

Nonlinear Dynamic Modeling and Active Vibration Control of a System with Fuel Sloshing

A. A. Jafari, A. M. Khoshnood, J. Roshanian

Abstract—Attitude control of aerospace system with liquid containers may face to a problem associate with fuel sloshing. The sloshing phenomena can degrade the stability of control system and in the worst case, interaction between the attitude control system and fuel vibration leading to resonance. In this paper, a full process of nonlinear dynamic modeling of an aerospace launch vehicle with fuel sloshing is given. Then, a new control system based on model reference adaptive filter is proposed and its algorithm is extracted. This controller implemented on the main attitude control system. Finally, numerical simulation of nonlinear model and control system is carried out to examine the performance of the new controller. Results of simulations show that the inconvenient effects of the fuel sloshing by augmenting this control system are reduced and attitude control system performs, satisfactorily.

Keywords—nonlinear dynamic modeling; fuel sloshing; vibration control; model reference; adaptive filter

I. INTRODUCTION

ONE of the main problem in the liquid containers refers to the vibrational behavior of the liquid and its interaction with the other components of system. This behavior called sloshing commonly exerts forces and moments on the container walls that can cause structural damage. This force is directly proportional to the slosh mass. Slosh mass forms a significant fraction of total mass of typical system, for this reason, slosh force can have significant affects on overall system dynamics. Specially, in systems that the closed loop control system is used, interaction between the control system and the sloshing, play an important role in the performance of system. Moreover, the effects of sloshing in the worst case can degrade the stability of the system. For investigation on the slosh phenomenon, authors have been usually considered two practical fields: the first one includes vibrational characteristics of the fuel slosh and the problem in which how to model this phenomenon. On the other hand, the authors have responded this question that which model can describe

the sloshing of fuel, accurately [1]. Meserole and Fortini have investigated the slosh dynamics in a Toroidal tank. They have extracted forces and characteristics of the sloshing as a hypothetic mechanical model [2]. Also, Faltinsen and Timohkha have proposed an asymptotic modal approximation of nonlinear resonant sloshing in a rectangular tank with small fluid depth [3].

In the second investigations, the scope of works has inclined to dynamic of multi body model. On the other word, the real aim of these researches leads to dynamic modeling of a main system simultaneously with fuel slosh at the same time. In fact, in this field of investigations the effects of sloshing phenomenon on the flight parameters are considered. In this way, Nichkawde et al have tried to driven a planner model for fuel slosh in an aerospace system [4]. They have assumed that the mechanical model of slosh behaves as a pendulum. In this case the equations of motion are nonlinear, so linearization for stability analysis is necessary.

It is noticeable that although common control systems implemented on the aerospace vehicle can satisfactorily execute attitude control, in the face of fuel sloshing and its inconvenient effects, one should improve this control system. Consequently, it must design a new control strategy for suppressing the destructive effects of the sloshing. In additional to the last activities, there are several works in which an active vibration control system has been designed for reducing the vibrational effects of the sloshing as well as dynamic modeling. In this way, it has been used various types of control approaches associate with suppression of sloshing vibration and recently these activities have considerably developed. From these activities, Krishnaswamy and Dan Bugajski have proposed a systematic output feedback design procedure for inner loop control of multi-body vehicles with fuel slosh [5]. Specifically, in their study the fuel sloshing has been modeled as a normal pendulum and the dynamic inversion-based nonlinear control algorithm has been conveniently presented. Moreover, in the field of control system design, Dong et al have proposed an estimation approach for propellant sloshing effect on Spacecraft [6]. In the other works, various methods of advanced controllers such as optimal, robust and adaptive controller have been employed and implemented [7].

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Although these approaches can reduce the destructive effects of fuel sloshing, the noticeable problem of vibration control of sloshing refers to uncertainty of its parameters due to the flight time. Considerably, this problem leads to requirement of a high performance estimator which identify the parameters of vibration through the flight time. For this reason, one of the aims of this paper is proposing an algorithm for estimating of vibration parameters and reducing its inconvenient effects.

