

# Magnetohydrodynamic Damping of Natural Convection Flows in a Rectangular Enclosure

M. Battira, and R. Bessaih

**Abstract**—We numerically study the three-dimensional magnetohydrodynamics (MHD) stability of oscillatory natural convection flow in a rectangular cavity, with free top surface, filled with a liquid metal, having an aspect ratio equal to  $A=L/H=5$ , and subjected to a transversal temperature gradient and a uniform magnetic field oriented in  $x$  and  $z$  directions. The finite volume method was used in order to solve the equations of continuity, momentum, energy, and potential. The stability diagram obtained in this study highlights the dependence of the critical value of the Grashof number  $Gr_{crit}$ , with the increase of the Hartmann number  $Ha$  for two orientations of the magnetic field. This study confirms the possibility of stabilization of a liquid metal flow in natural convection by application of a magnetic field and shows that the flow stability is more important when the direction of magnetic field is longitudinal than when the direction is transversal.

**Keywords**—Natural convection, Magnetic field, Oscillatory, Cavity, Liquid metal.

## I. INTRODUCTION

NATURAL convection of a conducting fluid of electricity contained in a cavity represents an adequate research subject, because of its presence in many industrial processes, especially during the process of crystal growth (Tagawa and Ozoe, 1997). The widespread use of this process in electronic and optical applications had, for consequence, an extended research towards the comprehension and the control of the natural convection in these systems.

With the application of an external magnetic field, it is possible to act on the flows without any physical contact, and thus to remove the fluctuations to control heat and mass transfers, in order to improve the quality of the crystal. For this purpose, the damping magnetic to control the flow induced by a temperature variation was used in several industrials applications [1]-[8]. Tagawa and Ozoe [1] numerically studied three-dimensional natural convection of a liquid metal in a cubic enclosure, under the action of a magnetic field applied, according to the three main directions.

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Benhadid and Henry [2] studied the effect of a magnetic field on the flow of liquid metal in a parallelepiped cavity, using a spectral numerical method. Bessaih et al. [3] numerically examined the effect of the electric conductivity of the walls and the direction of the magnetic field on the flow of Gallium. Their results show a considerable reduction in the intensity of the convection when the magnetic field increases. Juel and al. [4] had the results of a numerical and experimental study of the effect of the application of a magnetic field in the direction perpendicular to the convective flow of Gallium. Aleksandrova and Molokov [5] considered three-dimensional convection in a rectangular cavity subjected to a horizontal temperature gradient and a magnetic field, by an asymptotic model. The effectiveness of the application of the magnetic field depends considerably on the aspect ratio and the value of the Hartmann number. Hof and al. [6] presented an experimental study of the effect of the magnetic field on the natural convection stability in a rectangular cavity of square section, filled with a liquid metal. These authors founded that the vertical direction of the magnetic field is most effective for the suppression of oscillations. This is in good agreement with the work of Gelfgat and Bar-Yoseph [7].

In the present work, we present a three-dimensional numerical study on the critical value of the Grashof number  $Gr_{crit}$ , and the magnitude and orientation of a uniform magnetic field. Here, the geometry is the same considered by Xu et al. [8], which is filled with the liquid Gallium.

## II. GEOMETRY AND MATHEMATICAL MODEL

The geometry of the flow field analysed in this study is illustrated in Fig. 1. A liquid metal with a density  $\rho$ , a kinematics viscosity  $\nu$  and an electrical conductivity  $\sigma$ , fills a rectangular cavity of dimensions  $L \times H \times W$ , having an aspect ratio  $A=L/H=5$ , and submitted to a uniform magnetic field  $B_0$ . The magnetic field is applied separately in  $x$ -, and  $z$ -directions. The left wall is kept at a local hot temperature  $T_h$ , and the right wall is maintained at a local cold temperature  $T_c$  ( $T_h > T_c$ ). The upper surface of the cavity is free and the other walls are adiabatic. The fluid contained in the rectangular cavity is the Gallium whose Prandtl number equal to 0.02.

