

Analytical Crack Propagation Scenario for Gear Teeth and Time-Varying Gear Mesh Stiffness

Omar D. Mohammed, Matti Rantatalo, Uday Kumar

Abstract—In this paper an analytical crack propagation scenario is proposed which assumes that a crack propagates in the tooth root in both the crack depth direction and the tooth width direction, and which is more reasonable and realistic for non-uniform load distribution cases than the other presented scenarios. An analytical approach is used for quantifying the loss of time-varying gear mesh stiffness with the presence of crack propagation in the gear tooth root. The proposed crack propagation scenario can be applied for crack propagation modelling and monitoring simulation, but further research is required for comparison and evaluation of all the presented crack propagation scenarios from the condition monitoring point of view.

Keywords—Crack propagation, Gear tooth crack, Time varying gear mesh stiffness.

I. INTRODUCTION

THE dynamic behaviour of gear transmissions is sensitive to operation conditions like excessive applied load, insufficient lubrication, manufacturing errors or installation problems [1]. When an extreme stress concentration is generated in the gear tooth root, a crack may initiate and start propagating during operation. Therefore, for condition monitoring purposes, the status of the tooth damage can be quantified by the reduction in the time-varying gear mesh stiffness [1]-[3]. This reduction in the gear mesh stiffness affects the generated vibration, with the gear's dynamic response being dependent on the mesh stiffness of two mating gears [3].

Many studies have been performed to develop analytical methods for investigating the mesh stiffness of uncracked gears [4]-[8]. These methods are still in use today as their results have shown a good agreement with those obtained with FEM which is a more recent method used for this purpose. FEM is an efficient tool for calculating the mesh stiffness [9]-[11]. It is worth mentioning, however, that FEM needs a great deal of computation time, especially in cases which require mesh refinement, so that analytical methods can still be considered as an appropriate alternative option for this purpose.

Omar D. Mohammed is with Division of Operation, Maintenance, and Acoustic, Lulea University of Technology (LTU), 97187 Lulea, Sweden. E-mail: omar.mohammed@ltu.se, enggomar@hotmail.com.

Matti Rantatalo is with Division of Operation, Maintenance, and Acoustic, Lulea University of Technology (LTU), 97187 Lulea, Sweden. E-mail: Matti.Rantatalo@ltu.se.

Uday Kumar is with Division of Operation, Maintenance, and Acoustic, Lulea University of Technology (LTU), 97187 Lulea, Sweden. E-mail: Uday.Kumar@ltu.se.

Investigations of gear mesh stiffness have been carried out extensively for uncracked healthy gears, but for faulty gears, crack modelling and mesh stiffness evaluation are fields that are still currently being explored by many researchers [3].

A great deal of research work has been conducted to study the crack propagation path and the effect of the rim thickness on the crack direction [12]-[14]. It has been found that the path of the crack propagation is dependent on the “backup ratio”, which may be defined as the ratio of the rim thickness to the tooth height. In gears with high backup ratio, the direction of the crack path in the tooth root will be through the tooth. In gears with a low backup ratio, the crack path will be through the rim. The initial crack angle also affects the crack propagation path in such a way that, when the initial angle is low, the path of the crack may go through the rim, even in the case of a high backup ratio. Investigations of the reduction in the mesh stiffness due to damage or crack propagation have been carried out [15]-[17]. The fact that the mesh stiffness changes proportionally with the crack status could be used for severity assessment, but these investigations do not define the crack sizes which correspond to the amounts of stiffness reduction found. Calculation of the total gear mesh stiffness using crack modelling has been introduced. The effect of the crack propagation size on the mesh stiffness has been treated in [18], [19]. A crack model was presented by S. Zouari, M. Maatar, T. Fakhfakh, and H. Haddar [20], and they studied how the mesh stiffness and transmission error are affected by the position, direction, and depth of the crack using FEA (Ansys code). F. Chaari, T. Fakhfakh, and M. Haddar [1] presented an analytical approach to calculating the reduction in the total gear mesh stiffness due to crack depth propagation, as well as a model using FEM to verify the results obtained analytically. In all the above-mentioned works that have dealt with analytical stiffness modelling and calculation, the crack is assumed to extend through the whole tooth width with a uniform crack depth distribution along the tooth width. Z. Chen, and Y. Shao [2] presented an analytical approach to modelling the crack propagation along the tooth width with a non-uniform parabolic distribution, and then, studied the effect of this propagation on the time-varying gear mesh stiffness and the dynamic response. Based on the above-mentioned research, one can conclude that more than one scenario have been assumed when applying analytical methods to the investigation of crack propagation in the tooth root. The scenario of a constant or uniformly distributed crack depth through the whole tooth width was assumed first, after, which two more scenarios were presented by Chen, and Y. Shao [2]. The first assumed a constant crack depth with a varying crack length and a parabolic distribution along the tooth width, while the second assumed a constant crack length with a varying crack depth and a parabolic distribution.

