Novel and Different Definitions for Fuzzy Union and Intersection Operations

Aarthi Chandramohan, M. V. C. Rao

Abstract—This paper presents three new methodologies for the basic operations, which aim at finding new ways of computing union (maximum) and intersection (minimum) membership values by taking into effect the entire membership values in a fuzzy set. The new methodologies are conceptually simple and easy from the application point of view and are illustrated with a variety of problems such as Cartesian product of two fuzzy sets, max –min composition of two fuzzy sets in different product spaces and an application of an inverted pendulum to determine the impact of the new methodologies. The results clearly indicate a difference based on the nature of the fuzzy sets under consideration and hence will be highly useful in quite a few applications where different values have significant impact on the behavior of the system.

Keywords— Centroid, fuzzy set operations, intersection, triangular norms , triangular S-norms, union.

I. INTRODUCTION

FUZZY sets involve capturing, representing and working with linguistic notions-objects with unclear boundaries. It emerged as a new way of representing uncertainties. The membership values express the degrees to which each object is compatible with the properties or features distinctive to the collection [1]-[3]. A central concept in this framework is that of fuzzy sets and the operations involved with the fuzzy sets using maximum-minimum membership values.

There is a variety of methods available in the literature to calculate the union and intersection of fuzzy sets [5]-[7]. Each method is different and is applicable in a particular context. Indirectly all these methods do indicate the power of fuzzy set theory and also its flexibility to deal with a gamut of disparate situations. In section 2 all these methods are briefly listed for easy reference and place our contributions in proper perspective.

The primary purpose of this paper is to investigate the new methodologies of finding OR (union) and AND (intersection) membership values. We also present illustrative problems and discuss the effect of new methodologies on fuzzy set operations. We experiment with some problems to analyze their effects on defuzzified values.

The remainder of this paper is organized as follows. Section 2

briefly describes the fuzzy set operations. Section 3 focuses on the definitions of the three new methodologies. Section 4 illustrates the problems using the new definitions .In section 5 the results regarding the performance of new methodologies on the defuzzified values are presented. Finally, section 6 presents the conclusion.

II. SET THEORY OPERATIONS

To perform operations on sets means to combine, compare and aggregate sets. Set operations allow constructs that are of an utmost importance in any situation involving information and data processing [3].

The set theory operations that are discussed in this paper using max and min are as follows.

A. Union of fuzzy sets

The union of two fuzzy sets A and B with respective membership functions $f_A(x)$ and $f_B(x)$ is a fuzzy set [1] C, written as $C = A \cup B$, whose membership function is related to those of A and B by

$$f_c(x) = \text{Max}[f_A(x), f_B(x)], \ x \in X \qquad (1)$$

B. Intersection of fuzzy sets

The intersection of fuzzy sets A and B with respective membership functions $f_A(x)$ and $f_B(x)$ is a fuzzy set C, written as $C = A \cap B$, whose membership function is related to those of A and B by [1]

$$f_c(x) = \text{Min}[f_A(x), f_B(x)], \quad x \in X \qquad (2)$$

In the last two sections, union of fuzzy sets interpreted as logical "OR", referred to as triangular co-norms and the intersection of fuzzy sets modeled as logical "AND", referred to as triangular norms were introduced. Other operators [4] have also been suggested. They are compiled and presented in tables 1 and 2.

These operators are ordered as follows:

$$t_w \le t_1 \le t_{1.5} \le t_2 \le t_{2.5} \le t_3$$
$$s_3 \le s_{2.5} \le s_2 \le s_{1.5} \le s_1 \le s_w \qquad (3)$$

TABLE 1: LIST OF TRIANGULAR NORMS

T-Norms	Equations
Drastic product	$t_w(\mu_A(x), \mu_B(x)) = \begin{cases} \{\min\{\mu_A(x), \mu_B(x)\} \} if \max\{\mu_A(x), \mu_B(x)\} = 1 \\ 0 \, otherwise \end{cases}$
Bounded Difference	$t_1(\mu_A(x), \mu_B(x)) = \max\{0, \mu_A(x) + \mu_B(x) - 1\}$
Einstein Product	$t_{1.5}(\mu_A(x), \mu_B(x)) = \dfrac{\mu_A(x) * \mu_B(x)}{2 - [\mu_A(x) + \mu_B(x) - \mu_A(x) * \mu_B(x)]}$
Algebraic Product	$t_2(\mu_A(x), \mu_B(x)) = \mu_A(x) * \mu_B(x)$
Hamacher Product	$t_{2.5}(\mu_A(x), \mu_B(x)) = \dfrac{\mu_A(x) * \mu_B(x)}{[\mu_A(x) + \mu_B(x) - \mu_A(x) * \mu_B(x)]}$
Minimum	$t_3(\mu_A(x), \mu_B(x)) = \min\{\mu_A(x), \mu_B(x)\}$