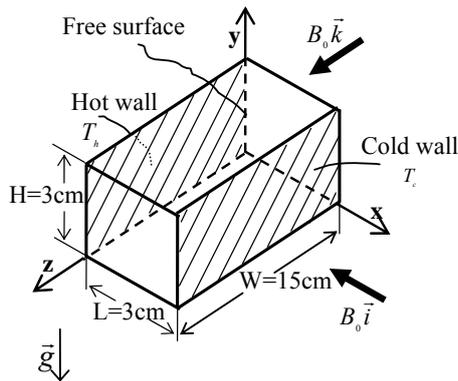


Fig. 1 The geometry of the problem

The interaction between the magnetic field and convective flow involves an induced electric current  $\vec{j}$ :

$$\vec{j} = \sigma [-\vec{\nabla}\phi + \vec{v} \wedge \vec{B}] \quad (1)$$

The divergence of Ohm's law  $\nabla \cdot \vec{j} = 0$ , produces the equation of the electric potential  $\phi$ :

$$\nabla^2 \phi = \vec{\nabla} \cdot (\vec{v} \wedge \vec{e}_s) \quad (2)$$

By Neglecting the induced magnetic field, the dissipation and Joule heating, and the Boussinesq approximation is valid; and using  $L$ ,  $v/L$ ,  $L^2/v$ ,  $\rho_0(v/L)^2$  and  $vB_0$ , and  $(T_h - T_c)$  as typical scales for lengths, velocities, time, pressure, potential, and temperature, respectively, the dimensionless governing equations for the conservation of mass, momentum, and energy, together with appropriate boundary conditions in the Cartesian coordinates system  $(x, y, z)$ , are written as follows:

$$\nabla \cdot \vec{v} = 0 \quad (3)$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\nabla p + \nabla^2 \vec{v} - Gr T \vec{e}_z + Ha^2 [(\vec{v} \times \vec{B}) \times \vec{B}] \quad (4)$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \frac{1}{Pr} \nabla^2 T \quad (5)$$

Where  $Gr = g\beta(T_h - T_c)L^3/v^2$  is the Grashof number,  $Ha = B_0 L \sqrt{\sigma/\mu}$  the Hartmann number, and  $Pr = \nu/\alpha$  the Prandtl number.

The initial conditions impose that the fluid is at rest and that the temperature distribution is zero, and that the electric potential is zero everywhere in the rectangular cavity. Thus, at  $t=0$ , we have:  $u=v=w=T=\phi=0$ .

At  $t>0$  the boundary conditions of the dimensionless quantities  $(u, v, w, T$  and  $\phi)$  are:

$$\text{At } x=0, \quad u=v=w=0, \quad T=1, \quad \frac{\partial \phi}{\partial x} = 0 \quad (6a)$$

$$\text{At } x=1, \quad u=v=w=0, \quad T=0, \quad \frac{\partial \phi}{\partial x} = 0 \quad (6b)$$

$$\text{At } y=0, \quad u=v=w=0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial \phi}{\partial y} = 0 \quad (6c)$$

$$\text{At } y=1, \quad \frac{\partial u}{\partial y} = v = \frac{\partial w}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = -Bi(T - T_c), \quad \frac{\partial \phi}{\partial y} = 0 \quad (6d)$$

$$\text{At } z=0, \quad u=v=w=0, \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial \phi}{\partial z} = 0 \quad (6e)$$

$$\text{At } z=5, \quad u=v=w=0, \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial \phi}{\partial z} = 0 \quad (6f)$$

The Biot number in equation (6d) is given by  $Bi = hL/k$ , where  $h$  is a heat transfer coefficient to the surroundings at the cold wall temperature, and  $k$  is the thermal conductivity of the fluid.

### III. NUMERICAL METHOD

The equations (2) – (5) with the boundary conditions (6a-6f) were solved by using the finite volume method [9]. Scalar quantities  $(P, T, \phi)$  were stored in the center of these volumes, whereas the vectorial quantities  $(u, v, \text{ and } w)$  are stored on the faces. For the discretisation of spatial terms, a second-order central difference scheme was used for the diffusion and convection parts of the equations (3-5), and the SIMPLER algorithm [9] was used to determine the pressure from continuity equation.