In this paper, for investigation on the effects of fuel sloshing on the characteristics of flight dynamic in a launch vehicle, a mechanical model is used. This mechanical model which is widely employed for fuel sloshing is considered as a mass-spring system. Consequently, the dynamic behavior of the overall model included flight dynamic and fuel sloshing in final form changes the problem of the single body dynamic modeling to the multi body dynamic one. Therefore, in this work, the model of fuel sloshing in a launch vehicle is driven in the point of view of a multi body system specially, in 6 degree of freedom (6-dof) form.

Continuously, in this study a new control strategy based on adaptive filtering is proposed for reducing the destructive effects of sloshing. In this control strategy after estimating the frequencies of vibration sources such as sloshing, one or more filters are employed to suppress the closed loop control system from destructive effects of vibration associated with these frequencies. Results of numerical simulation demonstrate that the dynamic model includes the frequency bound of sloshing and the control system successfully can reduce the effects of vibration.

This paper is organized as follows: the first section refers to introduction of modeling and control of sloshing. Dynamic modeling process of fuel sloshing and flight dynamic of overall system is expressed in the second section. In the third section vibration control strategy and its details are given. Simulation results and conclusion are expressed in the fourth and final section, respectively.

II. DYNAMIC MODELING STRATEGY

A. Dynamic modeling of vehicle with fuel sloshing

As stated in the last section the model of sloshing taken in this work is a mechanical model. By this assumption the overall dynamic model of the vehicle with 6-dof and one added mass-spring leads to dynamic modeling of a multi body system. In order to derive the equations of motions for a multi body system in this work, it is used floating frame of reference approach. This method is widely used for the system in which rigid body motion of the main system is noticeable in contrast with the other motions. Defining the body frames in floating frame of reference method is shown in figure (1). For fuel sloshing as illustrated in figure (2) one can express the displacement vector as:

$$\begin{aligned} E &= R + r \\ r &= r_o + e^s \end{aligned} \quad (1)$$

where E^s is the position of slosh mass in the inertial frame, R is the position of vehicle in the inertial frame, r is the position in the body frame and e^s is the displacement of slosh mass in the body frame.

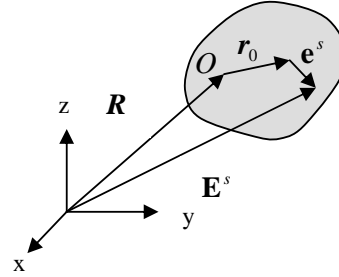


Fig. 1 Schematic of floating frame of reference approach.

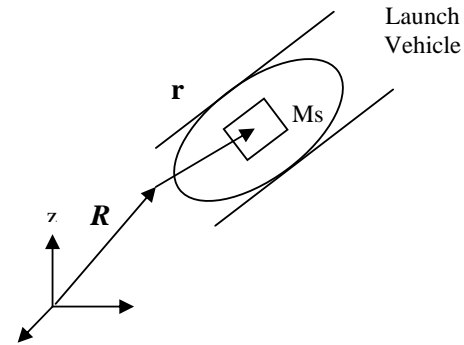


Fig. 2 Definition of vectors for stating displacement of sloshing mass

In multi body dynamic it is usually unavoidable to use of the Lagrangian approach. Specially, in this dynamic modeling it is better to use Quasi-coordinate method [8]. So, with regard to equation (1) and stating the velocity of the slosh parameters, the vehicle total kinetic energy, potential energy and dissipative energy, in vector form as:

$$T = \frac{1}{2} \int \frac{dE^T}{dt_1} \frac{dE}{dt_1} dm + \frac{1}{2} \int \frac{dE^{sT}}{dt_1} \frac{dE^s}{dt_1} dm^s \quad (2)$$

