# Exploiting Machine Learning Techniques for the Enhancement of Acceptance Sampling

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Abstract—This paper proposes an innovative methodology for Acceptance Sampling by Variables, which is a particular category of Statistical Quality Control dealing with the assurance of products quality. Our contribution lies in the exploitation of machine learning techniques to address the complexity and remedy the drawbacks of existing approaches. More specifically, the proposed methodology exploits Artificial Neural Networks (ANNs) to aid decision making about the acceptance or rejection of an inspected sample. For any type of inspection, ANNs are trained by data from corresponding tables of a standard's sampling plan schemes. Once trained, ANNs can give closed-form solutions for any acceptance quality level and sample size, thus leading to an automation of the reading of the sampling plan tables, without any need of compromise with the values of the specific standard chosen each time. The proposed methodology provides enough flexibility to quality control engineers during the inspection of their samples, allowing the consideration of specific needs, while it also reduces the time and the cost required for these inspections. Its applicability and advantages are demonstrated through two numerical examples.

**Keywords**—Acceptance Sampling, Neural Networks, Statistical Quality Control.

### I. INTRODUCTION

NOWADAYS, industries care more and more about the quality of their products, motivated by the general belief that enhancement of quality will help them increase their prestige and, consequently, their incomes. However, quality is a complex concept, for which different definitions have been already given. Generally speaking, quality is a metric of perfection. According to the international standard ISO 9000 [1], the interpretation of quality is the degree to which a set of inherent characteristics fulfills a certain set of requirements.

Statistical methods play a vital role in quality control and improvement. According to this research direction, quality is

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usually assured through the application of Statistical Quality Control (SQC) methods. Work reported in this paper focuses on Acceptance Sampling (AS), one of the three big categories of SQC, and more particularly on Acceptance Sampling by Variables (ASV). As reveals from the literature [2-3], in the majority of contexts and cases, ASV is difficult to be applied by unskilled staff. Moreover, products usually need to be checked for more than one variables at the same time (e.g. the diameter and the length of an axle); thus, the overall procedure has to be repeated as many times as the number of the variables, which in turn increases significantly the time needed for the complete product's inspection. Finally, the quality engineer who is responsible for the decision of acceptance or rejection of a product has to use standards that may not refer to the accurate value of the inspection parameters considered each time, thus resulting in a compromise with the values given by the standards (standards refer to ranges of values).

Aiming to enhance the above process, this paper proposes an innovative methodology for the decision of acceptance or rejection of a sample, by training the proper Artificial Neural Networks (ANNs) with data from different standards, for samples with one or more variables, and for different Acceptance Quality Levels (AQLs). Once trained, ANNs can give the necessary information for acceptance or rejection of the samples, providing closed-form solutions for any AQL required and removing the burden of table look-ups.

In the proposed methodology, feed-forward neural networks with one hidden layer are trained by a back propagation algorithm [4] using data sets from different standards. The back propagation algorithm has been widely recognized as an effective method for training feed-forward neural networks. It works well for many problems (e.g., classification or pattern recognition) [5-7]. Different sets of ANNs are used for different levels of inspections.

In the remaining of this paper, the theoretical background of AS is sketched and the proposed methodology is presented in detail. The applicability of our approach is then demonstrated through two numerical examples. Finally, a discussion about the advantages and limitations of the proposed methodology, as well as about future work directions, is given.

### II. ACCEPTANCE SAMPLING

### A. Background Issues

The purpose of AS is the acceptance or rejection of a product sample based on its adherence to a specific standard. Its adoption is motivated by the fact that the 100% inspection is not the most efficient method for the separation of the conforming and non-conforming products (based on economic criteria). The inspected samples can be raw materials, semifinished or finished products. Nowadays, that the quality systems aim for prevention and not for ex post facto inspection, AS is not appropriate for direct quality improvement; instead, it can be used for quality assurance. This is basically the reason why AS is being used less as a main methodology (as is the case in Statistical Process Control) and more as a helpful tool to ensure the quality of the products in a manufacturing process.

