International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:6, No:1, 2012

Perfect Plastic Deformation of a Circular Thin Bronze Plate due to the Growth and Collapse of a Vapour Bubble

M.T. Shervani-Tabar, M. Rezaee and E. Madadi Kandjani

Abstract—Dynamics of a vapour bubble generated due to a high local energy input near a circular thin bronze plate in the absence of the buoyancy forces is numerically investigated in this paper. The bubble is generated near a thin bronze plate and during the growth and collapse of the bubble, it deforms the nearby plate. The Boundary Integral Equation Method is employed for numerical simulation of the problem. The fluid is assumed to be incompressible, irrotational and inviscid and the surface tension on the bubble boundary is neglected. Therefore the fluid flow around the vapour bubble can be assumed as a potential flow. Furthermore, the thin bronze plate is assumed to have perfectly plastic behaviour. Results show that the displacement of the circular thin bronze plate has considerable effect on the dynamics of its nearby vapour bubble. It is found that by decreasing the thickness of the thin bronze plate, the growth and collapse rate of the bubble becomes higher and consequently the lifetime of the bubble becomes shorter.

Keywords—Vapour Bubble; Thin Bronze Plate; Boundary Integral Equation Method.

I. Introduction

DYNAMIC behaviour of a vapour bubble near different boundaries is an interesting and important problem for engineers. There are some experimental and numerical studies about the growth and collapse of a vapour bubble near a rigid boundary, a free surface and a compliant wall. Klaseboer *et al.* in a numerical study investigated the behaviour of the vapour bubble near an elastic surface [1]. They used a fluid-fluid interface for modelling the surface. Their numerical simulation showed that in the period of the growth and collapse of the vapour bubble near an elastic surface, a high pressure region develops around the bubble boundary and this may cause an annular jet formation and splitting of the bubble. Blake *et al.* in a study, investigated transient cavities near boundaries [2]. Experimental observations and numerical simulations showed that the liquid jet which is developed during the collapse phase of the bubble in the vicinity of a rigid boundary and in the absence of strong buoyancy forces, is directed towards the rigid boundary, whereas in the vicinity of a free surface the liquid jet migrates away from the free surface and is directed away from it [3]–[8].

This paper describes the behaviour of a vapour bubble near a circular thin bronze plate. This problem is very important in manufacturing industries in micro and macro scales. The advantages of this method is the elimination of mechanical parts in the deformation of thin metal plates.

M.T. Shervani-Tabar is with the Faculty of Mechanical Engineering at the University of Tabriz, 29 Bahman Boulevard, Tabriz, IRAN (Phone: +98-411-3393044; Fax: +98-411-3356026; e-mail: msherv@tabrizu.ac.ir)

M. Rezaee is with the Faculty of Mechanical Engineering at the University of Tabriz, 29 Bahman Boulevard, Tabriz, IRAN (Phone: +98-411-3392459; Fax: +98-411-3356026; e-mail: m_rezaee@tabrizu.ac.ir)

E. Madadi Kandjani is with the Faculty of Mechanical Engineering at the University of Tabriz, 29 Bahman Boulevard, Tabriz, IRAN (Phone: +98-914-4143295; e-mail: e-madadi87@ms.tabrizu.ac.ir)

II. Mathematical model of the bubble behaviour

For any sufficiently smooth function ϕ which satisfies the Laplace equation in the domain Ω having piecewise smooth surface S Green's integral formula governs the hydrodynamic behaviour of the liquid domain and is given as:

$$C(p)\phi(p)+\int_S \frac{\partial}{\partial n}\left[\frac{1}{|p-q|}\right]\phi(q)\mathrm{d}s = \int_S \left[\frac{1}{|p-q|}\right]\frac{\partial\phi(q)}{\partial n}\mathrm{d}s, \quad (1)$$

where S is the boundary of the liquid domain which includes the bubble boundary and the interface of the liquid domain with the thin bronze wall, ϕ is the velocity potential, $\frac{\partial\phi}{\partial n}$ is the normal velocity of the boundary, p is any point in the liquid domain or on the boundary and q is any point on the boundary. $C(p)$ is 2π when p is on the boundary and is 4π when p is in the liquid domain. The unsteady Bernoulli equation in its Lagrangian form is given as:

$$\frac{D\phi}{Dt} = \frac{P_\infty - P_b}{\rho} + \frac{1}{2}|\nabla\phi|^2 + g(z-h), \quad (2)$$

where t is time, P_∞ is pressure in the far field, P_b is pressure inside the bubble and ρ is the liquid density. In the absence of the gravity effect, the third term on the right hand side of the above equation will be neglected.