$$\begin{aligned} \Rightarrow T &= \frac{1}{2} m^s V^T V + \frac{1}{2} (\bar{\omega}^T S \bar{\omega}) m^s + \frac{1}{2} \frac{d\bar{e}^{sT}}{dt_c} \frac{d\bar{e}^s}{dt_c} m^s \\ &+ \frac{1}{2} \bar{\omega}^T G \bar{\omega} + \bar{\omega}^T \tilde{V}^T \int \bar{r}_{0c}^s dm^s + \bar{\omega}^T V^T \int \bar{e}^s dm^s \\ &+ V^T \frac{d\bar{e}^s}{dt_c} m^s + \bar{\omega}^T \left(\frac{d\bar{e}^s}{dt_c} \right)^T \bar{r}_{0c}^s m^s + \bar{\omega}^T \frac{d\bar{e}^{sT}}{dt_c} \bar{e}^s m^s \\ &+ \bar{\omega}^T D \bar{\omega} + \frac{1}{2} m V^T V + \frac{1}{2} (\bar{\omega}^T I \bar{\omega}) \end{aligned} \quad (3)$$

where E is the position of body in the inertial frame, T is kinetic energy, V is the velocity vector of the vehicle, $\bar{\omega}$ is the angular velocity vector, I the inertial momentums matrix

of vehicle, m^s , m are mass of sloshing part and mass of vehicle, respectively. Also, S, G and D are:

$$S = \tilde{r}_{0c}^s \tilde{r}_{0c}^{sT}$$

$$G = \tilde{e}^s \tilde{e}^{sT}$$

$$D = \left(\int \begin{bmatrix} \bar{z}^s \bar{e}_z^s + \bar{y}^s \bar{e}_y^s & -\bar{x}^s \bar{e}_y^s & -\bar{x}^s \bar{e}_z^s \\ 0 & \bar{z}^s \bar{e}_z^s & -\bar{y}^s \bar{e}_z^s \\ 0 & -\bar{z}^s \bar{e}_y^s & \bar{y}^s \bar{e}_y^s \end{bmatrix} dm^c \right)$$

Moreover, potential and dissipative energy of system is given as:

$$U = \frac{1}{2} [k_z (e_z^s)^2 + k_y (e_y^s)^2] \quad (4)$$

$$D^s = m^s [\mu_z^s \omega_z^s (\dot{e}_z^s)^2 + \mu_y^s \omega_y^s (\dot{e}_y^s)^2] \quad (5)$$

where K_z, K_y are stiffness magnitude of sloshing in z and y direction, μ_z^s, μ_y^s are damping parameters and ω_z^s, ω_y^s are frequencies of sloshing in the same directions. Approximately, the fuel sloshing has not any component in x direction. It is noticeable that the parameters of sloshing are extracted from equation related to fluid dynamic theories or experimental investigations. So, in the current paper this process is not given. Continuously, using quasi-coordinate Lagrangian method as the followings lead to derivation of the coupled rigid-slosh non-linear equation of motion:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{V}} \right) + \tilde{\omega} \frac{\partial L}{\partial V} &= F \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\omega}} \right) + \tilde{V} \frac{\partial L}{\partial V} + \tilde{\omega} \frac{\partial L}{\partial \omega} &= M \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{e}^s} \right) - \frac{\partial L}{\partial e^s} &= Q_e \end{aligned} \quad (6)$$

$$\begin{aligned} \ddot{e}^s m^s + \dot{V} m^s + (\tilde{r}_{0c}^{sT} \dot{\tilde{\omega}}^c) m^s + \tilde{e}^{sT} \dot{\tilde{\omega}}^c m^s \\ + \tilde{e}^{sT} \dot{\tilde{\omega}}^c m^s - [(\tilde{\omega}^{cT} \tilde{\omega}^c) \tilde{e}^s] m^s - \tilde{\omega}^{cT} V m^s \\ - \frac{d\tilde{e}^s}{dt_c} \tilde{\omega}^c m^s - \tilde{\omega}^{cT} \tilde{r}_{0c}^{sT} \tilde{\omega}^c m^s + \Omega \dot{e}^s + K e^s = 0 \end{aligned} \quad (7)$$