TABLE 2: LIST OF TRIANGULAR CO NORMS

T Co - norms	Equations
Drastic Sum	$s_w(\mu_A(x), \mu_B(x)) = \begin{cases} \{\max\{\mu_A(x), \mu_B(x)\} \} if \min\{\mu_A(x), \mu_B(x)\} = 0 \\ 1 \, otherwise \end{cases}$
Bounded Sum	$s_1(\mu_A(x), \mu_B(x)) = \min\{1, \mu_A(x) + \mu_B(x)\}$
Einstein Sum	$s_{1.5}(\mu_A(x), \mu_B(x)) = \dfrac{\mu_A(x) + \mu_B(x)}{1 + \mu_A(x) * \mu_B(x)}$
Algebraic Sum	$s_2(\mu_A(x), \mu_B(x)) = \mu_A(x) + \mu_B(x) - \mu_A(x) * \mu_B(x)$
Hamacher Sum	$s_{2.5}(\mu_A(x), \mu_B(x)) = \dfrac{\mu_A(x) + \mu_B(x) - 2 * \mu_A(x) * \mu_B(x)}{1 - \mu_A(x) * \mu_B(x)}$
Maximum	$s_3(\mu_A(x), \mu_B(x)) = \max\{\mu_A(x), \mu_B(x)\}$

C Cartesian product of Fuzzy sets

Let A be a fuzzy set on a universe X and B be a fuzzy set on a universe Y, then the Cartesian product between fuzzy sets A and B will result in a fuzzy relation R, which is contained within the Cartesian product space. The fuzzy relation R has membership function

$$\mu_{R(x,y)} = \mu_{A \times B(x,y)} = \text{Min}\left(\mu_{A(x)}, \mu_{B(y)}\right) \qquad (4)$$

D. Composition of fuzzy sets

Suppose R is a fuzzy relation on the Cartesian space $X \times Y$, S is a fuzzy relation on $Y \times Z$ and T is a fuzzy relation on $X \times$ Z, and then the fuzzy max-min composition is defined in terms of set-theoretic notation [1] in the following manner:

$$\mu_{T(x,z)} = \bigvee_{y \in Y}\left(\mu_{R(x,y)} \wedge \mu_{S(y,z)}\right) \qquad (5)$$

III. NEW METHODOLOGIES

This section introduces three new formulae for computing union and intersection of membership values for fuzzy set operations, which are a little different from the most commonly used ones.

A. First Methodology:

The membership function $\mu_c(x)$ for union of fuzzy sets A and B is defined as:

$$\mu_c(x) = \mu_A(x) \text{ OR } \mu_B(x) \tag{6}$$

The membership function $\mu_c(x)$ of the intersection of fuzzy sets A and B is defined as

$$\mu_c(x) = \mu_A(x) \text{ AND } \mu_B(x) \tag{7}$$

$$\begin{aligned}&(\mu_A(x) \, AND \mu_B(x)) = \\ &\text{Minimum} \{ \text{Minimum}(\mu_A(x), \mu_B(x)), \\ &1 - \text{Maximum}(\mu_A(x), \mu_B(x)) \}\end{aligned} \tag{8}$$

$$\begin{aligned}&(\mu_A(x) \, OR \mu_B(x)) \\ &= \text{Maximum} \{ \text{Maximum}(\mu_A(x), \mu_B(x)), \\ &\qquad 1 - \text{Minimum}(\mu_A(x), \mu_B(x)) \}\end{aligned} \tag{9}$$

1) Conditions/Limitations

Simple and easy to check conditions can easily be identified for these new union and intersection operations so that the application will certainly yield different and better values in the required sense.