III. Mathematical model of thin bronze plate

It is assumed that the thin bronze plate is clamped at its circular edge to a rigid wall. It is also assumed that the plate undergoes perfectly plastic deformation under dynamic loading of the growth and collapse of a vapour bubble which is generated due to a local energy input. The shape of the deformed plate at any instant is assumed to be a part of a sphere with its centre on the axisymmetric axis of the circular plate. The energy method is used to relate the dynamic loading of the bubble pulsation to the plate plastic deformation. Therefore on the interface of the fluid flow and the thin bronze plate it can be written:

$$\frac{\partial \phi}{\partial z} = \psi, \tag{3}$$

$$P(r,t) = -\rho \frac{\partial \phi}{\partial t} + P_\infty, \tag{4}$$

The equation of the deformation of the plate is given as:

$$\psi = \frac{\Delta y}{\Delta t}\left[1 - \left(1 - \frac{{R_{ps}}^2 - y^2}{{R_{ps}}^2 + y^2}\Delta y\right) \times \left(\frac{{R_{ps}}^2 - y^2}{2y} + 1\right)(1-\cos\theta)\right], \tag{5}$$

where R_{ps} is the radius of the thin bronze plate, y is the plate midpoint deflection, Δy is an incremental deflection and θ is the angle of each point on the thin bronze plate with respect to the axisymmetric axis.

IV. Discretization of the Problem

The boundary integral equation method is employed for numerical solution of the problem. The interfaces of the liquid domain with the thin bronze plate and the rigid wall are discretized by linear segments (see figure 1).

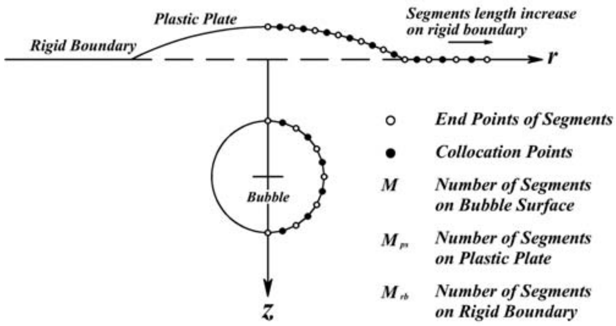

Fig. 1. Discretization of the surface of the bubble and the thin bronze plate surface

The interface of the liquid domain with the thin bronze plate is divided into M_{ps} segments while the interface of the liquid domain with the rigid wall is divided into M_{rb} elements. Therefore the total segments on the interfaces of the liquid domain with the thin bronze plate and the rigid wall is equal to N, where $N = M_{ps} + M_{rb}$. The segments on the plate are equal in size while the length of the segments on the rigid boundary could increase in the direction of the radial axis. The bubble boundary is discretized by cubic spline elements. The collocation points are located at the midpoint of each segment. The discretization of the interfaces of the liquid domain with the thin bronze plate and the rigid wall is extended up to physical infinity, where the growth and collapse of the bubble has not significant effect on the liquid flow.

V. Non-dimensionalising of the Problem

By employing maximum radius of the bubble as an independent variable for the length, the pressure difference driving the bubble collapse, $\Delta P = P_\infty - P_C$, as an independent variable for the pressure, $U = \sqrt{\frac{\Delta P}{\rho}}$ and $t = \frac{R_m}{\sqrt{\frac{\Delta P}{\rho}}}$ as independent variables for the velocity and time respectively, the problem under investigation has been non-dimensionalised.

VI. Evaluating the Initial Conditions of the Bubble

The process of the bubble expansion from its initial radius at $t = 0$ up to radius R can be assumed a polytropic process and the gas inside the bubble is assumed to be an ideal gas. By considering $\varepsilon = \frac{P_{ig}}{P_\infty - P_C}$, the strength parameter, which measures the strength of the initial high pressure inside the bubble that drives the bubble motion [9], the equation that describes oscillatory motion of the bubble is expressed as:

$$\mathrm{R}\ddot{\mathrm{R}} + \frac{3}{2}\dot{\mathrm{R}}^2 + 1 - \varepsilon\left(\frac{\mathrm{R}_0}{\mathrm{R}}\right)^{3\gamma} = 0, \tag{6}$$

The value for the initial radius is depended upon the value of the strength parameter, and with every specified values of the strength parameter a corresponding initial radius can be obtained (see Best [9]).