In all the previous research, no crack model has assumed that the crack propagates in both the crack depth direction and the tooth width direction.

The present paper uses a previously presented [2] analytical approach for evaluating the time-varying gear mesh stiffness by assuming that the crack propagates in the tooth root in both the crack depth direction and the tooth width direction, which is a more reasonable and realistic assumption for non-uniform load distribution cases.

II. GEAR MESH STIFFNESS MODELLING WITH A CRACK IN THE TOOTH ROOT

Gear mesh stiffness can be explained as a factor describing the stiffness produced between two engaged gears due to the stiffness of the meshing teeth of both gears. Gear mesh stiffness is a time varying factor during the operation of gears and is dependent on the number of engaged teeth, the applied load position and direction, the tooth geometry, the gear material specifications, and the profile errors and/or faults in the gear tooth [1].

An important observation was reported by D. Lewicki [12], namely that the paths of the crack depth propagation are smooth, continuous, and tend to be straight with a slight curvature. Therefore, the crack propagation path is assumed to be a straight line in the crack depth direction in [1], [2], [19]. In the crack modelling of the current study, the crack depth is simulated as propagating in a straight line.

A calculation model for gear mesh stiffness using a parabolic distribution of the tooth crack propagation is presented in [2] and can be described as consisting of two parts: firstly, determination of the mesh stiffness with a constant crack depth for a thin slice along the tooth width, and secondly, determination of the mesh stiffness for all the slices along the tooth width which have a non-uniform crack depth. Moreover, the effects of fillet foundation deflection and Hertzian contact are taken into account, as explained in Sections (II-C) and (II-D).

A. The Mesh Stiffness with a Constant Crack Depth for a Slice along the Tooth Width

The stiffness of one tooth is a combination of the bending, shear, and axial compressive stiffnesses, with all of them acting in the direction of the applied load.

By considering the tooth as a non-uniform cantilever beam with an effective length of 'd', see Fig. 1-d, the deflections under the action of the force can be determined, and then the stiffness can be calculated. Note that, in this part of the stiffness calculations, the crack is assumed to have a constant crack depth $q(z)$ for each slice dW along the tooth width, as explained in Figs. 1-a,b.

Based on the calculation of the potential energy stored in a meshing gear tooth, it is feasible to obtain the bending, shear, and axial compressive stiffnesses as follows [2]:

$$\frac{1}{K_b} = \int_0^d \frac{(x \cdot \cos(\alpha_1) - h \cdot \sin(\alpha_1))^2}{E \cdot I_x} dx \quad (1)$$

$$\frac{1}{K_s} = \int_0^d \frac{1.2 \cos^2(\alpha_1)}{G \cdot A_x} dx \quad (2)$$

$$\frac{1}{K_a} = \int_0^d \frac{\sin^2(\alpha_1)}{E \cdot A_x} dx \quad (3)$$

where the following notations are used:

K_b : Bending stiffness.

K_s : Shear stiffness.

K_a : Axial compressive stiffness.

h , h_c , h_x , x , dx , d , and α_1 are illustrated in Fig. 1-d. Note that α_1 varies with the gear tooth position.

E : Young's modulus.

G : Shear modulus, $G = \frac{E}{2(1+\nu)}$

ν : Poisson ratio.

I_x : Area moment of inertia,

$$I_x = \begin{cases} (1/12)(h_x + h_x)^3 dW, & h_x \leq h_q \\ (1/12)(h_x + h_q)^3 dW, & h_x > h_q \end{cases}$$

A_x : Area of the section of distance 'x' measured from the load application point,

$$A_x = \begin{cases} (h_x + h_x) dW, & h_x \leq h_q \\ (h_x + h_q) dW, & h_x > h_q \end{cases}$$

$$h_q = h_c - q(z) \cdot \sin(\alpha_c)$$

$q(z)$, and α_c are the crack depth and crack angle respectively, as shown in Fig. 1-b.