For $(\mu_A(x) \, AND \mu_B(x))$:

$$(\text{Minimum}(\mu_A(x) + \mu_B(x) + \text{Maximum}(\mu_A(x) + \mu_B(x))) > 1.0 \tag{10}$$

For $(\mu_A(x) \, OR \mu_B(x))$:

$$(\text{Minimum}(\mu_A(x) + \mu_B(x) + \text{Maximum}(\mu_A(x) + \mu_B(x))) < 1.0 \tag{11}$$

Clearly, it is a strict inequality. Equality sign corresponds to the case when the values are equal and hence should be clearly avoided because it reverts back to the usual max-min operations.

When these inequalities are not satisfied these operations are equivalent to the usual and most commonly used operations and hence do not yield different results. So one can definitely check and expect the nature of results in direct contrast with the usual.

2) Exceptions

The first methodology should be strictly avoided when

a)   *The membership values are equal*

b)   *Either of the membership values is 0 or 1.*

Consequently the usual union and intersection operations

should be used.

3) Example

Let A = [0.63, 0.001, 1] and B = [0.3, 0.002, 0.86] then

A OR B = [0.7, 0.002, 1]

Here the minimum values 0.001 and 0.002 are rounded off to first decimal place, which falls under the exceptional case and therefore the OR operation reverts back to the original method.

B. Second Methodology

$$\begin{aligned}&Harmonic\ Mean \le Geometric\ Mean \\ &\le Arithmetic\ Mean \le Root\ Mean\ Square\ (RMS)\end{aligned} \tag{12}$$

This is a well-known inequality useful in many contexts especially in areas like optimization, so it should be exploited here also.

$$(\mu_A(x) \, AND \, \mu_B(x)) = \text{Harmonic mean}(\mu_A(x), \mu_B(x)) \tag{13}$$
$$(\mu_A(x) \, OR \, \mu_B(x)) = \text{RMS}(\mu_A(x), \mu_B(x)) \tag{14}$$

This definition favors clearly a higher value for the minimum and smaller value for the maximum. But, however, it should be emphasized that both minimum and maximum values are clearly influenced by all the values in the set. Hence where such a situation is desirable, this method can be advantageously employed.

C. Third Methodology

The membership function $\mu_c(x)$ of the union of fuzzy sets A and B is defined as :

$$\mu_c(x) = \mu_A(x) \text{ OR } \mu_B(x) \tag{15}$$

The membership function $\mu_c(x)$ of the intersection of fuzzy sets A and B is defined as :

$$\mu_c(x) = \mu_A(x) \text{ AND } \mu_B(x) \tag{16}$$

$$\begin{aligned}&(\mu_A(x) \, AND \, \mu_B(x))_{THIRD} \\ &= \frac{\text{Minimum}(\mu_A(x), \mu_B(x))}{\text{Maximum}(\mu_A(x), \mu_B(x))}\end{aligned} \tag{17}$$

$$\begin{aligned}&(\mu_A(x) \, OR \, \mu_B(x))_{THIRD} \\ &= \text{Maximum}(\mu_A(x), \mu_B(x)) + \\ &\left[\begin{array}{l} (\mu_A(x) \, AND \, \mu_B(x))_{THIRD} \\ - \text{Minimum}(\mu_A(x), \mu_B(x)) \end{array} \right]\end{aligned} \tag{18}$$

This is clearly motivated by the fact that in order to maximize the effect of fuzzification or to take care of any arbitrary assignment of membership value, this operation seems to be most effective to arrive at reasonably meaningful results. This methodology again supports higher values for both minimum and maximum operations.

1) Conditions/Limitations

A condition is not really required, yet it can easily be identified that $(\mu_A(x)\text{AND }\mu_B(x))$ in Eq (17) should always be less than 1.0 so that the actual values of minimum and maximum get enhanced and still stay within limits. Therefore when higher values are of importance from the point of view of ultimate results this method can certainly be used with advantage.

2) Exceptions

The third methodology should be strictly avoided when

a) *The membership values are equal*

b) *Either of the membership values is 0 or 1.*

Consequently the usual union and intersection operations should be used.

3) Example

Let A = [0.6, 0.00099, 1] and B = [0.3, 0.001, 0.86] then A AND B = [0.5, 0.00099, 0.86].