VII. Numerical Implementation

At the beginning the bubble contains a very high pressure gas. The initial normal velocity of the bubble boundary is equal to zero. Consequently the initial distribution of the velocity potential on the bubble surface is equal to zero. The initial normal velocity of the thin bronze plate and consequently the velocity potential on the plate are also equal to zero. Δt is a variable time step and is given by:

$$\Delta t = min\left|\frac{\Delta\phi}{\frac{P_\infty - P_C}{\rho} + \frac{1}{2}(\nabla\phi)^2 + \delta^2 z}\right|, \tag{7}$$

where $\Delta\phi$ is some constant and is the maximum increment of the velocity potential on the bubble boundary between two successive time steps.

If the number of elements on the bubble boundary are M and the number of elements on its nearby boundary including the thin bronze plate and the rigid boundary are assumed to be N, therefore the discretized form of the Green's integral formula becomes a system of linear equations as:

$$2\pi\phi(p_i) + \sum_{j=1}^{M+N} \phi(q_j)\left\{\int_{S_j} \frac{\partial}{\partial n}\left(\frac{1}{|p_i - q_j|}\right)\right\} \mathrm{d}s = \sum_{j=1}^{M+N} \psi(q_j)\left\{\int_{S_j} \left(\frac{1}{|p_i - q_j|}\right)\right\} \mathrm{d}s \tag{8}$$

VIII. Numerical results and discussion

In this section, time dependent profiles of the bubble in the vicinity of a thin bronze plate are illustrated. The bubble is initially in its minimum volume. At the beginning, the velocity of the plate and consequently the velocity potential on the bubble boundary are zero and initial pressure inside the bubble is very high.

Figures 2 (A) and (B) illustrate successive profiles of the bubble in the vicinity of a thin bronze plate with the thickness of $th^* = 0.0094$ during its growth and collapse phases. The bubble is initially located at $\gamma = 1.5$ and is characterized by $\delta = 0$ and $\varepsilon = 10$. During the growth phase, the bubble expands almost spherically. But later during the collapse phase, the bubble elongates in the direction normal to the thin bronze plate at its midpoint. Later, far side of the vapour bubble from the thin bronze plate becomes flattened and a liquid jet develops on the bubble boundary at the point far from the thin bronze plate and is directed towards the plate.

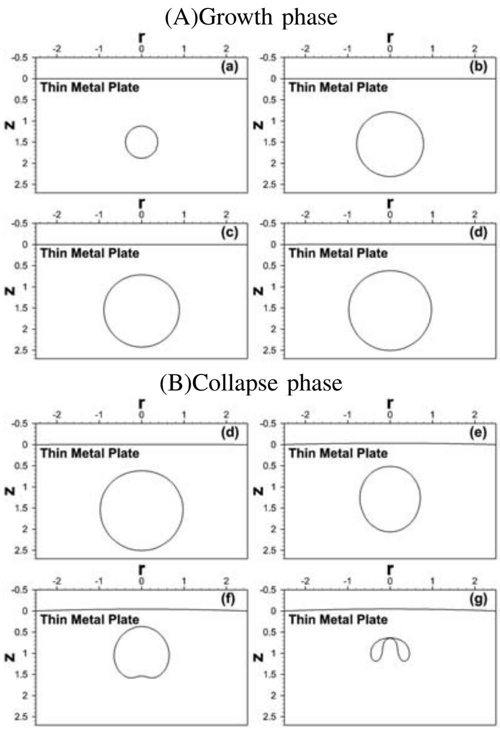

Fig. 2. Time dependent profiles of the vapour bubble during its growth and collapse phases near a thin bronze plate. The bubble is characterized by $\gamma = 1.5$, $\delta = 0$ and $\varepsilon = 10$. The thin bronze plate is characterized by the thickness of $th^* = 0.0094$. The non-dimensional times corresponding to the bubble successive profiles are: (A) Growth phase: $(a) = 0.00181$, $(b) = 0.45633$, $(c) = 0.65065$, $(d) = 1.46225$. (B) Collapse phase: $(d) = 1.46225$, $(e) = 2.16944$, $(f) = 2.40217$, $(g) = 2.59401$.