At a certain position z through the tooth width, we can find the stiffness of one slice resulting from the effect of all the stiffnesses calculated previously as follows:

$$K(z) = 1 / \left(\frac{1}{K_b} + \frac{1}{K_s} + \frac{1}{K_a} \right) \quad (4)$$

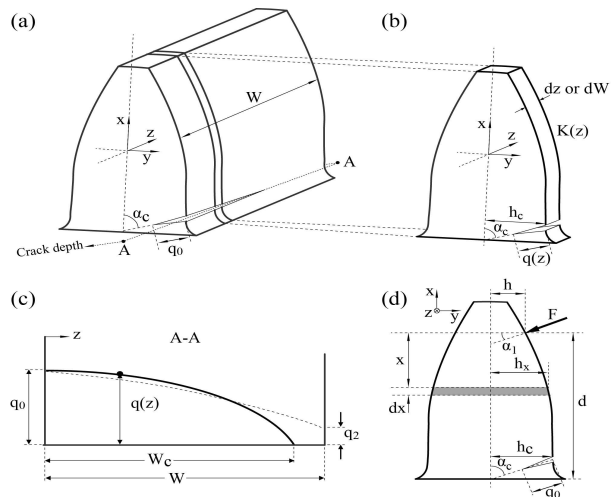


Fig. 1 Modelling of gear tooth crack. (a) Modelling of cracked tooth, (b) Modelling of one slice of width dW , (c) Crack depth distribution along the tooth width, and (d) Tooth notation [2]

B. The Mesh Stiffness for All the Slices along the Tooth Width

The mesh stiffness model presented in [2] divides the tooth width into thin slices to represent the crack propagation through the tooth width, as shown in Fig. 1-a. Consequently, for a small dW the crack depth is assumed to be a constant through the width for each slice, see Fig. 1-b. By integrating the stiffness of all the slices along the width, the stiffness of the entire tooth can be evaluated as follows:

$$K_{tp} = \int_0^W K(z) dz \quad (5)$$

In [2] it is assumed that the distribution of the crack depth follows a parabolic function along the tooth width, as shown in Fig. 1-c for the crack section A-A, which can be recognized in Fig. 1-a.

When the crack length is less than the whole tooth width,

$$q(z) = q_0 \sqrt{\frac{W_c - z}{W_c}}, z \in [0 - W_c] \quad (6)$$

$$q(z) = 0, z \in [W_c - W] \quad (7)$$

where W_c is the crack length, W is the whole tooth width, and q_0 is the max. crack depth, see Fig. 1-c.

When the crack length extends through the whole tooth width,

$$q(z) = \sqrt{\frac{q_0^2 - q_2^2}{W} z + q_2^2} \quad (8)$$

where q_2 is shown in Fig. 1-c.

C. The effect of Fillet Foundation Deflection on the Mesh Stiffness

P. Sainsot, P. Valex, and O. Duverger [21] studied the effect of fillet foundation deflection on the gear mesh stiffness, derived this deflection, and applied it for a gear body. The fillet foundation deflection can be calculated as follows [21]:

$$\delta_f = \frac{F \cdot \cos^2(\alpha_m)}{W \cdot E} \left\{ L^* \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* \left(1 + Q^* \tan^2(\alpha_m) \right) \right\} \quad (9)$$

where the following notations are used:

α_m is the pressure angle.

u_f and S_f are illustrated in Fig. 2.

L^* , M^* , P^* , and Q^* can be approximated using polynomial functions as follows [21]:

$$X_i^*(h_{fi}, \theta_f) = A_i / \theta_f^2 + B_i h_{fi}^2 + C_i h_{fi} / \theta_f + D_i / \theta_f + E_i h_{fi} + F_i \quad (10)$$

X_i^* represents the coefficients L^* , M^* , P^* , and Q^* .

$h_{fi} = r_f / r_{int}$.

r_f , r_{int} , and θ_f are illustrated in Fig. 2.

The coefficients A_i , B_i , C_i , D_i , E_i and F_i are given in Table I.

TABLE I
VALUES OF THE COEFFICIENTS OF EQ.(10)

	$A_i (\times 10^{-5})$	$B_i (\times 10^{-3})$	$C_i (\times 10^{-4})$	$D_i (\times 10^{-3})$	E_i	F_i
$L^*(h_{fi}, \theta_f)$	-5.574	-1.9986	-2.3015	4.7702	0.0271	6.8045
$M^*(h_{fi}, \theta_f)$	60.111	28.100	-83.431	-9.9256	0.1624	0.9086
$P^*(h_{fi}, \theta_f)$	-50.952	185.50	0.0538	53.300	0.2895	0.9236
$Q^*(h_{fi}, \theta_f)$	-6.2042	9.0889	-4.0964	7.8297	-0.1472	0.6904

Then the stiffness due to fillet foundation deflection can be obtained as:

$$\frac{1}{K_f} = \frac{\delta_f}{F} \quad (11)$$

For a pinion it could be denoted as K_{fp} .

