectively, a 2-dimensional VAR(1) with $N = 10,000$ induced with KL divergence (Eq. (6)) is simulated. The change points are set to occur at each $t_0 \times 1000$ (for $t_0 = 1, \dots, 9$) using Eq. (6).

$$x_{1,t} = \mu_1 + \phi_{11}x_{1,t-1} + \phi_{12}x_{2,t-1} + \varepsilon_{1,t}, \quad (10)$$

$$x_{2,t} = \mu_2 + \phi_{21}x_{1,t-1} + \phi_{22}x_{2,t-1} + \varepsilon_{2,t}, \quad (11)$$

where ε is the normally distributed white noise and $\varepsilon \sim N(0, \Sigma_\varepsilon)$. The simulated series is generated using the following parameters:

$$\Phi = \begin{pmatrix} 0.4 & -0.5 \\ -0.3 & 0.4 \end{pmatrix}, \mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \text{ and } \Sigma_\varepsilon = \begin{pmatrix} 1.0 & 0 \\ 0 & 1.0 \end{pmatrix}.$$

With $r = 0.02$ and $T = 5$, R is run for learning the 10,000 data using the SDVAR algorithm (the speed of the run time is highly depending on the processor of the computer

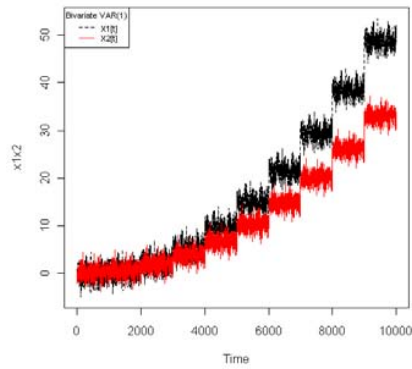


Fig. 2. Simulated VAR(1) series of jumping mean

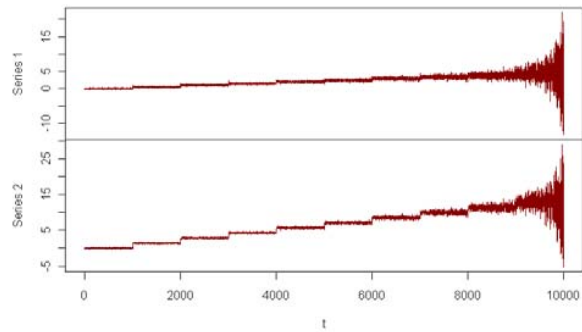


Fig. 5. Simulated VAR(1) series of jumping mean and varying variance

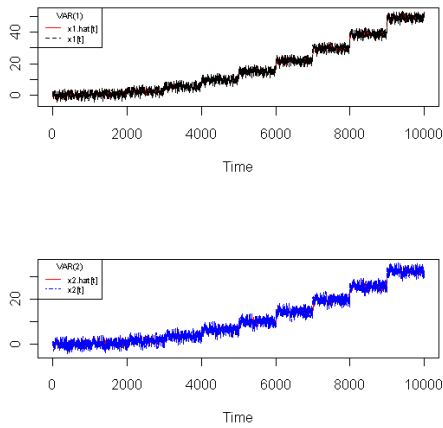


Fig. 3. Predicted series of jumping mean using the SDVAR algorithm

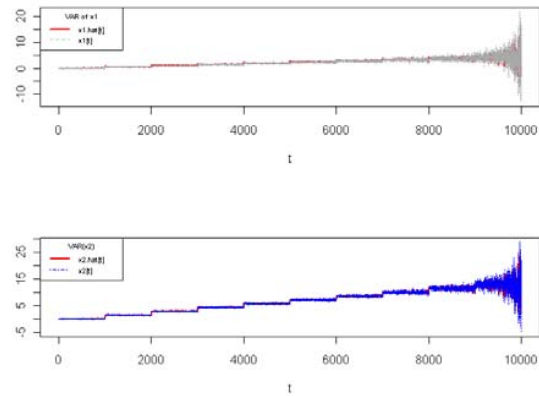


Fig. 6. Predicted series of jumping mean and varying variance using SDVAR algorithm