Figures 3 (A) and (B) illustrate the thin bronze plate deformation profiles in a proper scale. They show that during the growth phase of the bubble, the thin bronze plate deformation is small. While significant deformation takes place at the collapsing phase.

Figures 4 (A) and (B) illustrate the successive time dependent profiles of the bubble in the vicinity of a thin bronze plate with the thickness of $th^* = 0.0089$ during its growth and collapse phases. The bubble is initially located at $\gamma = 1.5$ and is characterized by $\delta = 0$ and $\varepsilon = 10$. The thin bronze plate is thinner than that of figure 2. During the growth phase of the vapour bubble near the thin bronze plate the bubble expands almost spherically, while in the collapse phase, the bubble elongates in the direction normal to the thin bronze plate at its midpoint. Later both sides of the bubble are flattened. At the latest stages of the collapse phase, two liquid jets develop on the top and bottom of the bubble boundary. These figures show that by decreasing the thickness of the thin bronze plate, the distance between centroid of the bubble at the final stages of its collapse phase and the thin bronze plate increases.

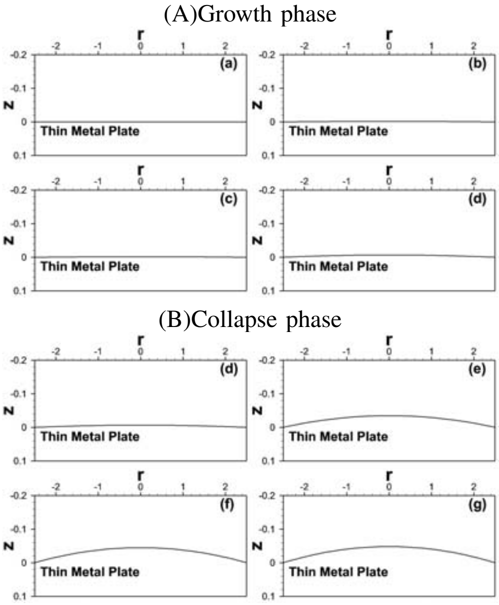

Fig. 3. The thin bronze plate deformation profiles in a proper scale at the non-dimensional times and bubble characteristics corresponding to that of figure 2.

Figures 5 (A) and (B) illustrate the thin bronze plate deformation in a proper scale. They show that the significant deformation takes place at the collapsing phase. The non-dimensional displacement of the thin bronze plate at its midpoint is about $z = 0.053$.

Figures 6 (A) and (B) illustrate the successive time dependent profiles of the bubble in the vicinity of a thin bronze plate with the thickness of $th^* = 0.008$ during its growth and collapse phases. The bubble is initially located at $\gamma = 1.5$ and is characterized by $\delta = 0$ and $\varepsilon = 10$. During the growth and collapse phases, the bubble centroid moves away from the thin bronze plate. Figure 6 (B) shows that the side of the bubble close to the thin bronze plate during its collapse phase is flattened and a liquid jet is developed on the flattened side of the bubble and is directed away from the plate.

Figures 7 (A) and (B) illustrate the thin bronze plate deformation profiles in a proper scale. It is shown that the significant deformation takes place during the collapse phase of the bubble.