$$\begin{aligned} \dot{V} m^s + \dot{V} m + \tilde{r}_{0c}^{sT} \dot{\tilde{\omega}}^c m^s + \tilde{\omega}^c \tilde{e}^s m^s \\ + \tilde{\omega}^c \tilde{e}^s m^s + \tilde{e}^s m^s + \tilde{\omega} [V m^s + V m \\ + \tilde{r}_{0c}^{sT} \tilde{\omega}^c m^s + \tilde{\omega}^c \tilde{e}^s m^s + \frac{d\tilde{e}^s}{dt_c} m^s] = F \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{1}{2} (\dot{S} + \dot{S}^T) \tilde{\omega}^c m^s + \frac{1}{2} (S + S^T) \dot{\tilde{\omega}}^c m^s \\ + \tilde{e}^{sT} \tilde{e}^{sT} \tilde{\omega}^c m^s + \tilde{e}^s \tilde{e}^{sT} \tilde{\omega}^c m^s \tilde{e}^s \tilde{e}^{sT} \dot{\tilde{\omega}}^c m^s \\ + \tilde{r}_{0c}^{sT} \dot{V} m^s + \tilde{e}^s V m^s + \tilde{e}^s \dot{V} m^s + \tilde{r}_{0c}^{sT} \ddot{e}^s m^s \\ + (\tilde{e}^s)^2 m^s + \tilde{e}^s \tilde{e}^s m^s + (\dot{D} + \dot{D}^T) \tilde{\omega}^c \\ + (D + D^T) \dot{\tilde{\omega}}^c + I \dot{\tilde{\omega}}^c + \tilde{V} [V m^s + V m \\ + \tilde{r}_{0c}^{sT} \tilde{\omega}^c m^s + \tilde{\omega}^c \tilde{e}^s m^s + \frac{d\tilde{e}^s}{dt_c} m^s] \\ + \tilde{\omega} \left[\frac{1}{2} (S + S^T) \tilde{\omega}^c m^s + \tilde{e}^s \tilde{e}^{sT} \tilde{\omega}^c m^s \right. \\ \left. + \tilde{r}_{0c}^{sT} V m^s + \tilde{e}^s V m^s + \tilde{r}_{0c}^{sT} \frac{d\tilde{e}^s}{dt_c} m^s \right. \\ \left. + \tilde{e}^s \frac{d\tilde{e}^s}{dt_c} m^s + (D + D^T) \tilde{\omega}^c + I \tilde{\omega}^c \right] = M \end{aligned} \quad (9)$$

where L is Lagrangian, F and M are external forces and momentums vector of vehicle, Ω is damping ratio matrix and K is stiffness matrix of fuel sloshing.

B. Linearization of equations of motion s

Linearization of the equations of motion (7-9) is done about equilibrium operating points considering common perturbation approach. Because in this paper the main aim refers to the influence of sloshing on attitude of the vehicle one can extract transfer function between angular velocities and actuator commands. In addition, as the vehicle is symmetric only sloshing effects in the pitch channel is considered. Therefore, transfer function between angular velocity of pitch channel and its actuator command with sloshing effects as:

$$\frac{q}{\delta} = \frac{a_{n1} s^3 + a_{n2} s^2 + a_{n3} s + a_{n4}}{a_{d1} s^4 + a_{d2} s^3 + a_{d3} s^2 + a_{d4} s + a_{d5}} \quad (10)$$

where q is angular velocity of vehicle, δ is deflection of actuator (thrust vector control) and a_{ni}, a_{di} are parameters associated with rigid dynamic of vehicle and fuel sloshing.

III. VIBRATION CONTROL STRATEGY

One of the best approaches to vibration control of systems is prevention of excitation. Consequently, to prevent the destructive effects of fuel oscillations, the main strategy used in this study is to protect the control loop from the feeding back of vibrational bias. In the other word, it is desirable that the measuring devices do not send any excitation feedback to the actuators. For implementing this strategy, the control

system should consist of an algorithm for estimation of the fuel sloshing frequencies and one or more filters. As the vibration frequencies vary in respect to time the challenging issue is to estimate these values. The approach used in this paper to estimate the bending frequencies are the model reference adaptive method which is explained in this section.