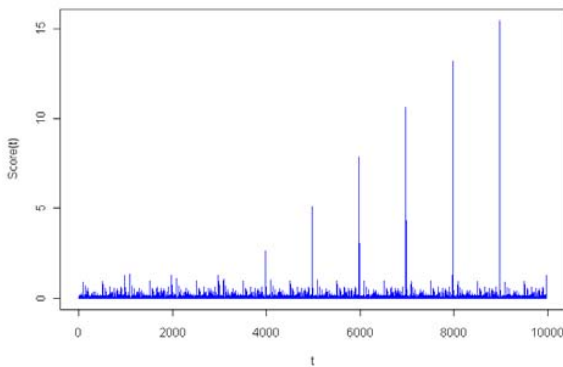


Fig. 4. Change points detected in the first simulated series

machine). In Figure 4, the plot shows the change points detected for the first simulated series. The sudden jumps are appropriately detected by the detection scheme using SDVAR learning model.

2) *Jumping mean with varying variance*: Following the previous simulation set-up for sudden jumps of mean at each $t_0 \times 1000$ (for $t_0 = 1, \dots, 9$) simulated data is generated with varying variance, as displayed in Figure 5. The variance of the white noise is set to change gradually over time as defined as follows:

$$\epsilon_{i,t} = \frac{0.1}{\frac{0.01+(N-t)}{N}}, \quad (12)$$

where $t = 1, \dots, N$ and $i = 1, 2$.

The predicted series using SDVAR are as given in Figure 6 where the parameters used in the simulation are as follows:

$$\Phi = \begin{pmatrix} 0.4 & -0.5 \\ -0.3 & 0.4 \end{pmatrix}, \mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \text{ and } \Sigma_\epsilon = \begin{pmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{pmatrix}.$$

The scores for the second simulated series is displayed in Figure 7 indicating the 7 change points detected but no change points detected after that. This simulation shows the capability of the detection scheme using SDVAR algorithm to accurately detect the sudden jumps in mean despite the

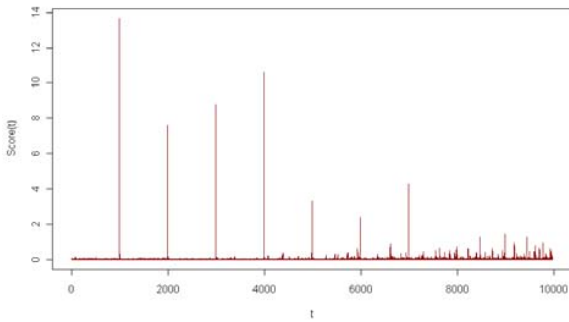


Fig. 7. Change points detected in the second simulated series

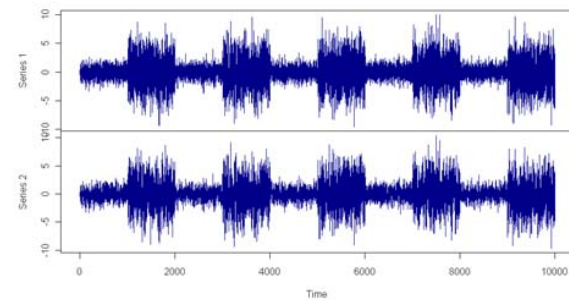


Fig. 8. Simulated VAR(1) series with jumping variance

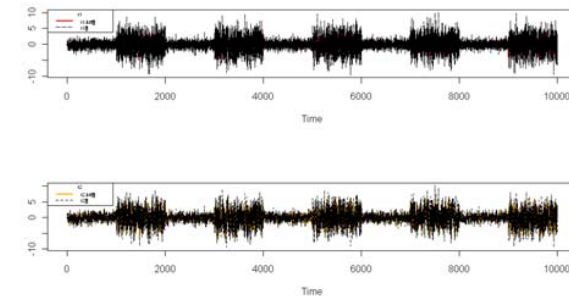


Fig. 9. Predicted series with jumping variance using SDVAR algorithm

changing of variance over time.