In order to examine the effect of the grid on the numerical solution, a number of grid sizes have been investigated for grid independence:  $32 \times 32 \times 52$ ,  $60 \times 60 \times 100$  and  $80 \times 80 \times 150$  nodes. By increasing the grid size from a  $60 \times 60 \times 100$  to  $80 \times 80 \times 150$  nodes, less than 2% change in computed values, was observed in Fig. 2. Therefore, the grid used has  $60 \times 60 \times 100$  nodes and was chosen after performing grid independency tests, since it is considered to have the best compromise between the computing time and the sufficient resolution in calculations. Calculations were carried out on a PC with CPU 3 GHz, thus the average computing time for a typical case was approximately of 24 hours.

### IV. RESULTS AND DISCUSSION

#### A. Code Validation with Experimental Data

The sentence of validation consisted in establishing some comparisons with experimental investigations presented in the literature [8]. We compared the temperature distribution  $T^*$  (in dimensional value) for various positions  $y$  with measurements obtained by Xu et al. [8] in a rectangular cavity, with stress-free top surface. We note easily that the computed values are in good agreement with measurements, in the absence of a magnetic field (Fig. 3(a)), and in the presence of a magnetic field of 300 Gauss, applied in the longitudinal direction (Fig. 3(b)).

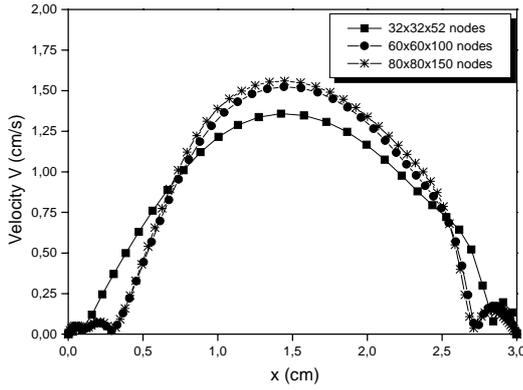
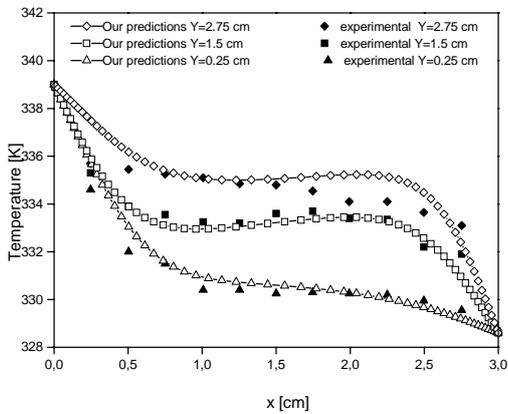
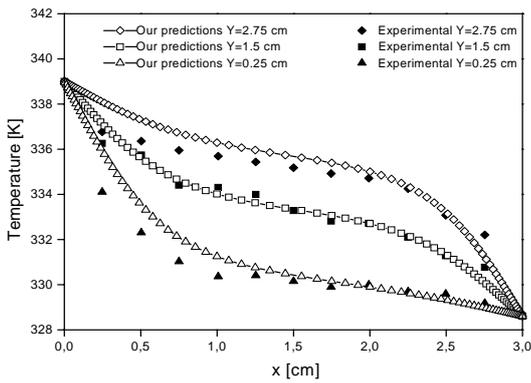


Fig. 2 Variation of the dimensional vertical velocity  $v$  vs.  $x$  for three grids.



