International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:3, No:11, 2009

# The Euler Equations of Steady Flow in Terms of New Dependent and Independent Variables

Peiangpob Monnuanprang

***Abstract*—**In this paper we study the transformation of Euler equations

$$\frac{\partial \vec{u}}{\partial t}+\left(\vec{u}\cdot\nabla\right)\vec{u}=-\frac{1}{\rho}\nabla P+\vec{f},\ \nabla\cdot\vec{u}=0,$$

where $\vec{u}(\vec{x},t)$ is the velocity of a fluid, $P(\vec{x},t)$ is the pressure of a fluid and $\rho(\vec{x},t)$ is density. First of all, we rewrite the Euler equations in terms of new unknown functions. Then, we introduce new independent variables and transform it to a new curvilinear coordinate system. We obtain the Euler equations in the new dependent and independent variables. The governing equations into two subsystems, one is hyperbolic and another is elliptic.

***Keywords*—**Euler equations, transformation, hyperbolic, elliptic

## I. Introduction

IN In the paper we consider the two-dimensional for flow of ideal incompressible fluid through the channel depicted in fig. 1. The incompressible Euler equations can be formulated in a convenient alternative manner, by introducing two scalar variables in place of the primitive variables the velocity $\vec{u}$ and the pressure $P$. The vorticity-stream function formulation has been a popular tools of computing two-dimensional incompressible flows $[1]$, $[2]$. Sometimes, it is convenient to use another two scalar variables which are differed from vorticity and stream function$[3]$, $[4]$. In this paper, we will use the modulus of the velocity and the flow angle (angle between the direction of velocity vector and the direction of $Ox$ axis) as two new unknown functions instead the primitive variables.

## II. Mathematical Formulation

The motion of a homogeneous ideal incompressible fluid is described by the Euler equations $[5]$.

$$\frac{\partial \vec{u}}{\partial t}+\left(\vec{u}\cdot\nabla\right)\vec{u}=-\frac{1}{\rho}\nabla P+\vec{f},\ \nabla\cdot\vec{u}=0,$$

where $\vec{u}(\vec{x},t)$ is the velocity of a fluid, $P(\vec{x},t)$ is the pressure of a fluid and $\rho(\vec{x},t)$ is density.

The Euler equations of the two-dimensional steady incompressible ideal fluid are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}, \qquad (1)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}, \qquad (2)$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \qquad (3)$$

where $u(x,y)$ and $v(x,y)$ are components of the velocity in the $x$ and $y$ direction respectively, $P(x,y)$ is the pressure of a fluid. Without loss of generality we set the density equal to one. $(\rho=1)$.

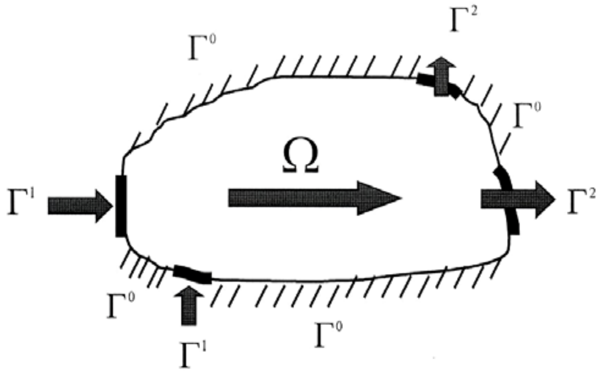



Fig. 1 Sketch of a domain.

Let us assume that flow occurs in the domain $\Omega$ depicted in Fig 1. We assume that the solid impermeable boundaries $\Gamma^0$ is described by curves which are given by equations in the natural form

$$k=k_i(l), \qquad i=1,2.$$

It is convenient to rewrite the Euler Equations in terms of new unknown functions $w(x,y)$ and $q(x,y)$ which are determined by the relations $u=w(x,y)\cos q(x,y)$ and $v=w(x,y)\sin q(x,y)$. Actually, $w(x,y)$ is the modulus of the velocity vector and problems $q(x,y)$ is the angle between the direction of the velocity vector and the $Ox$ axis. We will call $q$ as the flow angle. In this paper, we study the

Peiangpob Monnuanprang, Phranakhon Rajabhat University Changwattana Road, Bangkhen,Bangkok,10220,Thailand,. (corresponding author to provide phone:066-2522-6623; fax: 066-2522-6623; e-mail: peiangpb@math.sut.ac.th )

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:3, No:11, 2009

transformation for two kinds of the boundary value problem in Fig. 2.