Figure 8 illustrates the variation of the bubble volume with

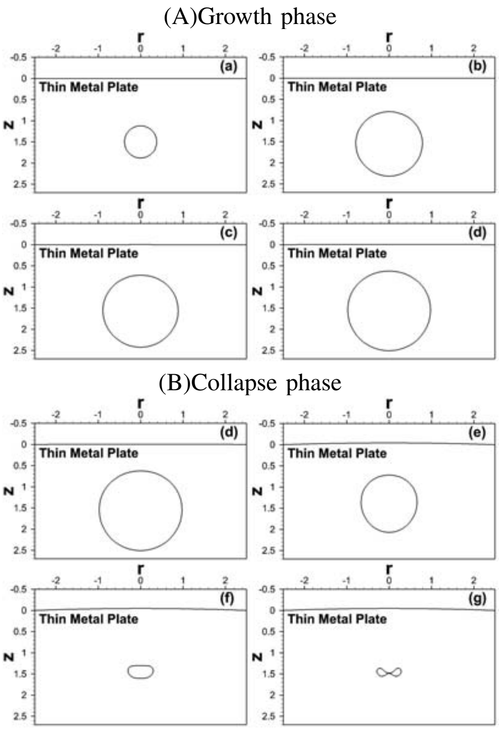

Fig. 4. Time dependent profiles of the vapour bubble during its growth and collapse phases near a thin bronze plate. The bubble is characterized by $\gamma = 1.5$, $\delta = 0$ and $\varepsilon = 10$. The thin bronze plate is characterized by the thickness of $th^* = 0.0089$. The non-dimensional times corresponding to the bubble successive profiles are: (A) Growth phase: $(a) = 0.00181$, $(b) = 0.45421$, $(c) = 0.64757$, $(d) = 1.45775$. (B) Collapse phase: $(d) = 1.45775$, $(e) = 2.14886$, $(f) = 2.29621$, $(g) = 2.31240$.

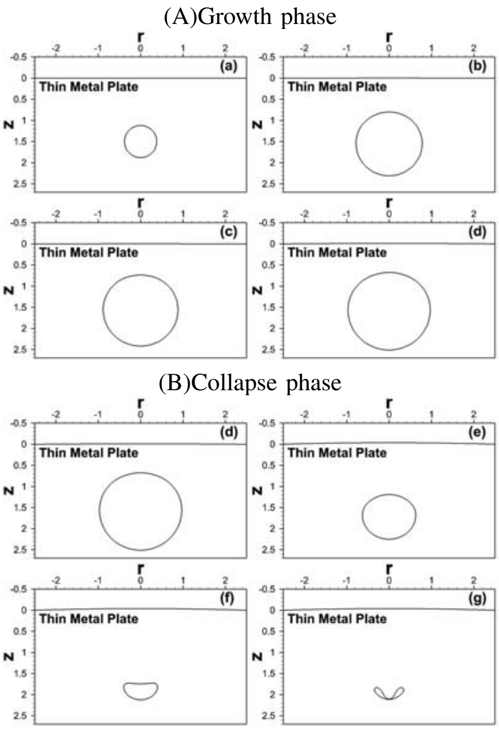

Fig. 6. Time dependent profiles of the vapour bubble during its growth and collapse phases near a thin bronze plate. The bubble is characterized by $\gamma = 1.5$, $\delta = 0$ and $\varepsilon = 10$. The thin bronze plate is characterized by the thickness of $th^* = 0.008$. The non-dimensional times corresponding to the bubble successive profiles are: (A) Growth phase: $(a) = 0.00181$, $(b) = 0.44909$, $(c) = 0.64009$, $(d) = 1.44566$. (B) Collapse phase: $(d) = 1.44566$, $(e) = 1.91041$, $(f) = 2.00156$, $(g) = 2.02447$.

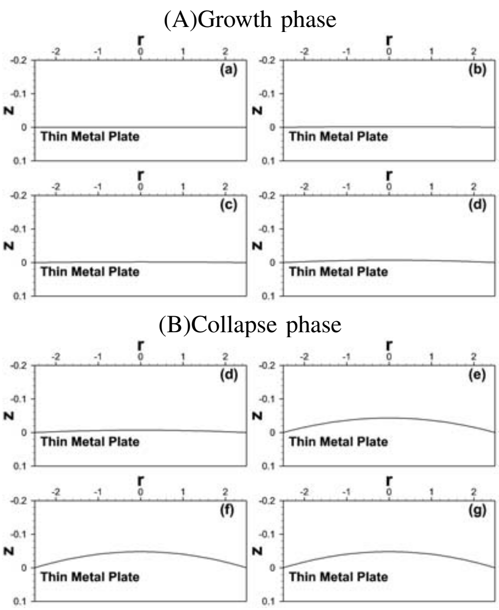

Fig. 5. The thin bronze plate deformation profiles in a proper scale at the non-dimensional times and bubble characteristics corresponding to that of figure 4.