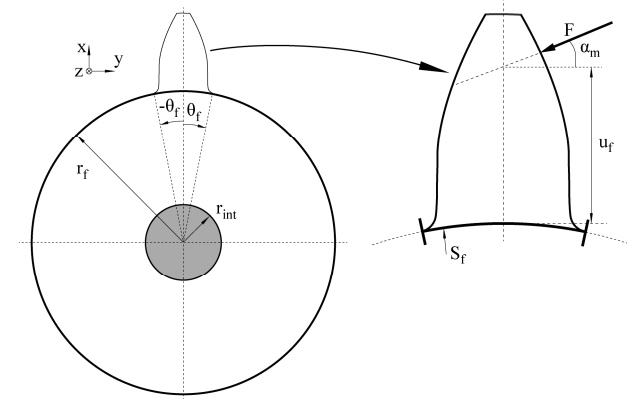


Fig. 2 Geometrical parameters for fillet foundation deflection [1]

D. The effect of Hertzian Contact on the Mesh Stiffness

D. C. H. Yang, and Z. S. Sun [22] ascertained that the stiffness of the Hertzian contact of two gears in mesh is constant during the whole contact period, and therefore has the same value at all the contact positions along the path of contact. The Hertzian contact stiffness K_h can be calculated as:

$$\frac{1}{K_h} = \frac{4(1 - \nu^2)}{\pi \cdot E \cdot W} \quad (12)$$

III. ANALYTICAL CRACK PROPAGATION SCENARIO

To study the reduction in the mesh stiffness due to crack propagation in the gear tooth root, several analytical crack propagation scenarios have been proposed. First, the scenario of a constant or uniformly distributed crack depth through the whole tooth width was assumed, which is considered as a correct assumption for uniform load distribution cases. After that, two more scenarios with a non-uniform parabolic crack distribution were presented in [2], the first of which assumed a constant crack depth with a varying crack length and a parabolic distribution along the tooth width, and the second assumed a constant crack length with a varying crack depth and a parabolic distribution.

In the former crack propagation scenarios then, it is assumed either that the crack propagates in the crack depth direction only with a constant crack length extending through the tooth width, or that the crack length propagates through the tooth width with the crack depth kept unchanged.

Practical experience shows that the load distribution along the tooth width is usually non-uniform [23], [24], see Fig. 3, with a number of factors causing the deviation from the ideal uniform load distribution, e.g. manufacturing errors, gear shaft misalignment, and deformations in the bearing and/or housing. Experimental tests of a one-sided load distribution have shown that the fatigue crack propagates in two directions; crack length, and crack depth [23]. It is more reasonable and realistic for non-uniform load distribution cases to assume the crack is propagating through both the crack depth direction and the crack length direction together.

As it is difficult to define analytically the exact crack length corresponding to a certain crack depth, and because the study of crack growth with time is out of the scope of this work, we have assumed propagation cases which are explained in Fig. 4 to study the effect of this propagation scenario on the reduction in the gear mesh stiffness. The propagation cases that have been assumed with this scenario are shown in Table II.

The values of q_2 in Table II have been obtained from the intersection of the parabolic lines with the vertical axis at points A, B, C, and D which are illustrated in Fig. 4, the section A-A can be recognized in Fig. 1-a. A high backup ratio with a crack angle of 70° is considered here.