The successful use of AS at the early stages of manufacturing can greatly reduce, and in some cases even eliminate, the need for extensive sampling inspection. Consequently, AS is being used at the beginning of the second stage of a manufacturing process, where the raw materials are inspected before entering the production line, at the heart of the second stage to check semi-finished products, and at the third stage to assure the quality of finished products (Fig. 1).

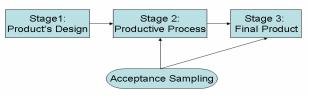


Fig. 1 Manufacturing Process Stages

AS includes two broad approaches, namely Acceptance Sampling by Attributes (ASA) and ASV [8]. The former is followed when the item inspection leads to a binary result (either the item is conforming or non-conforming), or the number of non-conformities in an item are counted. The latter is used when the item inspection leads to a continuous measurement, which is the case that occurs more often. The most important advantage of ASV is that variables reach the operating-characteristic curve with a smaller sample. This is due to the fact that the variables contain more information than the attributes, which refer to a breadth of values that fall in or out a range of specifications. Inspection of a smaller sample of variables is certainly associated with less cost.

One of the procedures associated to ASV, upon which the proposed methodology builds, is described below. A sample n is taken from a lot and the mean value  $\bar{x}$  of the sample, as well as the indicator  $Z_L = (\bar{x} - L)/\sigma$ , which gives the distance between and L in number of standards deviations are counted (where L refers to the lower specification limit and  $\sigma$  to the standard deviation of the variable). If the indicator  $Z_L$  is bigger or equal than a crucial distance k, then the sample is accepted;

otherwise, it is rejected. The distance k defines indirectly the percentage of non-conforming products, which is accepted with a defined probability.

It is noted that, in the methodology described above, the variability of the samples taken is unknown; also, in case of existence of upper and lower limit, the methodology is repeated for both limits. For the calculation of k, standardization sampling plan schemes are used. These schemes contain tables and diagrams for the choice of the parameters of the sampling plans, the AQL being the basic parameter. Another important parameter is the level of inspection, which defines the relation between the lot size N and the sample size n.

All inspections begin with the normal type; under specific circumstances, the type changes to the tightened or the reduced one. Tightened inspection is used when the sample's recent quality history has deteriorated (acceptance criteria are more stringent than those under normal inspection). On the other hand, reduced inspection is used when the sample's recent quality history has been exceptionally good (sample sizes are usually smaller than those under normal inspection). The switching procedures between the normal, tightened and reduced inspection types are standard-dependant (for instance, a detailed description of these procedures for the MIL STD 414 standard [9], which is used in the following sections of this paper, can be found in [10]).

# B. Acceptance Sampling Enriched with Machine Learning

Approaches employing machine learning techniques in AS are limited and mainly focused on the design of AS plans. One such approach is described in [11], where the authors suggest a Genetic Algorithm (GA) mechanism to reach a closed-form solution for the design of a double sampling plan. In order to design the double sampling plan, the operating-characteristic curve has to satisfy some specific criteria. As the parameters of the sampling plan have to be integers, the solution has to be optimal in each case. The GA mechanism is responsible for providing the optimal solution in contrast to the trial-and-error method that has been used so far. This approach seeks for the minimum sample number, even when the initial criteria are not satisfied. Its disadvantage is the relatively large number of the proposed solutions, from which the quality engineer has to decide the optimal one by changing the criteria that were predetermined at the beginning of the process.

A second approach attempts to find a closed-form solution for the design of a single-sampling plan for attributes [12]. The sample size and the acceptance number are determined by training ANNs with data from tabled sampling schemes, for normal, tightened and reduced inspections. The contribution of this approach is the design of the sampling plan for any required value of AQL and lot size. The methodology proposed in the next section of this paper elaborates this second approach.

### III. THE PROPOSED METHODOLOGY

As stated above, the proposed methodology aims to eliminate the difficulties of applying ASV. Its main purpose is to automate the process of reading the standards, while giving closed-form solutions for any AQL and for any level of inspection. The methodology can be repeated for as many variables as needed. It can handle different AQLs, for different upper or lower limits. The automation of the decision making process towards acceptance or rejection of a sample by variables also helps to reduce the time spending for inspection.