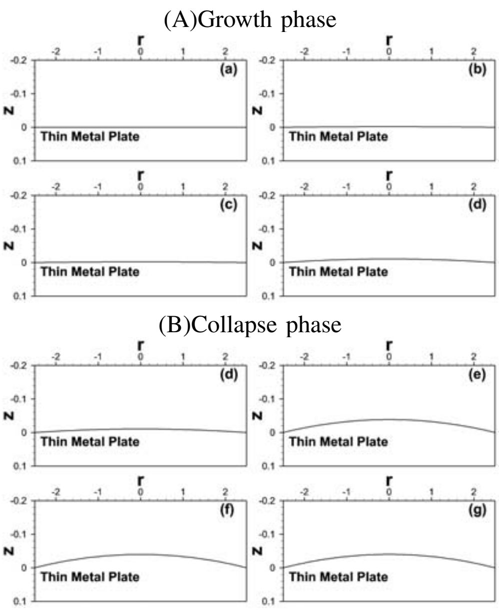

Fig. 7. The thin bronze plate deformation profiles in a proper scale at the non-dimensional times and bubble characteristics corresponding to that of figure 6.

respect to non-dimensional time during its growth and collapse phases near a rigid boundary and near a thin bronze plate with different thicknesses. This figure indicates that the lifetime of the bubble decreases by decreasing the thickness of the plate. This figure also shows that the vapour bubble in the vicinity of a thin bronze plate expands to a maximum volume which is smaller than that of near a rigid boundary. It is seen that as thickness of the thin bronze plate decreases, the maximum volume of the vapour bubble at the end of its growth phase becomes smaller.

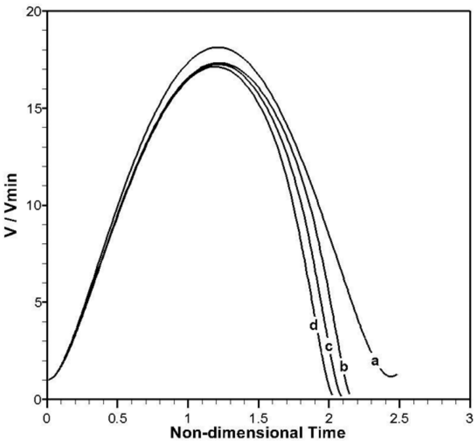

Fig. 8. Variation of the bubble volume with respect to non-dimensional time during its growth and collapse phases near a rigid boundary and near a thin bronze plate with different thicknesses. (a) Rigid boundary, $(b)th^* = 0.0086$, $(c)th^* = 0.0083$ and $(d)th^* = 0.008$. The bubble is characterized by $\gamma = 1.5$, $\delta = 0$ and $\varepsilon = 10$.

Figure 9 shows the movement of the bubble centroid with respect to non-dimensional time during its growth and collapse phases near a rigid boundary and near a thin bronze plate with different thicknesses. It should be noted that in the case of the bubble near a rigid boundary, the bubble centroid moves away from the rigid boundary during the growth phase, and during the collapse phase it moves rapidly towards the rigid boundary. It is interesting to note that during the growth and collapse phases of the vapour bubble in the vicinity of a thin bronze plate, the bubble centroid migrates away from the plate.

IX. Conclusion

In this paper, perfect plastic deformation of a thin bronze plate due to the pulsation of a vapour bubble is numerically investigated by using the boundary integral equation method. The numerical calculations are carried out by assuming no buoyancy force and no heat transfer inside the bubble. The plate is assumed to be circular with a specific radius which is attached to a rigid boundary at its edge.

Results indicate that the minute displacement of the thin bronze plate has significant effect on the dynamics of its nearby vapour bubble.

It is also found that by decreasing the thickness of the thin bronze plate, the growth and collapse rate of the bubble becomes higher and consequently the lifetime of the bubble becomes shorter.