Here the minimal value 0.00099 and 0.001 are rounded off to first decimal place, which falls under the exceptional case. Now the AND operation reverts back to the original method.

III. PROBLEM ILLUSTRATIONS

A. Problem1

Let A be a fuzzy set $\dfrac{0.2}{x_1}+\dfrac{0.5}{x_2}+\dfrac{1}{x_3}$ and B be a fuzzy set $\dfrac{0.3}{y_1}+\dfrac{0.9}{y_2}$ Then the fuzzy Cartesian product A × B is compiled in table 3.

It can clearly be seen from table 3 that the new methodologies yield higher values in general. Methodology one is a sort of combination of old min operation and the new definitions and hence is not distinctively higher but nonetheless much different and closer to the older methodologies delineated. In table 3, the membership values $X_3\ Y_1$ and $X_3\ Y_2$ fall under exception in the first and third methodology.

TABLE 3: CARTESIAN PRODUCT A × B

Type of Triangular norms	$\mu_{\text{AxB(x,y)}}$
Drastic product	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.2 & 0.2\\ x_2 \ 0.3 & 0.5\\ x_3 \ 0.3 & 0.9\end{bmatrix}\end{array}$
Bounded difference	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.5 & 0.1\\ x_2 \ 0.2 & 0.4\\ x_3 \ 0.3 & 0.9\end{bmatrix}\end{array}$
Einstein Product	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.04 & 0.16\\ x_2 \ 0.11 & 0.428\\ x_3 \ 0.3 & 0.9\end{bmatrix}\end{array}$
Algebraic Product	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.06 & 0.18\\ x_2 \ 0.15 & 0.45\\ x_3 \ 0.3 & 0.9\end{bmatrix}\end{array}$
Hamachar Product	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.136 & 0.195\\ x_2 \ 0.23 & 0.47\\ x_3 \ 0.3 & 0.9\end{bmatrix}\end{array}$
Min operation	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.2 & 0.2\\ x_2 \ 0.3 & 0.5\\ x_3 \ 0.3 & 0.9\end{bmatrix}\end{array}$
First Methodology	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.2 & 0.1\\ x_2 \ 0.3 & 0.1\\ x_3 \ 0.3 & 0.9\end{bmatrix}\end{array}$
Second Methodology	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.24 & 0.33\\ x_2 \ 0.38 & 0.64\\ x_3 \ 0.46 & 0.95\end{bmatrix}\end{array}$
Third Methodology	$\begin{array}{c}\quad y_1 \quad y_2\\ x_1\begin{bmatrix}0.67 & 0.22\\ x_2 \ 0.60 & 0.56\\ x_3 \ 0.30 & 0.9\end{bmatrix}\end{array}$

TABLE 4: COMPOSITION OF FUZZY SETS R AND S USING DIFFERENT FUZZY OPERATORS

Type of t - norm and s - norm	Resultant set $\mu_{T(x,z)} = X \times Z$
Bounded difference– Bounded sum composition	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0.6 & 0.5 & 0 \\ x_2 & 0.7 & 0.5 & 0 \end{array}$
Drastic product- drastic sum composition	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 0 \end{array}$
Einstein product- Einstein sum composition :	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0.63 & 0.609 & 0.311 \\ x_2 & 0.71 & 0.61 & 0.28 \end{array}$
Algebraic product- Algebraic sum composition	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0.648 & 0.623 & 0.355 \\ x_2 & 0.731 & 0.625 & 0.392 \end{array}$
Hamacher product- Hamacher sum composition	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0.673 & 0.638 & 0.416 \\ x_2 & 0.741 & 0.616 & 0.3947 \end{array}$
Max-Min Composition	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0.7 & 0.6 & 0.5 \\ x_2 & 0.8 & 0.6 & 0.6 \end{array}$
First Methodology	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0.1 & 0.3 & 0.8 \\ x_2 & 0.1 & 0.8 & 0.8 \end{array}$
Second Methodology	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0.57 & 0.62 & 0.42 \\ x_2 & 0.61 & 0.61 & 0.38 \end{array}$
Third Methodology	$\begin{array}{ccc} & z_1 & z_2 & z_3 \\ x_1 & 0.84 & 0.98 & 1.00 \\ x_2 & 0.92 & 0.94 & 0.86 \end{array}$