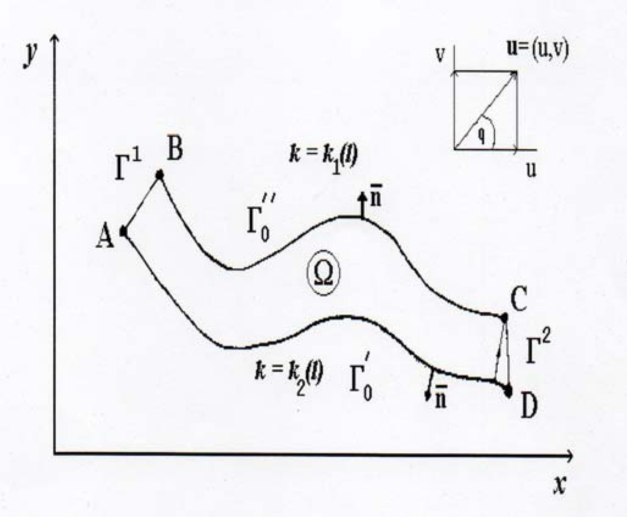



Fig. 2.Sketch of a physical domain.

Problem 1 boundary conditions:

Impermeable boundaries $AD$ and $BC$ : $(x, y) \in \Gamma^0$

$$\vec{u} \cdot \vec{n} = 0. \tag{4}$$

Inflow part $AB : (x, y) \in \Gamma^1$

$$q = q_1(x, y), w = w_1(x, y). \tag{5}$$

Outflow part $CD : (x, y) \in \Gamma^2$

$$q = q_2(x, y). \tag{6}$$

Problem 2 boundary conditions:

Impermeable boundaries $AD$ and $BC$ : $(x, y) \in \Gamma^0$

$$\vec{u} \cdot \vec{n} = 0. \tag{7}$$

Inflow part $AB : (x, y) \in \Gamma^1$

$$q = q_1(x, y), w = w_1(x, y). \tag{8}$$

Outflow part $CD : (x, y) \in \Gamma^2$

$$\begin{gathered} P = P_2(x, y), \\ \vec{u} \cdot \vec{n} > 0. \end{gathered} \tag{9}$$

The problem 2 differs from the problem 1 in the boundary conditions on the outflow part $CD$. On $CD$, only pressure and condition for sign of normal component of the velocity vector are given.

## III. EULER EQUATIONS IN TERMS OF NEW UNKNOWN FUNCTION $\omega(x, y)$ AND $q(x, y)$

We eliminate the pressure from the Euler equations by eliminating the mixed derivatives. Taking the derivatives in $(1)$ and $(2)$ with respect to $y$ and $x$, respectively, we obtain

$$\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial x}\right)+u\frac{\partial^2 u}{\partial x\partial y}+\left(\frac{\partial v}{\partial y}\right)\left(\frac{\partial u}{\partial y}\right)+v\frac{\partial^2 u}{\partial y^2}=-\frac{\partial^2 P}{\partial x\partial y},$$

$$\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x}\right)+u\frac{\partial^2 v}{\partial x^2}+\left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right)+v\frac{\partial^2 v}{\partial x\partial y}=-\frac{\partial^2 P}{\partial y\partial x}.$$

Then, we eliminate the terms containing the pressure by subtracting these two equations and use the condition

$$\frac{\partial^2 P}{\partial x\partial y}=\frac{\partial^2 P}{\partial y\partial x},$$

We have

$$\begin{aligned}&\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial x}\right)+u\frac{\partial^2 u}{\partial x\partial y}+\left(\frac{\partial v}{\partial y}\right)\left(\frac{\partial u}{\partial y}\right)+v\frac{\partial^2 u}{\partial y^2}\\&-\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x}\right)-u\frac{\partial^2 v}{\partial x^2}-\left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right)-v\frac{\partial^2 v}{\partial x\partial y}=0.\end{aligned} \tag{10}$$

Substitution of $u = w(x, y)\cos q(x, y)$ and $v = w(x, y)\sin q(x, y)$ into continuity $(3)$ yields

$$\frac{\partial w}{\partial x}\cos q - w\sin q\frac{\partial q}{\partial x}+\frac{\partial w}{\partial y}\sin q + w\cos q\frac{\partial q}{\partial y}=0. \tag{11}$$

