International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:6, No:1, 2012

# Design of an Augmented Automatic Choosing Control by Lyapunov Functions Using Gradient Optimization Automatic Choosing Functions

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*Abstract*—In this paper we consider a nonlinear feedback control called augmented automatic choosing control (AACC) using the gradient optimization automatic choosing functions for nonlinear systems. Constant terms which arise from sectionwise linearization of a given nonlinear system are treated as coefficients of a stable zero dynamics. Parameters included in the control are suboptimally selected by expanding a stable region in the sense of Lyapunov with the aid of the genetic algorithm. This approach is applied to a field excitation control problem of power system to demonstrate the splendidness of the AACC. Simulation results show that the new controller can improve performance remarkably well.

*Keywords*—augmented automatic choosing control, nonlinear control, genetic algorithm, zero dynamics.

## I. Introduction

GENERALLY, it is easy to design the optimal control laws for linear systems, but it is not so for nonlinear systems, though they have been studied for many years[1]∼[8]. One of most popular and practical nonlinear control laws is synthesized by applying a linearization method by Taylor expansion truncated at the first order and the linear optimal control method. This is only effective in a small region around steady state points or in almost linear systems[1]∼[3].

Another nonlinear control called an automatic choosing control (ACC) has been studied [6]. This controller is effective in nonlinear systems with high nonlinearity and wider regions. But constant terms, which generally appear in equations when linearized by Taylor expansion, lead the controller to have bias at the origin, so the resulting ACC must be modified by bothersome unbiased nonlinear functions in view of stability.

To overcome these weakness, in this paper we consider an augmented automatic choosing control (AACC) for nonlinear systems[7][8] and its design procedure is as follows.

Assume that a system is given by a nonlinear differential equation. Choose a separative variable, which makes up nonlinearity of the given system. The domain of the variable is divided into some subdomains. On each subdomain, the system equation is linearized by Taylor expansion around a suitable point so that a constant term is included in it. This constant term is treated as a coefficient of a stable zero dynamics. The given nonlinear system approximately makes up a set of augmented linear systems, to which the optimal linear control theory is applied to get the linear quadratic (LQ) controls[2]. These LQ controls are smoothly united by gradient optimization automatic choosing functions to synthesize a single nonlinear feedback controller.

This controller is of a structure-specified type which has some parameters, such as the number of division of the domain, regions of the subdomains, points of Taylor expansion, and gradients of the automatic choosing function. These parameters must be selected optimally so as to be just the controller's fit. Since they lead to a nonlinear optimization problem, we are able to solve it by using the genetic algorithm (GA)[9] suboptimally. In this paper the suboptimal values of these parameters are selected by maximizing a stable region in the sense of Lyapunov.

This approach is applied to a field excitation control problem of power system, which is Ozeki-Power-Plant of Kyushu Electric Power Company in Japan, to demonstrate the splendidness of the AACC. Simulation results show that the new controller using the GA is able to improve performance remarkably well.

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## II. Augmented Automatic Choosing Control Using Zero Dynamics

Assume that a nonlinear system is given by

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbf{D} \tag{1}$$

where $\cdot = d/dt, \quad x = [x[1], \cdots, x[n]]^T$ is an $n$-dimensional state vector, $u = [u[1], \cdots, u[r]]^T$ is an $r$-dimensional control vector, $f : \mathbf{D} \to R^n$ is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, $g(x)$ is an $n \times r$ driving matrix with $g(0) \neq 0$, $\mathbf{D} \subset R^n$ is a domain , and $T$ denotes transpose.

Considering the nonlinearity of $f$, introduce a vector-valued function $C : \mathbf{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let $D$ be a domain of $C^{-1}$. For example, if $x[2]$ is the element which has the highest nonlinearity in $f$, then

$$C(x) = x[2] \in D \subset R \quad (L = 1)$$

(see Section IV). The domain $D$ is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \leq i \leq M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L} [a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.

Introduce a stable zero dynamics :

$$\dot{x}[n+1] = -\sigma_i x[n+1] \quad (2)$$

$$(x[n+1](0) \simeq 1, \quad 0 < \sigma_i < 1).$$

Eq.(1) combines with (2) to form an augmented system

$$\dot{\mathbf{X}} = \bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u \quad (3)$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \quad \in \mathbf{D} \times R$$

$$\bar{f}(\mathbf{X}) = \begin{bmatrix} f(x) \\ -\sigma_i x[n+1] \end{bmatrix}, \bar{g}(\mathbf{X}) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}.$$

We assume a cost function being

$$J = \frac{1}{2}\int_0^{\infty} \left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T R u\right) dt \quad (4)$$

where $\mathbf{Q} = \mathbf{Q}^T > 0,\ R = R^T > 0$, and the values of these matrices are properly determined based on engineering experience.