TABLE 5: A x B COMPUTED USING DIFFERENT T NORMS

Type of T norm	Resultant set $A \times B = 2 \times 6 =$
Drastic product	$\left\{ \frac{0}{5} + \frac{0.6}{6} + \frac{0}{7} + \frac{0.8}{10} + \frac{1}{12} + \frac{0.7}{14} + \frac{0}{15} + \frac{0.8}{18} + \frac{0}{21} \right\}$
Bounded Difference	$\left\{ \frac{0.4}{5} + \frac{0.6}{6} + \frac{0.3}{7} + \frac{0.8}{10} + \frac{1}{12} + \frac{0.7}{14} + \frac{0.6}{15} + \frac{0.8}{18} + \frac{0.5}{21} \right\}$
Einstein product	$\left\{ \frac{0.44}{5} + \frac{0.6}{6} + \frac{0.37}{7} + \frac{0.8}{10} + \frac{1}{12} + \frac{0.7}{14} + \frac{0.615}{15} \right.$ $\left. + \frac{0.8}{18} + \frac{0.53}{21} \right\}$
Hamacher Product	$\left\{ \frac{0.52}{5} + \frac{0.6}{6} + \frac{0.47}{7} + \frac{0.8}{10} + \frac{1}{12} + \frac{0.7}{14} + \frac{0.66}{15} \right.$ $\left. + \frac{0.8}{18} + \frac{0.59}{21} \right\}$
Min t norm	$\left\{ \frac{0.6}{5} + \frac{0.6}{6} + \frac{0.6}{7} + \frac{0.8}{10} + \frac{1}{12} + \frac{0.7}{14} + \frac{0.8}{15} + \frac{0.8}{18} + \frac{0.7}{21} \right\}$
First Method	$\left\{ \frac{0.2}{5} + \frac{0.6}{6} + \frac{0.3}{7} + \frac{0.8}{10} + \frac{1}{12} + \frac{0.7}{14} + \frac{0.8}{15} + \frac{0.8}{18} + \frac{0.2}{21} \right\}$
Second Method	$\left\{ \frac{0.69}{5} + \frac{0.75}{6} + \frac{0.65}{7} + \frac{0.89}{10} + \frac{1}{12} + \frac{0.82}{14} + \frac{0.8}{15} \right.$ $\left. + \frac{0.89}{18} + \frac{0.75}{21} \right\}$
Third Method	$\left\{ \frac{0.75}{5} + \frac{0.6}{6} + \frac{0.86}{7} + \frac{0.8}{10} + \frac{1}{12} + \frac{0.7}{14} + \frac{0.8}{15} \right.$ $\left. + \frac{0.8}{18} + \frac{0.88}{21} \right\}$

B. Problem 2

Let R be a fuzzy set $\begin{array}{cc} & y_1 & y_2 \\ x_1 & 0.7 & 0.5 \\ x_2 & 0.8 & 0.4 \end{array}$

and S be a fuzzy set $\begin{array}{ccc} & z_1 & z_2 & z_3 \\ y_1 & 0.9 & 0.6 & 0.2 \\ y_2 & 0.1 & 0.7 & 0.5 \end{array}$

then $X \times Z$ the Cartesian product is compiled in table 4.

As in problem 1, again second and third methodologies yield higher and different values in direct contrast with first methodology which yields lower values. In table 4, for the first methodology, the membership values $X_1 Z_1$ and $X_1 Z_2$ and $X_1 Z_3$ fall under exception.

C. Problem 3

Let A be a fuzzy set "approx 2" $\left\{\frac{0.6}{1} + \frac{1}{2} + \frac{0.6}{7} + \frac{0.8}{3}\right\}$ and B be a fuzzy set "approx 6" $\left\{\frac{0.8}{5} + \frac{1}{6} + \frac{0.7}{7}\right\}$ then A x B is computed using different fuzzy operators and are listed in table 5. As in problem 1, again second and third methodologies yield higher and different values in direct contrast with first methodology which yields lower values.

IV. APPLICATION

This section presents the classic inverted pendulum problem to illustrate the effect of new methodologies [3].

A fuzzy controller is designed and analyzed for the simplified version of an inverted pendulum problem.