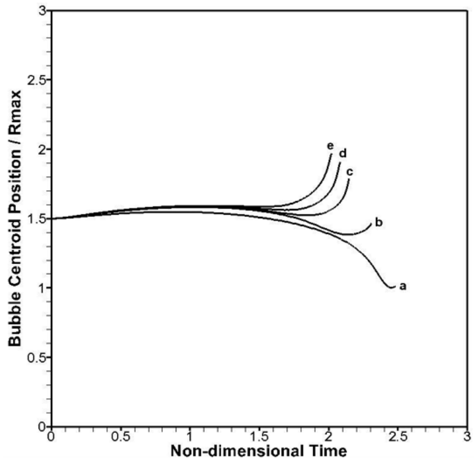

Fig. 9. Movement of the bubble centroid with respect to non-dimensional time during its growth and collapse phases near a rigid boundary and near a thin bronze plate with different thicknesses. (a) Rigid boundary, $(b)th^* = 0.0089$, $(c)th^* = 0.0086$, $(d)th^* = 0.0083$ and $(e)th^* = 0.008$. The bubble is characterized by $\gamma = 1.5$, $\delta = 0$ and $\varepsilon = 10$.

It is shown that the centroid of the bubble migrates towards the rigid boundary, while for a plate with proper thicknesses, it moves away from the thin bronze plate.

References

[1] E. Klaseboer, C. K. Turangan and B. C. Khoo, *Dynamic behaviour of a bubble near an elastic infinite interface*,International Journal of Multiphase Flow, 2006, vol. 32, pp. 1110-1122.
[2] J. R. Blake, B. B. Taib and G. Doherty, *Transient cavities near boundaries Part 2. Free surface*,Journal of Fluid Mechanics, 1987, vol. 181, pp. 197-212.
[3] J. R. Blake, *The Kelvin impulse: application to cavitation bubble dynamics.*,The ANZIAM Journal, 1988, vol. 30, pp. 127-146.
[4] J. R. Blake and P. Cerone, *A note on the impulse due to a vapour bubble near a boundary.*,The ANZIAM Journal, 1982, vol. 23, pp. 383-393.
[5] J. R. Blake and D. C. Gibson, *Growth and collapse of a vapour cavity near a free surface.*,Journal of Fluid Mechanics, 1981, vol. 111, pp. 123-140.
[6] P. Cerone and J. R. Blake, *A note on the instantaneous streamlines, pathlines and pressure contours for a cavitation bubble near a boundary.*,The ANZIAM Journal, 1984, vol. 26, pp. 31-44.
[7] G. L. Chahine, *Interaction Between an Oscillating Bubble and a Free Surface.*,Journal of Fluids Engineering, 1977, vol. 99, pp. 709-716.
[8] D. C. Gibson and J. R. Blake, *Growth and Collapse of Cavitation Bubbles Near Flexible Boundaries.*,Australasian Conference on Hydraulics and Fluid Mechanics, 1980, pp. 283-286.
[9] J. P. Best, *The dynamics of underwater explosions.*,PhD thesis, University of Wollongong, 1991.

M. T. Shervani-Tabar , Ph.D, He received his B.Sc. (Eng) in Mechanical Engineering in 1984 from University of Tabriz, Iran, and also received his M.Sc. (Eng) in Mechanical Engineering in 1989 from University of Tabriz. He received his PhD in 1995 from University of Wollongong, Australia. He is currently a professor in the Department of Mechanical Engineering, University of Tabriz, Iran. His research interests include cavitation and bubble dynamics, computational fluid dynamics and experimental study and visualization of cavitation in hydraulic machineries.

M. Rezaee , Ph.D, received the BSc. degree in Mechanical Engineering from University of Tabriz, Tabriz, Iran, in 1991, and the MSc and Ph.D degrees in Applied Mechanics from the Tarbiat Modaress University, Tehran, Iran, in 1995 and 2001, respectively. He is currently an associate professor in the Department of Mechanical Engineering, University of Tabriz, Tabriz, Iran. His areas of interest include Dynamics, Vibration Analysis, Damage Detection of Mechanical Systems and Control. He currently teaches graduate and under graduate courses in Dynamics, Vibration and Control theory and conducts research on Nonlinear Dynamic System, Random Vibration, and Damage Detection of structures and mechanical systems. He served as a reviewer for some journals and conferences and is the author of more than forty publications.

E. Madadi Kandjani , M.Sc., received the B.Sc. degree in Mechanical Engineering from Islamic Azad University of Tabriz, Tabriz, Iran, in 2008, and the M.Sc. degree in Mechanical Engineering (Energy Conversion) from the University of Tabriz, Tabriz, Iran, in 2011. His areas of interest are numerical investigations of fluid flows, and growing and collapsing of a vapour bubble interacting with different boundaries.