The schematic of the new control system is shown in figure (3). The main block of the control system is a gain scheduled proportional–integral–derivative (PID) commonly used for rigid body attitude control. The next block is the dynamic model of the fuel sloshing and dynamic of pitch channel of the vehicle. In the feedback of this system, in addition to IMU, there is an augmented adaptive controller which performs the vibration control strategy. In this way, one can employ a kind of IIR filter as follows:

$$H(z) = \frac{N(z)}{D(z)} = \frac{1 + (1+a)K_0Z^{-1} + Z^{-2}}{1 + K_0(a+\lambda)Z^{-1} + \lambda Z^{-2}} \quad (11)$$

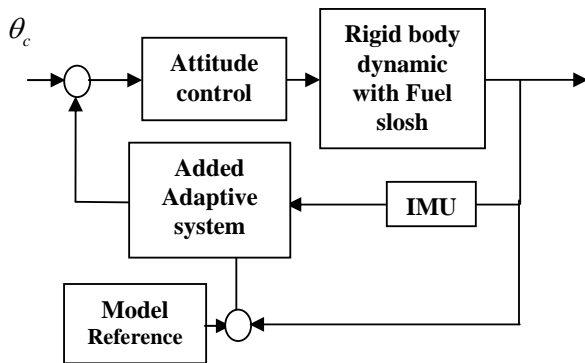


Fig. 3. Bloch diagram of attitude control system and added adaptive controller

Where K_0 and λ are the center frequency of the filter and a constant parameter, respectively. Also, a is a constant parameter which can equal to one [9].

One of the practical approaches for estimating characteristics of vibrational systems is model reference method [10]. In this approach, a closed loop control system is constructed for the rigid body dynamic without vibrational bias, and then, this closed loop stable system is applied as a model reference for the actual system with vibrational source. Now, one can employ an algorithm such as Gauss-Newton to minimize the root mean square of the error between the actual and reference model [9] as the followings:

error between the actual and reference model [9] as the followings:

$$x(n) = \frac{1}{D(z)} u(n) \quad (12)$$

$$y(n) = x(n) + 2K_0x(n-1) + x(n-2) \quad (13)$$

$$e = y - y_m$$

$$\Rightarrow E(n) = \sum_{k=0}^n \lambda^{n-k} e^2(k) = \quad (14)$$

$$\sum_{k=0}^n \lambda^{n-k} [x(k) + 2K_0x(k-1) + x(k-2) - y_m]^2$$

where λ is the forgetting factor that puts less weight on the past data. One can introduce new variables as:

$$A(k) = 2x(k-1)$$

$$B(k) = x(k) + x(k-2) - y_m \quad (15)$$

$$\Rightarrow E(n) = \sum_{k=0}^n \lambda^{n-k} [A(k)k_0 + B(k)]^2$$

Differentiating the cost function with respect to K_0 and making this zero, the value of $K_0(n)$ to minimize $E(n)$ can be obtained as equation (16). Also, this equation can be expressed as the following recursive form by using the weighted least square algorithm:

$$k_0(n) = -\frac{N(n)}{D(n)} = -\frac{\sum_{k=0}^n \lambda^{n-k} A(k)B(k)}{\sum_{k=0}^n \lambda^{n-k} A^2(k)} \quad (16)$$

It is important to say that the stability of this algorithm depend on the stability of the filter. Hence, if the parameters of the filter are chosen or estimated with regard to stable bounds, the stability of overall adaptive system is ensured.