(a)

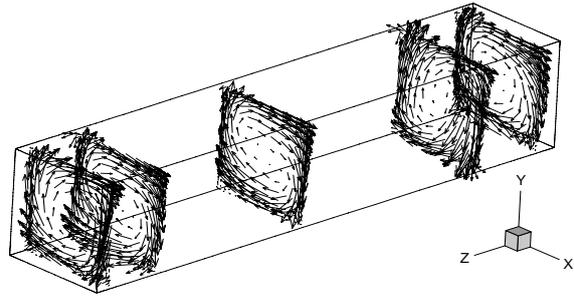


(b)

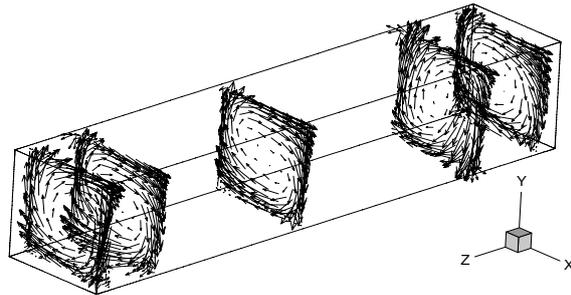
Fig. 3 Validation of the code with experimental measurements (Xu et al., 2006), for  $A=5$ . (a) Without magnetic field, (b) with magnetic field

*B. Flow Fields*

Fig. 4 (a) and Fig. 4(b) show the velocity field of flows in the rectangular cavity for both orientations of the magnetic field. We can see that the fluid moves from the hot wall (at  $x=0$ ) towards the cold wall (at  $x=W$ ). As the fluid moves away from the hot wall, the magnitude of velocity increases until the fluid approaches the cold wall. In this region, the magnitude of the velocity is reduced. Both figures show a single rotating cell. At the top of the cavity, the fluid flow circulates from the hot wall towards the cold wall.



(a)



(b)

Fig. 4 Velocity vectors with (a) transversal magnetic field  $B_x$ , (b) longitudinal magnetic field  $B_z$ .

*C. Oscillatory Solution with Magnetic Field*

In this section, we determine the physical instabilities within the flow from natural convection of a low Prandtl number fluid ( $Pr = 0.02$ ), contained in a rectangular cavity of an aspect ratio  $A=5$ . This flow is subjected to a magnetic field oriented in  $x$ -, and  $z$ - directions. Then, the determination of physical instabilities is reduced to the determination of the critical value of the Grashof number  $Gr_{crit}$  characterizing the subjacent flow, starting from which flow becomes oscillatory. It is noted that, the results obtained at given Grashof number were successively used for computation at the following  $Gr$  number.

From point of view of the dynamic systems, when a system re-enters in instability it presents to the beginning an oscillatory or periodic character, then because of the

bifurcation phenomenon, this system will become quasi-periodic, and finally it re-enters in chaos (or turbulence). The Grashof numbers characterizing the periodic flows are the critical numbers: transition from steady (Fig. 5) to time-dependent flow (Fig. 6 (a-d)).

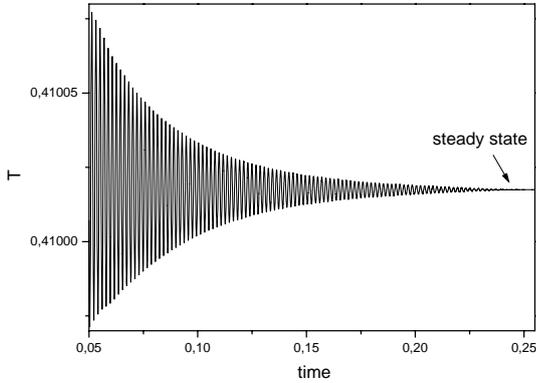
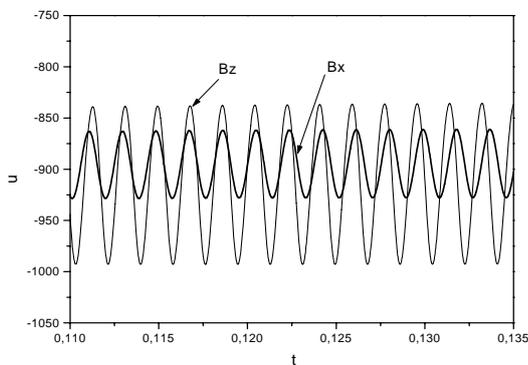
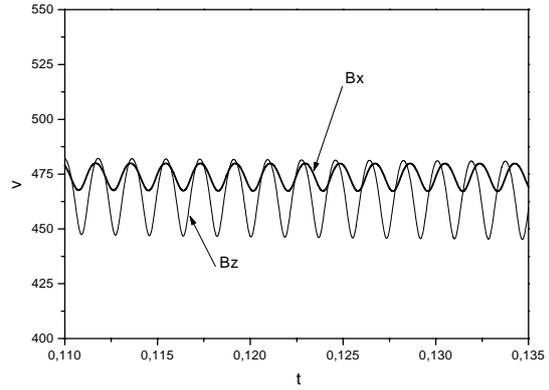