The proposed methodology considers the design of an AS plan as given, and goes ahead, looking if a random sample is accepted or rejected according to this plan. As soon as the quality engineers have designed the proper AS plan, they have to decide if a random sample is accepted or rejected. The application of ANNs in this case will give the information required in order to facilitate the abovementioned decision making process.

According to the proposed methodology, the crucial distance k of a random sample is calculated by ANNs. ANNs are trained in a way that the functional relationship between two discrete input attributes (sample size and AQL) and the output k is approximated. The same procedure can be repeated for different standards and levels of inspection, so that different sets of ANNs can be used in each case. Whatever standard is chosen each time, the corresponding ANNs are developed.

TABLE I
MASTER TABLE FOR NORMAL AND TIGHTENED INSPECTION FOR PLANS BASED
ON VARIABILITY UNKNOWN (STANDARD DEVIATION METHOD) MIL STD 414

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (normal inspection)													
		.04	.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00
		k	k	k	k	k	k	k	k	k	k	k	k	k	k
В	3		T			1	T	T	•		1.12	.958	.765	.566	.341
C	4			Ш				*	1.45	1.34	1.17	1.01	.814	.617	.393
D	5					*	*	1.65	1.53	1.40	1.24	1.07	.874	.675	.455
E	7			11	₩	2.00	1.88	1.75	1.62	1.50	1.33	1.15	.955	.755	.536
F	10	•	*	*	2.24	2.11	1.98	1.84	1.72	1.58	1.41	1.23	1.03	.828	.611
G	15	2.64	2.53	2.42	2.32	2.20	2.06	1.91	1.79	1.65	1.47	1.30	1.09	.886	.664
Н	20	2.69	2.58	2.47	2.36	2.24	2.11	1.96	1.82	1.69	1.51	1.33	1.12	.917	.695
1	25	2.72	2.61	2.50	2.40	2.26	2.14	1.98	1.85	1.72	1.53	1.35	1.14	.936	.712
J	30	2.73	2.61	2.51	2.41	2.28	2.15	2.00	1.86	1.73	1.55	1.36	1.15	.946	.723
K	35	2.77	2.65	2.54	2.45	2.31	2.18	2.03	1.89	1.76	1.57	1.39	1.18	.969	.745
L	40	2.77	2.66	2.55	2.44	2.31	2.18	2.03	1.89	1.76	1.58	1.39	1.18	.971	.746
M	50	2.83	2.71	2.60	2.50	2.35	2.22	2.08	1.93	1.80	1.61	1.42	1.21	1.00	.774
N	75	2.90	2.77	2.66	2.55	2.41	2.27	2.12	1.98	1.84	1.65	1.46	1.24	1.03	.804
0	100	2.92	2.80	2.69	2.58	2.43	2.29	2.14	2.00	1.86	1.67	1.48	1.26	1.05	.819
P	150	2.96	2.84	2.73	2.61	2.47	2.33	2.18	2.03	1.89	1.70	1.51	1.29	1.07	.841
Q	200	2.97	2.85	2.73	2.62	2.47	2.33	2.18	2.04	1.89	1.70	1.51	1.29	1.07	.845
		.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00	
	Acceptable Quality Levels (tightened inspection)														

The proposed methodology begins with the training of the proper ANNs by the different tables of a standard, for each level of inspection. More specifically, feed-forward neural networks with one hidden layer are trained by a back propagation algorithm. In the examples illustrated below, the MIL STD 414 standard has been used [9]. An example of the standard MIL STD 414 is shown in Table I.

The easyNN-plus software [13] has been used for the training of the neural networks, with data of different AQLs and different inspection levels. Tests with learning rates from 0.1 to 0.001 are performed to reach the one that is more suitable for each case. The learning rate specifies the

magnitude change of weights during back propagation training. In the hidden layer, one starts with a large number of nodes and, successively, some nodes are being removed until the performance is within a specific acceptable level [14].

For the implementation of the proposed methodology, the choice of the proper standard as well as the inspection level have to be determined. After the determination of the general inspection parameters, more information has to be inserted. This includes: the sample size n, the number of variables, the mean and the  $\sigma$  for each of the variables, and the AQL for the upper and lower limits (or any of the two that exists), for all variables.