TABLE II
DATA FOR THE PROPAGATION CASES OF THE PROPOSED SCENARIO

Case	q_0 (mm)	W_c (mm)	q_2 (mm)	$\alpha_c = 70^\circ$
1	0	0	0	
2	0.2	5	0	
3	0.4	10	0	
4	0.6	15	0	
5	0.8	20	0	
6	1.0	20	0.45	
7	1.2	20	0.7	
8	1.4	20	0.925	
9	1.6	20	1.14	

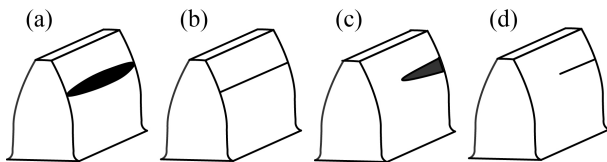


Fig. 3 Illustration of different load distribution cases. (a) Uniform load distribution, (b) Uniform load distribution represented analytically, (c) Non-uniform or one-sided load distribution, and (d) Non-uniform load distribution represented analytically

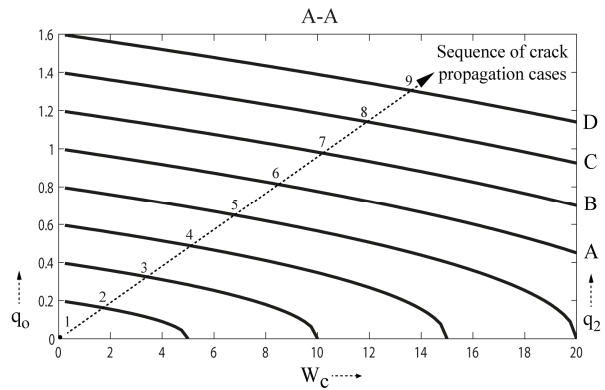


Fig. 4 Crack section A-A shows the crack propagation according to the proposed scenario

IV. TOTAL MESH STIFFNESS

After calculating the stiffness of a cracked pinion tooth, K_{tp} , due to bending, shear, and axial compression, and then calculating the stiffness due to the fillet foundation deflection, K_{fp} , we can perform the same calculations for an uncracked mating gear tooth to find K_{tg} and K_{fg} . Thus, the total mesh stiffness for one meshing tooth pair is:

$$K_t = \frac{1}{\frac{1}{K_{tp}} + \frac{1}{K_{fp}} + \frac{1}{K_{tg}} + \frac{1}{K_{fg}} + \frac{1}{K_h}} = K_{e1} \quad (13)$$

where K_{e1} is the total mesh stiffness for the first tooth pair.

In cases where there are two tooth pairs in contact, the same calculations are repeated for the second tooth pair to find K_{e2} . Then we can obtain the equivalent stiffness for meshing as follows:

$$K_e = K_{e1} + K_{e2} \quad (14)$$

In the mesh stiffness calculation, one needs to know the load position relative to the teeth in contact along the path of contact. Note that the faulty tooth position should be changed through the entire contact length for investigating the time-varying mesh stiffness. To obtain the locations of contact points along the path of contact, the contact zones can be determined from the contact ratio and the base circular pitch [25].

$$\text{Basically, C.R} = \frac{PTH}{P_b} \quad (15)$$

where C.R is the contact ratio, PTH is the path of contact, and P_b is the base circular pitch.

In low contact ratio gearing, the load is transmitted by a single pair of teeth for a part of the engagement period, and by two pairs of teeth during the rest of the period. At the beginning of contact there are two pairs in contact continuing to mesh for a specified period, and then one pair goes out of contact, while the other pair continues to mesh alone, until a new pair comes into action [26].

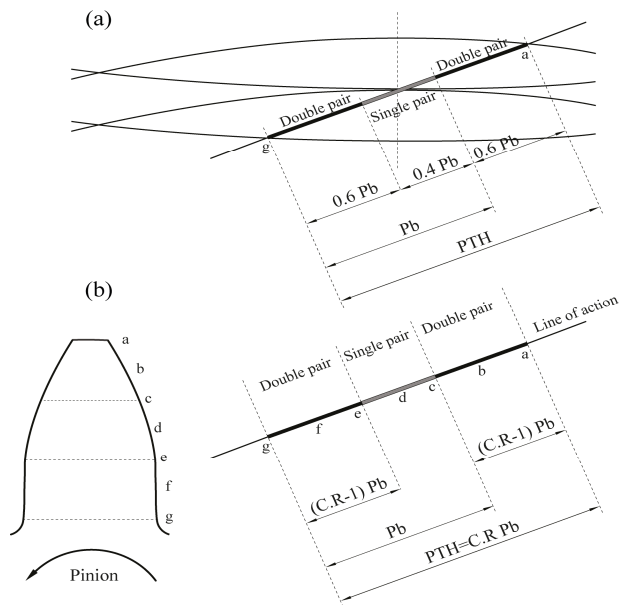


Fig. 5 Contact zones along the path of contact. (a) Contact zones for a contact ratio of 1.6, (b) Contact zones for low contact ratio gears

For example, a contact ratio of (1.6) means that the path of contact is equal to 1.6 times the base circular pitch. The path of contact is divided into three zones, as shown in Fig. 5-a, and at the beginning of the path of contact, there are two pairs continuing to mesh for 60% of the base circular pitch along the path of contact. Then one pair go out of contact and the other pair continue to mesh alone for the remaining 40% of the base circular pitch, after which another new pair come into contact for a further 60% of the base circular pitch. In general, the contact zones along the line of action for a low contact ratio can be found as described in Fig. 5-b.