IV. SIMULATION RESULTS

In the present paper, motion of an aerospace launch vehicle is analyzed by an 7-dof simulation in Matlab/Simulink environment. On the other word, the model consists of 6-dof for rigid body motion and one dof associated with fuel sloshing. It is noticeable to state that although in this work only one mode of sloshing is considered, the approach in modeling as well as control system design can extend for more mode of vibration without changing in the basic relations. The 7-dof nonlinear time dependent simulations are carried out with a sampling time of 0.01 sec and the transformed equations of motion are integrated numerically by using the fourth-order Runge-Kutta method.

Simulation results are given in figures (4)-(10). In figure (4) the effect of sloshing on the pitch channel of vehicle in nonlinear model is illustrated. The existence of oscillation arisen from sloshing is considerably shown in this situation. Because, at the first time of flight the fuel tank is full the fuel sloshing usually has no affects on the system in this time. Moreover, behavior of pitch channel in linear model is demonstrated in figure (5). Considerably, in this figure oscillations arisen from sloshing are clear. After implementing

the control system designed in the previous section, simulation results shows conveniently responses. However, figure (6) is related to performance of estimation algorithm. Estimation of vibration frequency of fuel sloshing is demonstrated in this figure. In fact, this performance is implemented using the error between reference and actual model. The behavior of state variable introduced for displacement of fuel sloshing is shown in figure (7). On the other hand, this figure illustrates the displacement of active mass associated with sloshing due to the flight time. Finally, performance of overall controller system consists of rigid controller, estimation algorithm and adaptive filter in the pitch channel is shown in figures (8, 9). Figure (8) shows the error between reference and actual model which is closes to zero and figure (9) depicts the output of pitch channel due to the application of the adaptive controller and without this added adaptive system. In this response reducing of the sloshing effects is considerably demonstrated. On the other words, the suppression of the destructive vibration associated with sloshing implements, satisfactorily.

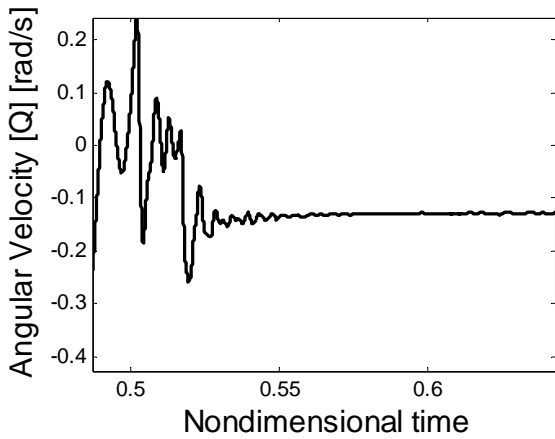


Fig. 4 effects of fuel sloshing on the angular velocity in nonlinear model

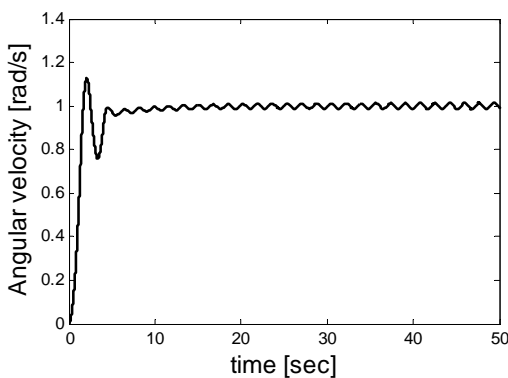


Fig. 5 effects of fuel sloshing on the angular velocity in linear model.