### IV. NUMERICAL EXAMPLES

Two numerical examples are given to better illustrate the applicability of the proposed methodology. The first one concerns a case of a product with one variable, which has two specification limits. The results of the proposed methodology are compared with those of the standard selected. The second one concerns a case of a product with two variables, both of them having only upper specification limit. For this case, the exact AQL value chosen is not included in the standard, in order to prove that the proposed methodology can handle any value of AQL and sample size, thus satisfying any request of the quality engineers to meet specific needs.

**Example 1.** A lot of 600 axles has to be inspected for acceptance or rejection. The basic variable for the decision is the axle's diameter that follows the normal distribution with unknown variability. Let the specifications of the case under consideration define lower limit L=44.8 mm and upper limit U=45.2 mm. The upper limit is more important, which is reflected in the values of  $AQL_L$  and  $AQL_U$ : let  $AQL_L$ =1.5 and  $AQL_U$ =0.65. From the measurement of a sample of n=20 axles, it was counted that  $\overline{x}$  =44.96 cm with  $\sigma$ =0.09 cm. The sample is checked for acceptance or rejection based on the criteria of MIL STD 414. Finally, let the level of inspection being the normal one.

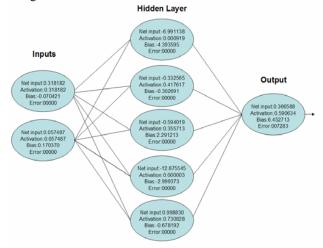


Fig. 2 The developed 2-5-1 feed-forward ANN

According to the proposed methodology, ANNs have to be trained based on the criteria of MIL STD 414 for ASV and the data contained in the tables of this standard. After a number of tests, the learning rate was settled to 0.01, the number of hidden layer nodes to 5 and the target error below 0.001. As the associated ANNs are trained with the above data, the values of  $k_L$  and  $k_U$  are recorded ( $k_L$  and  $k_U$  stand for lower and upper limit, respectively). The trained ANN by easyNN-plus gives  $k_L$ =1.687 and  $k_U$ =1.971 for the data of this case.

The interconnection of the nodes of the trained ANN, together with the associated data, are illustrated in Fig. 2.

Fig. 3 shows a part of the training set's errors (in the case described here, the training set consisted of 99 data tuples, while the target error was set to 0.001; all tuples were below the target error).

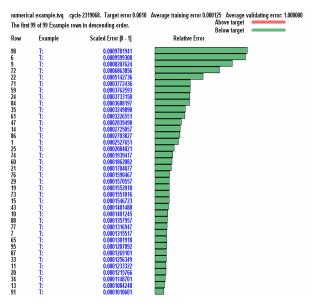


Fig. 3 Example errors of the developed 2-5-1 Feed-forward ANN

After the calculation of  $k_L$  and  $k_U$  from the trained ANN, the values of  $Z_L$  and  $Z_U$  are computed. It is:

$$Z_L = (\overline{x} - L) / \sigma = (44.96-44.8)/0.09=1.78$$

$$Z_U = (U - \overline{x})/\sigma = (45.2-44.96)/0.09 = 2.67$$

Based on the procedure described in Section II, and since  $Z_L > k_L$  and  $Z_U > k_U$ , the sample is accepted.

For the same problem, values taken directly from the MIL STD 414 standard (without using the proposed methodology) are  $k_L$ '=1.69 and  $k_U$ '=1.96. From the comparison with the values taken from the trained ANNs ( $k_L$  and  $k_U$ ), it becomes obvious that the percentage of deviation is very small.

**Example 2.** A lot of a specific type of cable is being checked against two variables. The first one is the product's resistance, which should not exceed  $UI=60 \Omega/km$ , and the

second one is its diameter, which should not exceed U2=30 cm. Both variables follow the normal distribution with unknown variability. From the lot, according to the criteria of MIL STD 414, a sample of 10 cables is taken. The mean value of resistance is measured to  $\overline{x}=56.85~\Omega/\mathrm{km}$  with  $\sigma=1.8~\Omega/\mathrm{km}$  and the mean value of the diameter is measured to  $\overline{x}=25~\mathrm{cm}$  with  $\sigma=2~\mathrm{cm}$ . The sample is checked for acceptance or rejection based on the criteria of MIL STD 414. The product's resistance is considered more important than its diameter, which is reflected in the values of  $AQL_{UI}$  and  $AQL_{U2}$ : let  $AQL_{UI}=0.7~\mathrm{and}~AQL_{U2}=0.9$ . The level of inspection is the normal one.