On the other hand, substitution of the expressions $u = w(x, y)\cos q(x, y)$ and $v = w(x, y)\sin q(x, y)$ into $(10)$ gives us the following equation

$$\begin{aligned}&-4\cos q w\sin q\frac{\partial w}{\partial y}\frac{\partial q}{\partial x}-4w\cos q\sin q\frac{\partial q}{\partial y}\frac{\partial w}{\partial x}-4w(\cos q)^2\frac{\partial w}{\partial x}\frac{\partial q}{\partial x}\\&+2w^2\cos q\sin q\left(\frac{\partial q}{\partial x}\right)^2-w\cos q\sin q\left(\frac{\partial^2 w}{\partial x^2}\right)-4w^2(\cos q)^2\frac{\partial q}{\partial y}\frac{\partial q}{\partial x}\\&-2w^2\cos q\sin q\frac{\partial^2 q}{\partial x\partial y}+4w(\cos q)^2\left(\frac{\partial q}{\partial y}\right)\left(\frac{\partial w}{\partial y}\right)-2w^2\cos q\sin q\left(\frac{\partial q}{\partial y}\right)^2\\&+w\cos q\sin q\frac{\partial^2 w}{\partial y^2}-w^2\frac{\partial^2 q}{\partial y^2}-\left(\frac{\partial w}{\partial y}\right)\left(\frac{\partial w}{\partial x}\right)-w\frac{\partial^2 w}{\partial x\partial y}+2\cos q\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial w}{\partial x}\\&+2w(\cos q)^2\frac{\partial^2 w}{\partial x\partial y}+\cos q\sin q\left(\frac{\partial w}{\partial y}\right)^2-\cos q\sin q\left(\frac{\partial w}{\partial x}\right)^2-w^2(\cos q)^2\frac{\partial^2 q}{\partial x^2}\\&+w^2(\cos q)^2\frac{\partial^2 q}{\partial y^2}+2w^2\frac{\partial q}{\partial y}\frac{\partial q}{\partial x}+w\frac{\partial w}{\partial x}\frac{\partial q}{\partial x}-3w\frac{\partial q}{\partial y}\frac{\partial w}{\partial y}=0.\end{aligned} \tag{12}$$

Differentiating $(11)$ with respect to $x$ and $y$ , we obtain

$$\begin{aligned}&\cos q\frac{\partial^2 w}{\partial x^2}-2\sin q\frac{\partial w}{\partial x}\frac{\partial q}{\partial x}-w\cos q\left(\frac{\partial q}{\partial x}\right)^2-w\sin q\frac{\partial^2 q}{\partial x^2}+\sin q\frac{\partial^2 w}{\partial x\partial y}\\&+\cos q\frac{\partial w}{\partial y}\frac{\partial q}{\partial x}+\cos q\frac{\partial w}{\partial x}\frac{\partial q}{\partial y}-w\sin q\frac{\partial q}{\partial y}\frac{\partial q}{\partial x}+w\cos q\frac{\partial^2 q}{\partial x\partial y}=0,\end{aligned}$$

$$\begin{aligned}&\cos q\frac{\partial^2 w}{\partial x\partial y}-\sin q\frac{\partial w}{\partial x}\frac{\partial q}{\partial x}-\sin q\frac{\partial w}{\partial y}\frac{\partial q}{\partial x}-w\cos q\frac{\partial q}{\partial y}\frac{\partial q}{\partial x}-w\sin q\frac{\partial^2 q}{\partial x\partial y}\\&+\sin q\frac{\partial^2 w}{\partial y^2}+2\cos q\frac{\partial w}{\partial y}\frac{\partial q}{\partial y}-w\sin q\left(\frac{\partial q}{\partial y}\right)^2+w\cos q\frac{\partial^2 q}{\partial y^2}=0.\end{aligned}$$