3) *Jumping variance*: The last simulation deals with data with jumping variance using KL as defined in Eq. (7) and the parameters used are:

$$\Phi = \begin{pmatrix} 0.4 & -0.5 \\ -0.3 & 0.4 \end{pmatrix}, \mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \text{ and } \Sigma_\epsilon = \begin{pmatrix} 1.0 & 0.1 \\ 0.1 & 1.0 \end{pmatrix}.$$

However, at $t_0 \times 1000$ ($t_0 = 1, \dots, 9$), the variance is changing from 1 to 9 when t_0 is odd and changing from 9 to 1 when t_0 is even (refer to Figure 8). The SDVAR algorithm used to learn the model yields the predicted values for each variate, x_1 and x_2 (Figure 9).

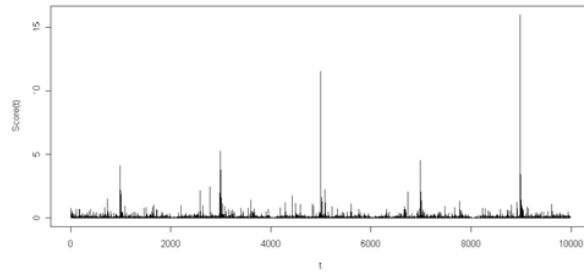


Fig. 10. Change points detected in the third simulated series

In Figure 10, SDVAR used in the detection scheme is correctly detecting the change points that occur when the variance suddenly jumps from 1 to 9. However no change points detected when the variance suddenly falls from 9 to 1.

B. Case Study

A sample of 6-day CDR data is used as a training data in obtaining the user profiles of the subscribers. The CDR is collected from a PBX at one of telecommunications companies in Malaysia. The network growth profiles used as instantiations to detect change points in the CDR for the unique subscribers are duration-to-network and cost-to-network ratios (where the network growth is defined from the growth of destinations), as shown in Figure 11. One user account's profiles is used for this case study where the growth profiles are developed in 10 minutes interval for the whole six days.

Using bivariate VAR(1) process of the two selected profile measurements, *mar.est* function from *mar* package in *R* is applied to estimate the model. The detection scheme used as described in Figure 1 where the scoring methods used are logarithmic scores and quadratic scores in the first and second detection stage, respectively. Figure 12 exhibits the plot of the series using SDVAR algorithm as the model learning module. The scores result of the joint densities of both variates defined in the VAR(1) model indicate that the 78th of the 10-minute interval is identified as change point (Figure 13).

In the context of CDR the change point might show a sudden deviation from the normal behaviour of the call of the respective account. This may be an indicative of suspicious act of fraudulent that has occurred in the sample CDR series. Further investigation indicate that the call duration growth in the 78th interval is found to be two times higher than in the 77th interval. In contrast, cost growth profile shows a strange behaviour which is not reflecting the long call made at $t=78$ of the corresponding user (Figure 12).

As a purpose of an example for this case study, the quadratic loss function is used in both of the detection stages [14] and the results are as shown in Figure 14. Both variates, duration-to-network growth and cost-to-network growth show sudden jumps (change points) at time $t=78$ (Figure 14), as found in the first example using the joint probability density of both referenced profiles.