Using the ANN already trained in the previous example (the same standard as well as the same level of inspection are being used), the values of  $k_{UI}$  and  $k_{U2}$  are recorded ( $k_{UI}$  and  $k_{U2}$  are associated to the product's resistance and diameter, respectively). The trained ANN by easyNN-plus gives  $k_{UI}$ =1.812 and  $k_{U2}$ =1.746. The values of  $Z_{UI}$  and  $Z_{U2}$  are:

$$Z_{UI} = (UI - \overline{x}) / \sigma = (60-56.85)/1.8 = 1.75$$

$$Z_{U2} = (U2 - \overline{x}) / \sigma = (30-25)/2 = 2.5$$

Based on the procedure described in Section II, and since  $Z_{UJ} < k_{UJ}$  and  $Z_{UZ} > k_{UZ}$ , the sample is not accepted.

The proposed methodology calculates the exact value of distance k for any required value of AQL in contrast to the table look-up method used so far. For this case, according to the standard (and not applying the proposed methodology), both values of  $AQL_{UI}$  (0.7) and  $AQL_{U2}$  (0.9) are compromised to the value 1, which gives  $k_{UI}$ '=1.72 and  $k_{U2}$ '=1.72. Even though different AQLs have been used for the resistance and the diameter to show that the former is more important than the latter, the same values of distance k are finally applied. This is not correct and may conclude to a false decision (as  $Z_{UI} > k_{UI}$ '). ANNs that calculated a closed-form solution for  $AQL_{UI}$ =0.7 resulted that the resistance is not accepted  $(Z_{UI} < k_{UI})$ . Instead, if the compromised value of the standard were used, then a sample with not acceptable quality would have passed the quality control.

# v. Discussion - Conclusion

This paper proposes an innovative methodology for ASV, which plays an important role in the overall quality assurance of an enterprise. As soon as an ASV plan is designed, the quality engineers have to decide (based on the criteria of an appropriate standard) if the samples taken from a lot are accepted or rejected. The proposed methodology alleviates the burden of standards' table schemes look-ups and gives closed-form solutions for any AQL value, even for those that are not included in the standards. Moreover, it does not include complex and excessive calculations, and gives straightforward solutions, thus making quality inspection a much easier task, as far as effort, cost and time are concerned. In addition, the proposed methodology can be exploited by laymen, since its

application does not require any deep knowledge in ANNs and sampling inspection procedures.

In all sampling plan schemes, sampling plans are available for a finite number of AQL values and sample sizes. Table II shows how the AQL values are compromised to those of the standard. In the proposed methodology, no compromise is required. The distance k (which is the key factor of accepting or rejecting a sample based on the ASV procedure that is used in this paper) is calculated for any discrete AQL value, thus reducing the possibility of false decision after compromising with a standard.

TABLE II CONVERSION OF AOL VALUES

CONVERSION OF AQL VALUES							
Desired value of AQL (%)	Compromised with the standard value of AQL (%)						
0 to 0.109	0.10						
0.110 to 0.164	0.15						
0.165 to 0.279	0.25						
0.280 to 0.439	0.40						
0.440 to 0.699	0.65						
0.700 to 1.09	1.00						
1.10 to 1.64	1.50						
1.65 to 2.79	2.50						
2.80 to 4.39	4.00						
4.40 to 6.99	6.50						
7.00 to 10.90	10.00						

In any case, a limitation of the proposed methodology, which is inherited from the ASV procedure adopted (as described in Section II), is that the quality variables under inspection have to follow the normal distribution.

Future work directions concern the enrichment of the proposed methodology with other machine learning techniques to further aid the overall quality assurance process. For instance, decision trees could be used to diagnose the causes of the non-conformities of a sample that is rejected, and indicate the appropriate interventions in the manufacturing process.

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