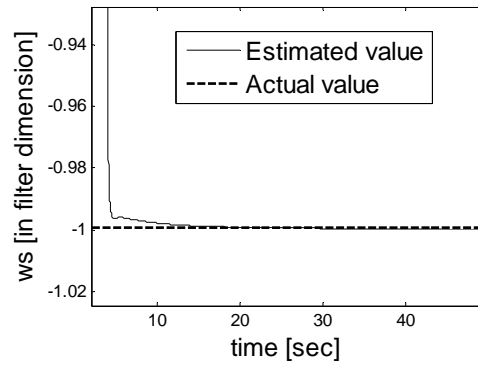


Fig. 6 Estimation of frequency of sloshing in flight time

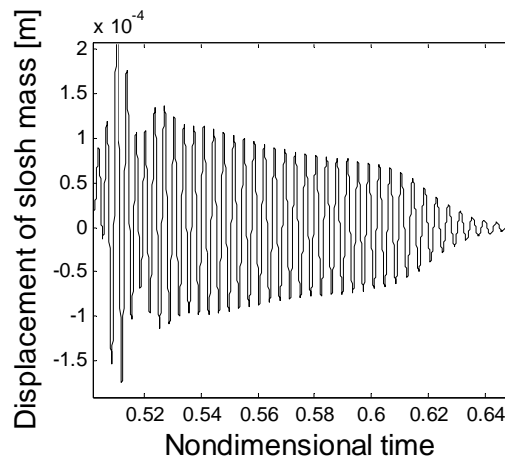


Fig. 7 Displacement of slosh mass in one of the flight condition

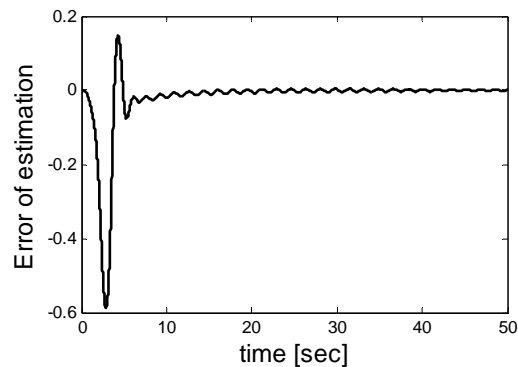


Fig. 8 The error between reference and actual model

V. CONCLUSIONS

Attitude control of aerospace system with liquid fuel may face to a problem associate with fuel sloshing. The sloshing phenomena can degrade the stability of control system and in the worst case leading to resonance. In this paper, a full process about nonlinear dynamic modeling of an aerospace launch vehicle specially, in vector form is given. In continuous, after linearization of nonlinear model transfer function between angular velocity and actuator command is driven. In fact, this function shows the effects of fuel sloshing

on the attitude control system of vehicle. In the final step, a new control system based on model reference adaptive filter is proposed and its algorithm is extracted. This controller implemented on the main attitude control system. Finally, numerical simulation of nonlinear model and control system is carried out to examine the performance of the new controller. Results of simulations show that the inconvenient effects of fuel sloshing by augmenting this control system are reduced and attitude control system performs, satisfactorily.

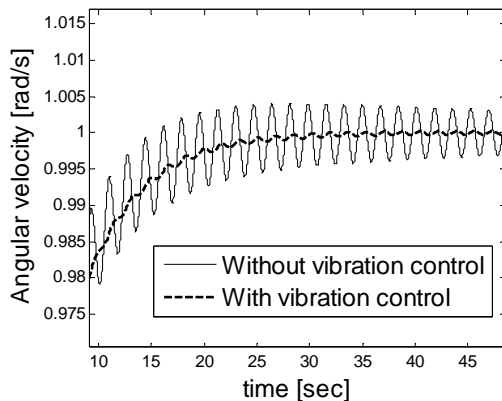


Fig. 9 Output of pitch channel due to the application of the adaptive controller and without this system

APPENDIX

Notations (dynamic model)

- (\bullet) Assign for vectors expressed in the body coordinate
- $(\tilde{\bullet})$ Definition of a cross matrix by a vector
- R Position vector $[x, y, z]$
- V Velocity vector $[u, v, w]$
- ω Angular velocity vector $[p, q, r]$
- e^s Displacement of sloshing $[0, e_y, e_z]$
- I Mass momentum of inertia
- $[I_x, 0, 0; 0, I_y, 0; 0, 0, I_z]$
- m Mass of vehicle
- m^s Active mass of fuel sloshing
- ω^s Frequency of sloshing in each direction
- μ^s Damping ratio of sloshing in each direction

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