Fig. 5 Time evolution of the dimensionless temperature  $T$  for  $Gr=7.6 \times 10^6$  and  $Ha=0$

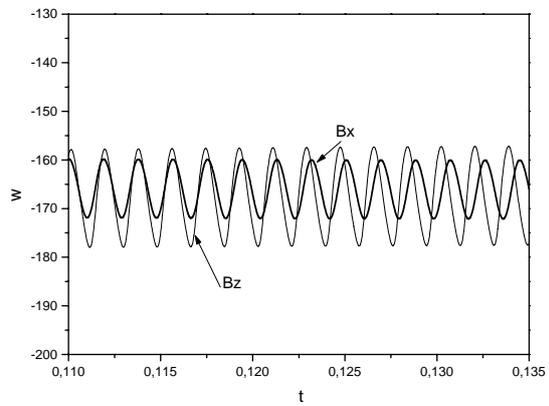
The oscillatory aspect (periodic) of the temporal evolutions of three dimensionless components velocity  $u$  (Fig. 6(a)) and  $v$  (Fig. 6 (b)) and  $w$  (Fig. 6 (c)) and dimensionless temperature  $T$  (Fig. 6 (d)) for  $Ha=7.5$ , recorded with a probe, indicates that oscillatory instabilities start and the flow bifurcates towards an unstable regime. By comparing the amplitudes of oscillations, we can notice that these amplitudes present different magnitudes, according to the points from recordings. This oscillatory behavior of the various parameters translates the existence of a continuous change of the flow structures (with periodicity). Also, we can notice clearly that the amplitudes of oscillations of the temperature (Fig. 6 (d)) are smaller than those of  $u$  (Fig. 6 (a)),  $v$  (Fig. 6 (b)) and  $w$  (Fig. 6 (c)). This can be interpreted by the domination of the conductive heat transfer mode in this type of low Prandtl number fluids flow. Also, we can notice on these figures, that the value of  $Gr_{crit}$  when the field is transverse is lower than that when the field is longitudinal. The amplitudes of oscillations are larger when the field is transverse.



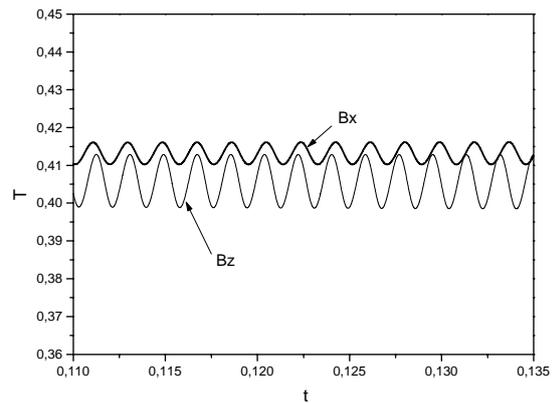
(a)



(b)



(c)



(d)

Fig. 6 Time evolutions vs. two orientations of the magnetic field (Bx and Bz)  $Ha=5$ : (a) dimensionless velocity  $u$ , (b) dimensionless velocity  $v$ , (c) dimensionless velocity  $w$ , (d) dimensionless temperature  $T$

#### D. Stability Diagram

In absence of the magnetic field ( $Ha=0$ ), the oscillatory mode begins at the critical value of the Grashof number  $Gr_{crit} = 7.7 \times 10^6$ . The flow stability appears when the direction of the magnetic field is longitudinal than when the direction of the magnetic field is transverse (Fig. 7). The increase of with the Hartmann number  $Ha$  is clearly seen if the magnetic field is longitudinal. In the case when the magnetic field is transverse, a reduction is noticed at  $Ha = 5$ , and beyond this value, the variation of  $Gr_{crit}$  increases with  $Ha$ .