V. ONE-STAGE GEAR MODELLING

A single-stage spur gear model has been adopted in the present research work for simulations performed to investigate the time-varying gear mesh stiffness of two mating gears. The main gear modelling parameters that are used in this paper have been taken from [1], and can be seen in Table III. Here the term '*pinion*' refers to the smaller gear which is a driven gear in our case, and the term '*gear*' refers to the larger gear, which is a driver gear in our case.

TABLE III
PARAMETERS OF GEAR-PINION SET [1]

Parameter	Gear	Pinion
Number of teeth	30	25
Module (mm)	2	2
Teeth width (mm)	20	20
Contact ratio	1.63	1.63
Rotational speed (rpm)	2000	2400
Pressure angle (deg.)	20	20
Young's modulus, E (N/mm ²)	2×10^5	2×10^5
Poisson's ratio	0.3	0.3

VI. PROGRAMMING AND COMPUTER SIMULATION

The mesh stiffness calculations have been programmed using Matlab code to investigate the time-varying gear mesh stiffness. In this code the crack propagation can be modelled with the crack propagation scenario that has been proposed in this paper, and according to the mesh stiffness calculations that have been explained in Section II. To find the varying load positions relative to the teeth in contact, AutoCAD software has been used for teeth simulation relative to the path of contact.

VII. RESULTS AND DISCUSSION

The time-varying gear mesh stiffness for all the studied cases is shown in Fig.6. It is obvious in the figure that the mesh stiffness decreases as the crack propagates; in the figure, the direction of the arrow shows the sequence of the crack propagation cases of the proposed scenario. We can observe that the loss of mesh stiffness is much higher on the left-hand side than on the right-hand side of the figure, and that is because the pinion, which has a cracked tooth is considered as the driven gear in the studied model. Therefore, if we reverse the direction of the gear rotation and consider the pinion as the driver, then the time-varying mesh stiffness plots will be reversed as well.

Since the proposed crack propagation scenario is more realistic for non-uniform load distribution cases and can be applied for crack propagation modelling and monitoring simulation, further research should be conducted to analyze the effect of this scenario and the other scenarios on the possibility of performing fault diagnosis. When crack propagation is present, it is not feasible to extract the fault from the generated vibration all the time, and the possibility of extracting the fault depends on how much the mesh stiffness parameter is decreased. When the amount of mesh stiffness loss due to crack propagation affects the dynamic behaviour of the system, it is possible to detect the fault by monitoring and analyzing the dynamic response.

VIII. CONCLUSIONS

In this paper an analytical approach has been used for investigating the time varying gear mesh stiffness with the presence of crack propagation in the gear tooth root. The proposed crack propagation scenario is more reasonable and realistic for non-uniform load distribution cases and can be applied for crack propagation modelling and monitoring simulation. Nevertheless, further research is required for comparison and evaluation of all the presented crack propagation scenarios from the condition monitoring point of view.

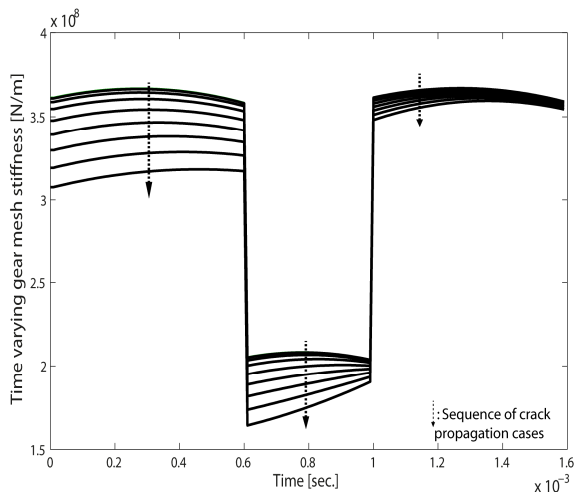


Fig. 6 Reduction in time varying mesh stiffness according to the proposed scenario

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