The differential equation governing the system is given below

$$-ml\frac{d^2\theta}{dt^2} + mlg(\sin\theta) = \tau = \mu(t) \qquad (18)$$

where m is the mass of the pole located at the tip point of the pendulum, l is the length of the pendulum, θ is the deviation angle from vertical in the clock wise direction, $\tau = \mu(t)$ is the torque applied to the pole in the counterclockwise direction, t is time and g is gravitational acceleration constant.

If $x_1 = \theta$ and $x_2 = \frac{d\theta}{dt}$, as start variables, the state space representation for the nonlinear system is given by

$$\frac{dx_1}{dt} = x_2 \text{ and } \frac{dx_2}{dt} = \left(\frac{g}{l}\right)x_1 - \left(\frac{1}{ml^2}\right)u(t) \qquad (19)$$

If x_1 is measured in degrees and x_2 is measured in degrees per second, by choosing $l = g$ and $m = \frac{180}{\pi \cdot g^2}$, then

$$x_1(k+1) = x_1(k) + x_2(k) \qquad (20)$$
$$\text{and } x_2(k+1) = x_1(k) + x_2(k) - u(k) \qquad (21)$$

The universes of discourse for the two variables are assumed to be $-2° \le x_1 \le 2°$ and $-5dps \le x_2 \le 5dps$.

Three membership functions for x_1 are constructed for the values positive (P), Zero (Z) and negative (N), shown in figure 1. Then three membership functions for x_2 are constructed for the values positive (P), Zero (Z) and negative (N), shown in figure 2.

Nine rules are constructed in a 3×3 FAM table and shown in table 6. The entries in this table are control actions $u(k)$. To start the simulation, the following crisp initial conditions are chosen $x_1(0) = 1°$ and $x_2(0) = -4dps$ only first cycle of simulation is conducted to show the effects of the new methodologies.

To partition the control space (output), five membership functions for $u(k)$ are constructed on its universe, which is $-24 \le u(k) \le 24$, shown in figure 3.
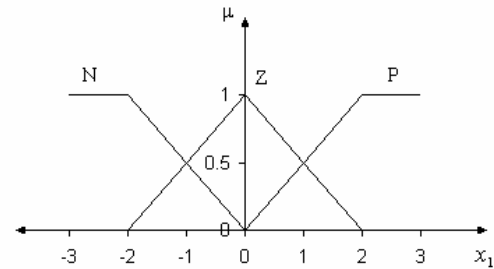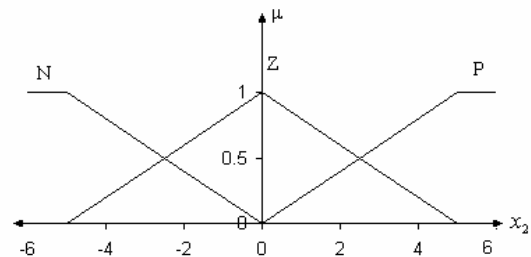

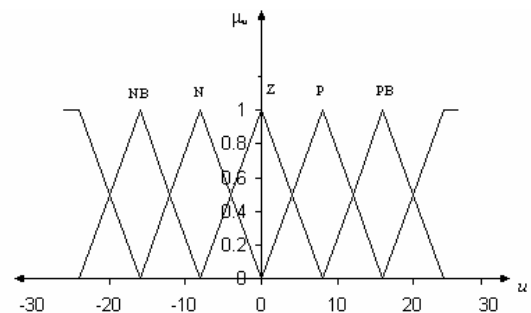Fig 1. Input x_1 Partitioned

Fig 2. Input x_2 Partitioned

Fig 3. Output $u(k)$ Partitioned into seven Partitions

TABLE 6: FAM TABLE

X_1 \ X_2	P	Z	N
P	PB	P	Z
Z	P	Z	N
N	Z	N	NB

New methodologies are used for fuzzy operation "AND" and are compared with the most commonly [8-10] used fuzzy operator (max-min).