ISSN: 2517-9934
Vol:3, No:11, 2009

Then, we use these two equations together to find $\frac{\partial^2 q}{\partial x \partial y}$ and $\frac{\partial^2 w}{\partial x \partial y}$. After that, substitute the value of mixed derivatives into $(12)$, we get the following

$$
\begin{aligned}
&-w^2\frac{\partial^2 q}{\partial y^2}-\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}-3w\frac{\partial w}{\partial y}\frac{\partial q}{\partial y}-2w\cos q\sin q\frac{\partial q}{\partial y}\frac{\partial w}{\partial x}-2w^2(\cos q)^2\frac{\partial q}{\partial x}\frac{\partial q}{\partial y}\\
&-w\frac{\partial q}{\partial x}\frac{\partial w}{\partial x}+2w(\cos q)^2\frac{\partial q}{\partial y}\frac{\partial w}{\partial y}-w^2\cos q\sin q\left(\frac{\partial q}{\partial y}\right)^2-2w(\cos q)^2\frac{\partial w}{\partial x}\frac{\partial q}{\partial x}\\
&+w^2\cos q\sin q\left(\frac{\partial q}{\partial x}\right)^2-2w\cos q\sin q\frac{\partial w}{\partial y}\frac{\partial q}{\partial x}+2(\cos q)^2\frac{\partial w}{\partial y}\frac{\partial w}{\partial x}\\
&+\cos q\sin q\left(\frac{\partial w}{\partial y}\right)^2-\cos q\sin q\left(\frac{\partial w}{\partial x}\right)^2-w^2\frac{\partial^2 q}{\partial x^2}+w^2\frac{\partial q}{\partial x}\frac{\partial q}{\partial y}=0.
\end{aligned} \quad (13)
$$

To eliminate the terms underlined, it is convenient to use continuity $(11)$. The multiplication of $(11)$ by $\frac{\partial w}{\partial x}\sin q$ gives us the value of $\left(\frac{\partial w}{\partial x}\right)^2\cos q\sin q.$ The multiplication these values, of $(11)$ by $w\cos q\frac{\partial q}{\partial x}$ gives us the value of $w(\cos q)^2\frac{\partial w}{\partial x}\frac{\partial q}{\partial x}.$ The multiplication of $(11)$ by $w\sin q\frac{\partial q}{\partial y}$ give us the value of $w\cos q\sin q\frac{\partial w}{\partial x}\frac{\partial q}{\partial y}.$ Then the substituting these values, $\left(\frac{\partial w}{\partial x}\right)^2\cos q\sin q, \left(\frac{\partial w}{\partial y}\right)^2\cos q\sin q, w(\cos q)^2\frac{\partial w}{\partial x}\frac{\partial q}{\partial x}$ and $w\cos q\sin q\frac{\partial w}{\partial x}\frac{\partial q}{\partial y}$, into $(13)$ instead of the terms underlined, after simplification, we obtain

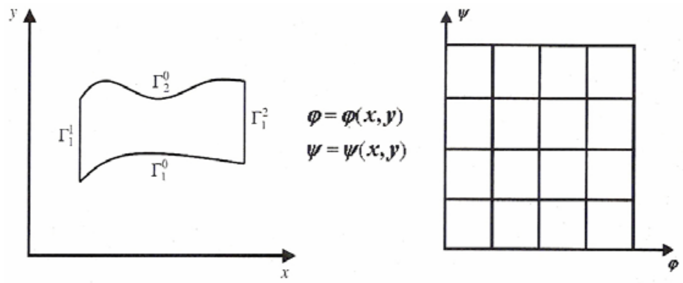
$$\frac{\partial}{\partial x}\left(w^2\frac{\partial q}{\partial x}\right)+\frac{\partial}{\partial y}\left(w^2\frac{\partial q}{\partial y}\right)=0. \quad (14)$$

All above algebraic manipulation are done by MAPLE program.

## IV. TRANSFORMATION FROM CARTESIAN COORDINATES $(x, y)$ TO GENERALIZED CURVILINEAR COORDINATES $(\varphi, \psi)$

The computation of flow fields in and around complex shapes such as ducts, engine, complete aircraft or automobiles, etc., involves computational boundaries that do not coincide with coordinate lines in a physical domain. For finite difference methods, the imposition of boundary conditions for such problems motivate the introduction of a mapping or transformation from physical $(x, y)$ domain to a generalized curvilinear coordinate space. The generalize coordinate domain is constructed so that a computational boundary in a physical domain coincides with a coordinate line in a generalized coordinate space.