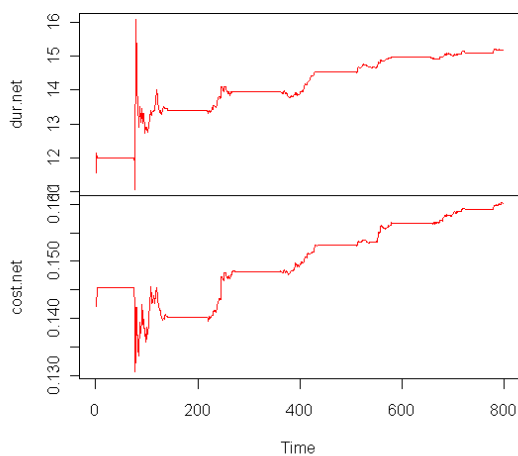


Fig. 11. Duration-Network growth (top) and Cost-Network growth series of the 10-minute interval

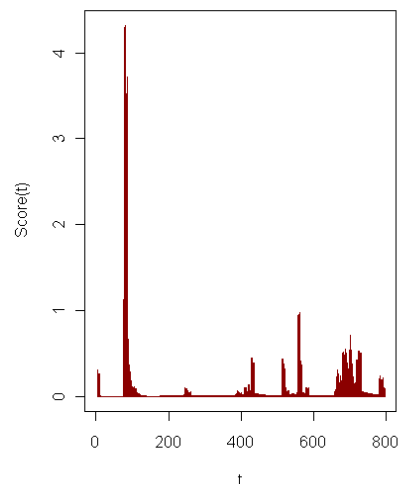


Fig. 13. Change point detected in the CDR using logarithmic loss scores (first stage) and quadratic loss scores (second stage)

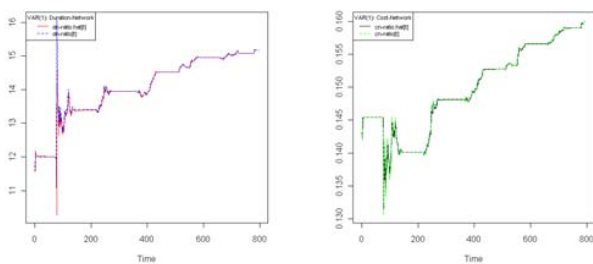


Fig. 12. Duration-Network growth (top) and Cost-Network growth series learned using SDVAR

VI. CONCLUSION

In this paper, an algorithm for learning a vector autoregressive process is proposed for detecting change points in time series. The SDVAR algorithm is used as a learning module for change point analysis from nonstationary time series data in online manner, based on the approach and change detection framework of [1]. The approach is part of research in detecting fraudulent acts from a CDR. The algorithm is validated using simulation and case study. Findings from the three different types of simulated vector autoregressive series with jumping mean, jumping mean and varying variance and jumping variance, the detection scheme using the SDVAR algorithm is capable of detecting multiple change points. These change points are referred as sudden jumps in the context of the mean and variance of the series which are defined by Kullback-Leibler divergence. The case studies using the 6-day CDR data indicate the change points are appropriately detected in both of the instantiations used in the VAR(1) model. The case study attempts to test the capability of the scoring method used in the detection stages. The use of either logarithmic or quadratic

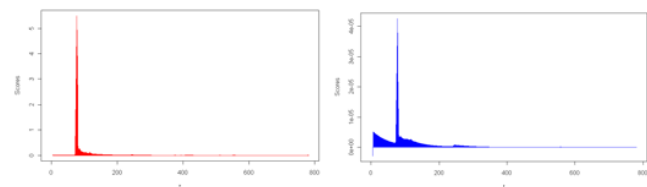


Fig. 14. Change points detected for both Duration-Network growth (left-hand side) and Cost-Network growth profiles using quadratic loss scores in both detection stages

score in this paper shows that both are correctly detecting the sudden jumps in the sample CDR data. The change points detected in the CDR may lead to an alarming stage of detecting suspicious activities as indicative of the fraudulent calls. Such calls will be quarantined for further investigation by fraud officers of the telecommunications companies.