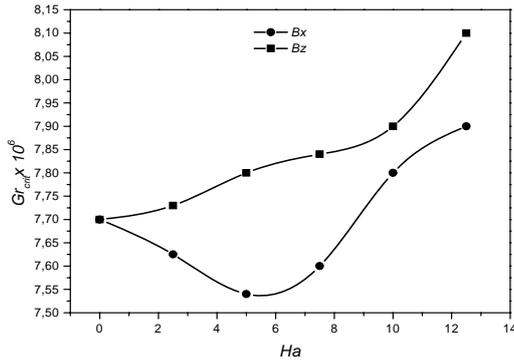


Fig. 7 Stability diagram  $Gr_{crit}-Ha$  for two orientations of the magnetic field (Bx and Bz)

#### V. CONCLUSION

A three-dimensional numerical study of a low Prandtl number fluid flow inside a rectangular cavity, which is subjected to a uniform magnetic field, has been carried out. The geometry considered here is related to crystal growth by a horizontal Bridgman configuration. The finite-volume method was used to discretize the mathematical model.

In the absence of a magnetic field, the results obtained show that the flow is steady for  $Gr = 7.6 \times 10^6 < Gr_{crit}$ , and becomes oscillatory for the critical value of the Grashof number,  $Gr_{crit} = 7.7 \times 10^6$ . The application of a magnetic field causes a change of the flow and thermal fields, and consequently stabilizes the convective flow. The flow stability appears when the direction of the magnetic field is longitudinal than when the direction of the magnetic field is transverse. The dependences of the critical value of the Grashof number with the Hartmann number for two orientations of the magnetic field was recapitulated in the stability diagram. Finally, the results obtained in this study will allow the research workers and industrialists to know the oscillatory modes in the presence of a magnetic field, in order to improve quality of the crystal.

TABLE I  
NOMENCLATURE

Symbol	Quantity	Unity
A	aspect ratio = L/H	
$\vec{B}$	dimensionless magnetic flux density vector	
$B_0$	uniform magnetic flux density	T
Bi	Biot number = hL/k	
$\vec{g}$	acceleration due to gravity	m/s <sup>2</sup>
Gr	Grashof number = $g\beta(T_s - T_c)L^3/\nu^2$	
$\vec{e}_s$	unitary vector of the direction of $\vec{B}$	
H	height of the cavity	M
Ha	Hartmann number = $B_0 L \sqrt{\sigma/\mu}$	
h	heat transfer coefficient	W/m <sup>2</sup> .K
$\vec{j}$	electric current density	A/m <sup>2</sup>
K	thermal conductivity of the fluid	W/m.K
$L_x$	length of the cavity	M
P	dimensionless pressure	
Pr	Prandtl number = $\nu/\alpha$	
T	dimensionless temperature	
$T^*$	temperature	K
$T_c$	ambient temperature	K
t	dimensionless time	
$\vec{v}$	dimensionless velocity vector	
u, v, w	dimensionless velocities in x-, y-, and z-directions, respectively	
W	width of the cavity	m
x, y, z	dimensionless transversal, vertical and longitudinal coordinates, respectively	
$\alpha$	thermal diffusivity of the fluid	m <sup>2</sup> /s
$\beta$	thermal expansion coefficient of the fluid	K <sup>-1</sup>
$\mu$	dynamic viscosity	Pa.s
$\nu$	kinematic viscosity of the fluid	m <sup>2</sup> /s
$\rho$	density of the fluid	kg/m <sup>3</sup>
$\sigma$	electric conductivity	$\Omega^{-1}.m^{-1}$
$\varphi$	electric potential	V

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