The use of generalized coordinates implies that a distorted region in a physical domain is mapped into a rectangular region in the generalized coordinate space as shown in Fig.3.




a) Physical domain b) Computational domain

Fig. 3 Physical and computational domain

Next, we introduce new independent variables $\varphi$ and $\psi$. We will choose $\psi$ which is similar to a stream function and $\varphi$ as an independent function which is similar to the potential. It is assumed that there is a unique, single-valued relationship between the generalized coordinates and the physical coordinates which can be written as

$$\varphi=\varphi(x, y), \psi=\psi(x, y) \quad (15)$$

and by implication

$$x=x(\varphi,\psi), y=y(\varphi,\psi).$$

The specific relationship is given by the equations for total differentials of $\varphi$ and $\psi$, respectively

$$d\varphi=\varphi_x dx+\varphi_y dy=\frac{\cos q}{\phi}dx+\frac{\sin q}{\phi}dy, \quad (16)$$

$$d\psi=\psi_x dx+\psi_y dy=-cw\sin q dx+cw\cos q dy. \quad (17)$$

In (16) and (17), $c$ is a constant, and $\phi(x, y)$ is a new unknown function. These values are chosen such that the new variables $(\varphi,\psi)$ are functionally independent, i.e. the Jacobian is not equal to zero

$$J(x, y)=\frac{\partial(\varphi,\psi)}{\partial(x, y)}=\frac{cw}{\phi}\neq 0.$$

Equation (16) has to deter mine unique function $\varphi(x, y)$. It means that the mixed derivatives are equal, i.e.

$$\frac{\partial^2\varphi(x, y)}{\partial x\partial y}=\frac{\partial^2\varphi(x, y)}{\partial y\partial x}. \quad (18)$$

Substitution of $\frac{\partial\varphi}{\partial y}$ and $\frac{\partial\varphi}{\partial x}$ from (16) into (18) gives the equation

$$\frac{\partial}{\partial x}\left(\frac{\sin q(x,y)}{\phi(x,y)}\right)=\frac{\partial}{\partial y}\left(\frac{\cos q(x,y)}{\phi(x,y)}\right). \qquad (19)$$

Equation (19) may be used as an additional equation for the new unknown function $\phi(x,y)$. From (16) and (17), we have the value of partial derivatives

$$\frac{\partial\varphi}{\partial y}=\frac{\sin q}{\phi(x,y)};\frac{\partial\varphi}{\partial x}=\frac{\cos q}{\phi(x,y)};$$
$$\frac{\partial\psi}{\partial x}=-cw\sin q;\frac{\partial\psi}{\partial y}=cw\cos q. \qquad (20)$$

To transform the system of (11), (14) and (19) to new independent variables, we need to know the values of $\frac{\partial x}{\partial\varphi},\frac{\partial x}{\partial\psi},\frac{\partial y}{\partial\varphi}$ and $\frac{\partial y}{\partial\psi}$. It is easy to show that

$$\begin{bmatrix}\frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y}\\ \frac{\partial\psi}{\partial x} & \frac{\partial\psi}{\partial y}\end{bmatrix}\begin{bmatrix}\frac{\partial x}{\partial\varphi} & \frac{\partial x}{\partial\psi}\\ \frac{\partial y}{\partial\varphi} & \frac{\partial y}{\partial\psi}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

Really, we have

$$dx=\frac{\partial x}{\partial\varphi}d\varphi+\frac{\partial x}{\partial\psi}d\psi,$$
$$dy=\frac{\partial y}{\partial\varphi}d\varphi+\frac{\partial y}{\partial\psi}d\psi$$

or in a matrix from

$$\begin{bmatrix}dx\\ dy\end{bmatrix}=\begin{bmatrix}\frac{\partial x}{\partial\varphi} & \frac{\partial x}{\partial\psi}\\ \frac{\partial y}{\partial\varphi} & \frac{\partial y}{\partial\psi}\end{bmatrix}\begin{bmatrix}d\varphi\\ d\psi\end{bmatrix}.$$

Solving this matrix equation, for the right-hand column matrix, we have

$$\begin{bmatrix}d\varphi\\ d\psi\end{bmatrix}=\begin{bmatrix}\frac{\partial x}{\partial\varphi} & \frac{\partial x}{\partial\psi}\\ \frac{\partial y}{\partial\varphi} & \frac{\partial y}{\partial\psi}\end{bmatrix}^{-1}\begin{bmatrix}dx\\ dy\end{bmatrix}.$$

This matrix from can be compared with the matrix form

$$\begin{bmatrix}d\varphi\\ d\psi\end{bmatrix}=\begin{bmatrix}\frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y}\\ \frac{\partial\psi}{\partial x} & \frac{\partial\psi}{\partial y}\end{bmatrix}\begin{bmatrix}dx\\ dy\end{bmatrix}.$$