REFERENCES

- [1] J. Takeuchi and K. Yamanishi, "A unifying framework for detecting outliers and change points from time series," *IEEE Transactions on Knowledge and Data Engineering*, vol. 18(4), pp. 482–492, 2006.
- [2] G. Weiss, *Networking and Telecommunications: Concepts, Methodologies, Tools and Applications*, ch. Data Mining in the Telecommunications Industry, pp. 486–491. IGI Global, 2010.
- [3] Y. Kawahara, T. Yairi, and K. Machida, "Change-point detection in time-series data based on subspace identification," in *Seventh IEEE International Conference on Data Mining (ICDM)*, pp. 559–564, 2007.
- [4] C. Erdman and J. Emerson, "A fast bayesian change point analysis for the segmentation," *Bioinformatics*, vol. 24(19), p. 21432148, 2008.
- [5] P. Chu and X. Zhao, "Bayesian change-point analysis of tropical cyclone activity: The central north pacific case," *Journal of Climate*, vol. 17(24), pp. 4893–4901, 2004.
- [6] V. Moskvina and A. Zhigljavsky, "An algorithm based on singular spectrum analysis for change-point detection," *Communication in Statistics: Simulation & Computation*, vol. 32(2), pp. 319–352, 2003.

- [7] H. Zhang, R. Dantu, and J. Cangussu, "Change point detection based on call detail records," in *IEEE International Conference on Intelligence and Security Informatics, 2009 (ISI 09)*, 2009.
- [8] B.-K. Yi, N. Sidiropoulos, T. Johnson, H. Jagadish, C. Faloutsos, and A. Biliris, "Online data mining for co-evolving time sequences," in *Proceedings of the 16th International Conference in Data Engineering*, 2000.
- [9] H. Lütkepohl, *Introduction to multiple time series analysis*. Berlin, New York: Springer-Verlag, 1993.
- [10] W. Wei, *Time Series Analysis: Univariate and Multivariate Methods*. Addison-Wesley Publishing Company, Inc., 1990.
- [11] D. Kazakos and P. Papantoni-Kazakos, "Spectral distance measuring between gaussian processes," *IEEE Trans. Automat. Contr. AC-25*, pp. 950–959, 1980.
- [12] B. Biller and B. Nelson, "Modeling and generating multivariate time-series input processes using a vector autoregressive technique," *ACM Transactions on Modeling and Computer Simulation*, vol. 13, no. 3, pp. 211–237, 2003a.
- [13] B. Biller and B. Nelson, "Online companion to modelling and generating multivariate time series input processes using a vector autoregressive technique," tech. rep., Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston Illinois, 2003b.
- [14] F. A. Saaid, D. Nur, and R. King, "Change points detection of vector autoregressive model using sdvar algorithm," in *Fifth Annual ASEARC Conference*, (University of Wollongong, Australia), pp. 18–21, 2–3 February 2012.



Robert King is a Lecturer in Statistics in the School of Mathematical and Physical Sciences. Dr Robert King researches in the areas of distributional shape, machine learning, data mining and socio-spatial statistics. His PhD is from Queensland University of Technology.



Fatimah Almah Saaid is a Senior Researcher at Telekom Malaysia Research & Development and she has been working with the company since 1998. She received her BSc. and MSc. (Research) degrees in Statistics from National University of Malaysia in 1997 and 2002 respectively. She is currently pursuing her PhD study at the School of Mathematical and Physical Sciences of The University of Newcastle Australia. Her research interests include time series analysis and telecommunications network planning.



Darfiana Nur is a Lecturer in Statistics in the School of Mathematical and Physical Sciences. Dr Darfiana Nur received her BMath degree from The University of Gadjahmada (UGM) Indonesia in 1988, MSc (Research) from The University of Western Australia in 1993 and her PhD in Statistics from Curtin University of Technology Australia in 1999 respectively. After completing her doctorate, she became a postdoctoral researcher in the School of Mathematical Sciences at Queensland University of Technology in 1999-2001 and at The University

of Newcastle in 2001-2003 respectively, before taking up her present appointment at The University of Newcastle in 2003. Her research interests include nonlinear time series analysis (ergodicity or estimation aspects) and modelling, focusing on Smooth Threshold Autoregressive models; Markov Chain Monte Carlo (MCMC) related algorithms and its convergence diagnostics; DNA sequence modelling in Bioinformatics; Bayesian Hidden Markov Models (HMMs) in Genetics.