Using max-min operator,

If $(x_1 = P)$ and $(x_2 = Z)$, then $(u = P)$ min (0.5, 0.2) = 0.2(P)
If $(x_1 = P)$ and $(x_2 = N)$, then $(u = Z)$ min (0.5, 0.8) = 0.5(Z)
If $(x_1 = Z)$ and $(x_2 = Z)$, then $(u = Z)$ min (0.5, 0.2) = 0.2(Z)
If $(x_1 = Z)$ and $(x_2 = N)$, then $(u = N)$ min (0.5, 0.8) = 0.5(N)

Using First methodology,

If $(x_1 = P)$ and $(x_2 = Z)$, then $(u = P)$ min (0.5, 0.2) = 0.2(P)
If $(x_1 = P)$ and $(x_2 = N)$, then $(u = Z)$ min (0.5, 0.8) = 0.2(Z)
If $(x_1 = Z)$ and $(x_2 = Z)$, then $(u = Z)$ min (0.5, 0.2) = 0.2(Z)
If $(x_1 = Z)$ and $(x_2 = N)$, then $(u = N)$ min (0.5, 0.8) = 0.2(N)

Using Second methodology,

If $(x_1 = P)$ and $(x_2 = Z)$, then $(u = P)$ min (0.5, 0.2) = 0.2857(P)
If $(x_1 = P)$ and $(x_2 = N)$, then $(u = Z)$ min (0.5, 0.8) = 0.6153(Z)
If $(x_1 = Z)$ and $(x_2 = Z)$, then $(u = Z)$ min (0.5, 0.2) = 0.2857(Z)
If $(x_1 = Z)$ and $(x_2 = N)$, then $(u = N)$ min (0.5, 0.8) = 0.6153(N)

Using Third methodology,

If $(x_1 = P)$ and $(x_2 = Z)$, then $(u = P)$ min (0.5, 0.2) = 0.4(P)
If $(x_1 = P)$ and $(x_2 = N)$, then $(u = Z)$ min (0.5, 0.8) = 0.625(Z)
If $(x_1 = Z)$ and $(x_2 = Z)$, then $(u = Z)$ min (0.5, 0.2) = 0.4(Z)
If $(x_1 = Z)$ and $(x_2 = N)$, then $(u = N)$ min (0.5, 0.8) = 0.625(N)

The FAM table will produce a membership function for the control action $u\ (k)$. This membership function is defuzzified using centroid method and the results are listed in table 7.

TABLE 7: DEEFUZZIFIED OUTPUT AND INITIAL CONDITIONS

Type of fuzzy operator	Centroid output	Initial condition
Max-min	-1.9	- 3, -1.1
First Methodology	0	-3,-3
Second Methodology	-1.5	-3, -1.5
Third Methodology	-0.959	-3,-2.041

V. CONCLUSION

Three different definitions for union (maximum) and intersection (minimum) operations are given. A practical example included also indicates the power and efficacy of these methods which hold a lot of promise. The results are appealing and useful and they are encouraging for future adoptions. All the three methods give different but close enough values. It is believed that these definitions will be sought after in a variety of applications to further widen the gamut of applications.

REFERENCES

[1] George J.Klir and B. O Yuan, "Fuzzy sets, Fuzzy logic and Fuzzy systems-Selected papers by Lotfi A. Zadeh," 1996. World Scientific Publication Co Pvt Ltd
[2] E.Cox, "The Fuzzy Systems Handbook," Second Edition, AP Professional, New York (1998).
[3] Timothy J.Ross, "Fuzzy Logic with Engineering Applications", McGraw Hill.1997
[4] Witold Pedrycz and Fernando Gomide, "An Introduction to Fuzzy sets-Analysis and Design," 1998.
[5] Zimmerman H.J, "Fuzzy set theory and its applications," Second Edition, Kluwer academic publishers
[6] B.Kovalerchuk and V.Taliansky, "Comparison of empirical and computed values of fuzzy conjunction," Fuzzy sets and systems, 46:49-53, 1992.
[7] U.Thole, H.J.Zimmerman, and P.Zysno, "On the suitability of minimum and product operators for the intersection of fuzzy sets," Fuzzy sets and systems, 2:167-180, 1979.
[8] H.J.Zimmerman and P.Zysno, "Latent connectives in human decision making," Fuzzy sets and systems, 4:37-51, 1980.
[9] Harald Dyckhoff and Witold Pedrycz, "Generalized means as model of compensative connectives," Fuzzy sets and systems, 14:143-154, 1984.
[10] Masaharu Mizumoto, "Pictorial representation of fuzzy connectives, parti: cases of t-norms, t-co norms and averaging operators," Fuzzy sets and systems 31:217-242, 1989.