Therefore

$$\begin{bmatrix}\frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y}\\ \frac{\partial\psi}{\partial x} & \frac{\partial\psi}{\partial y}\end{bmatrix}=\begin{bmatrix}\frac{\partial x}{\partial\varphi} & \frac{\partial x}{\partial\psi}\\ \frac{\partial y}{\partial\varphi} & \frac{\partial y}{\partial\psi}\end{bmatrix}^{-1}$$

Following the standard rules for finding the inverse matrix, this equation is written as follows

$$\begin{bmatrix}\frac{\partial x}{\partial\varphi} & \frac{\partial x}{\partial\psi}\\ \frac{\partial y}{\partial\varphi} & \frac{\partial y}{\partial\psi}\end{bmatrix}=\frac{\begin{bmatrix}\frac{\partial\psi}{\partial y} & -\frac{\partial\varphi}{\partial y}\\ -\frac{\partial\psi}{\partial x} & \frac{\partial\varphi}{\partial x}\end{bmatrix}}{\begin{vmatrix}\frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y}\\ \frac{\partial\psi}{\partial x} & \frac{\partial\psi}{\partial y}\end{vmatrix}}$$

or

$$\begin{bmatrix}\frac{\partial x}{\partial\varphi} & \frac{\partial x}{\partial\psi}\\ \frac{\partial y}{\partial\varphi} & \frac{\partial y}{\partial\psi}\end{bmatrix}=\frac{1}{J}\begin{bmatrix}\frac{\partial\psi}{\partial y} & -\frac{\partial\varphi}{\partial y}\\ -\frac{\partial\psi}{\partial x} & \frac{\partial\varphi}{\partial x}\end{bmatrix} \qquad (21)$$

where the Jacobian $J$ is defined as

$$J=\frac{\partial(\varphi,\psi)}{\partial(x,y)}=\begin{vmatrix}\frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y}\\ \frac{\partial\psi}{\partial x} & \frac{\partial\psi}{\partial y}\end{vmatrix}=\frac{cw\cos^2 q}{\phi}+\frac{cw\sin^2 q}{\phi}=\frac{cw}{\phi}\neq 0.$$

Since the Jacobian $J\neq 0,$ then $\phi$ and $w$ are not equal to zero. Finally, we can rewrite (21) in the form

$$\frac{\partial x}{\partial\varphi}=\frac{1}{J}\psi_y=\phi\cos q,\frac{\partial x}{\partial\psi}=-\frac{1}{J}\varphi_y=-\frac{\sin q}{cw},$$
$$\frac{\partial y}{\partial\varphi}=-\frac{1}{J}\psi_x=\phi\sin q,\frac{\partial y}{\partial\psi}=\frac{1}{J}\varphi_x=\frac{\cos q}{cw}. \qquad (22)$$

The first step: We transform the continuity (3). Substitution of $u=w\cos q$ and $v=w\sin q$ into this equation yields

$$\frac{\partial w\cos q}{\partial x}+\frac{\partial w\sin q}{\partial y}=0$$

or

$$\cos q\frac{\partial w}{\partial x}-w\sin q\frac{\partial q}{\partial x}+\sin q\frac{\partial w}{\partial y}+w\cos q\frac{\partial q}{\partial y}=0. \qquad (23)$$

## V. RESULTS

By using the chain rule, we have the formulas to change partial derivatives

$$\begin{aligned}\frac{\partial(\cdot)}{\partial x}&=\frac{\partial(\cdot)}{\partial\varphi}\frac{\partial\varphi}{\partial x}+\frac{\partial(\cdot)}{\partial\psi}\frac{\partial\psi}{\partial x}=\frac{\cos q}{\phi}\frac{\partial(\cdot)}{\partial\varphi}-cw\sin q\frac{\partial(\cdot)}{\partial\psi},\\ \frac{\partial(\cdot)}{\partial y}&=\frac{\partial(\cdot)}{\partial\varphi}\frac{\partial\varphi}{\partial y}+\frac{\partial(\cdot)}{\partial\psi}\frac{\partial\psi}{\partial y}=\frac{\sin q}{\phi}\frac{\partial(\cdot)}{\partial\varphi}+cw\cos q\frac{\partial(\cdot)}{\partial\psi}.\end{aligned}\qquad(24)$$

$$\frac{1}{\phi}\frac{\partial w}{\partial\varphi}+cw^2\frac{\partial q}{\partial\psi}=0,$$

$$\frac{\partial}{\partial\varphi}\left(\frac{1}{w}\right)=c\phi\frac{\partial q}{\partial\psi}.\qquad(25)$$

The second step: We have to use the condition

$$\frac{\partial^2\varphi}{\partial x\partial y}=\frac{\partial^2\varphi}{\partial y\partial x}.\qquad(26)$$

Substitution of and from equation yields

$$\frac{\partial}{\partial y}\left(\frac{\cos q}{\phi}\right)=\frac{\partial}{\partial x}\left(\frac{\sin q}{\phi}\right)\qquad(27)$$

or

$$-\phi\sin q\frac{\partial q}{\partial y}-\cos q\frac{\partial\phi}{\partial y}=\phi\cos q\frac{\partial q}{\partial x}-\sin q\frac{\partial\phi}{\partial x}.$$

Substituting (24) into (27) and making simplifications, we get the following equation in the term of new variables

$$\frac{\partial\phi}{\partial\psi}=-\frac{1}{cw}\frac{\partial q}{\partial\varphi}.\qquad(28)$$

The third step: Substituting partial derivatives from (24) into equation

$$\frac{\partial}{\partial x}\left(w^2\frac{\partial q}{\partial x}\right)+\frac{\partial}{\partial y}\left(w^2\frac{\partial q}{\partial y}\right)=0,$$

We obtain

$$\frac{\partial}{\partial x}\left(w^2\frac{\cos q}{\phi}\frac{\partial q}{\partial\varphi}-cw^3\sin q\frac{\partial q}{\partial\psi}\right)+\left(w^2\frac{\sin q}{\phi}\frac{\partial q}{\partial\varphi}+cw^3\cos q\frac{\partial q}{\partial\psi}\right)=0.$$

By simplifying, we get then the equation in the terms of new variables as follows

$$\frac{\partial}{\partial\varphi}\left(\frac{w^2}{\phi}\frac{\partial q}{\partial\varphi}\right)+c^2w\phi\frac{\partial}{\partial\psi}\left(w^3\frac{\partial q}{\partial\psi}\right)=0.\qquad(29)$$

In order to transform (1),(3) and (26) to (25), (28) and (29), a program by MAPLE.

## VI. CONCLUSION

The Euler equations are expressed in terms of new unknown functions which are the flow angle (angle between the direction of the velocity vector and direction of the $Ox$ axis) and the modulus of the velocity vector. The new independent variables are used to transform the physical domain to the canonical computational domain. Then, the governing equations into two subsystems, one is hyperbolic

$$\frac{\partial}{\partial\varphi}\left(\frac{1}{w}\right)=c\phi\frac{\partial q}{\partial\psi},$$

$$\frac{\partial\phi}{\partial\psi}=-\frac{1}{cw}\frac{\partial q}{\partial\varphi}$$

and another is elliptic

$$\frac{\partial}{\partial\varphi}\left(\frac{w^2}{\phi}\frac{\partial q}{\partial\varphi}\right)+c^2w\phi\frac{\partial}{\partial\psi}\left(w^3\frac{\partial q}{\partial\psi}\right)=0.$$

## ACKNOWLEDGMENT

I wish to record our thanks to Prof. Dr. Nikolay Moshkin for giving the idea , Phranakhon Rajabhat University and Thailand Research Fund for financial supporting this work.

## REFERENCES

[1] P.J. Roache, *Computational Fluid Dynamics*. Hermosa Publishers, Albuquerque, 1976.
[2] L. Quartapelle, . *Numerical Solution of the Incompressible Navier Stokes Equations*. Berlin. Germany,1993.
[3] I.L Osipov, V.P. Pashenko, and A.V. Shippillin, "Calculation of inviiscous gas flow within channel with essential change of geometry,"*J.Comm. Math. Phy*, vol 18,1978, pp. 964-973.
[4] P.Monnuanprang, "Numerical Method For The Incompressible Euler Equations," *IAENG International Journal of .Applied Mathematics*, vol.38, Sep. 2008, pp. 125-128.
[5] C.K. Batchelor, *An Introduction to Fluid Dynamics*. Cambridge: Cambridge University , 1970.