On each $D_i$, the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point $\hat{X}_i \in C^{-1}(D_i)$ and $\hat{X}_0 = 0$ (see Fig. 1):

$$f(x) + g(x)u \simeq A_i x + w_i + B_i u \qquad \text{on} \quad C^{-1}(D_i) \quad (5)$$

where

$$A_i = \partial f(x)/\partial x^T|_{x=\hat{X}_i}\ ,\ w_i = f(\hat{X}_i) - A_i\hat{X}_i\ ,$$
$$B_i = g(\hat{X}_i).$$

Make an approximation of (3) by

$$\dot{\mathbf{X}} = \bar{A}_i\mathbf{X} + \bar{B}_i u \qquad \text{on} \quad C^{-1}(D_i) \times R \quad (6)$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$

An application of the linear optimal control theory[2] to (4) and (6) yields

$$u_i(\mathbf{X}) = -R^{-1}\bar{B}_i^T\mathbf{P}_i\mathbf{X} \quad (7)$$

where the $(n+1)\times(n+1)$ matrix $\mathbf{P}_i$ satisfies the Riccati equation :

$$\mathbf{P}_i\bar{A}_i + \bar{A}_i^T\mathbf{P}_i + \mathbf{Q} - \mathbf{P}_i\bar{B}_i R^{-1}\bar{B}_i^T\mathbf{P}_i = 0. \quad (8)$$

Introduce a gradient optimization automatic choosing function :

$$I_i(x) = \prod_{j=1}^{L}\Big\{1 - \frac{1}{1+\exp\left(2N_{1i}\left(C_j(x) - a_{ij}\right)\right)} - \frac{1}{1+\exp\left(-2N_{1i}\left(C_j(x) - b_{ij}\right)\right)}\Big\} \quad (9)$$

where $N_{1i}$:positive real value, $-\infty \le a_{ij}\ ,\ b_{ij} \le \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero(see Fig. 2).

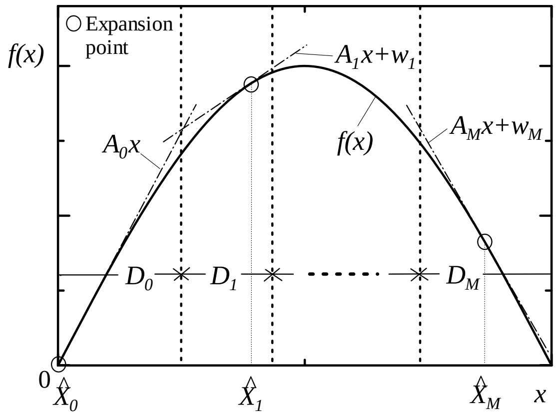


Fig. 1 Sectionwize linearization

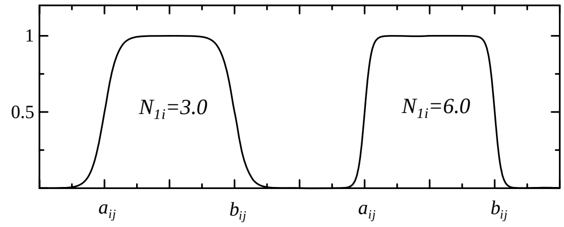


Fig. 2 Automatic Choosing Function($N_{1i}$=3.0, 6.0)

Uniting $\{u_i(\mathbf{X})\}$ of (7) with $\{I_i(x)\}$ of (9), we have an augmented automatic choosing control

$$u(\mathbf{X}) = \sum_{i=0}^{M} u_i(\mathbf{X})I_i(x). \quad (10)$$

## III. Parameter Selection by GA

Introduce a Lyapunov function candidate:

$$V(\mathbf{X}) = \mathbf{X}^T\Pi(\mathbf{X})\mathbf{X} \quad (11)$$

where

$$\Pi(\mathbf{X}) = \sum_{i=0}^{M}\mathbf{P}_i\Pi_i(x)\ ,$$

$$\Pi_i(x) = \eta_i\prod_{j=1}^{L}\Big\{1 - \frac{1}{1+\exp\left(2N_2\left(C_j(x) - a_{ij}\right)\right)} - \frac{1}{1+\exp\left(-2N_2\left(C_j(x) - b_{ij}\right)\right)}\Big\} \quad (12)$$

where $\mathbf{P}_i$ satisfies the Riccati equation (8). $N_2$ and $\eta_i$ are positive real values. By the Lyapunov's direct method, the equilibrium point 0 is uniformly stable on a connected set:

$$\mathbf{D}_V = \left\{x \in \mathbf{D} : V(\mathbf{X}) < \gamma,\ \dot{V}(\mathbf{X}) < 0\right\}$$

where

$$\gamma = \inf\left\{V(\mathbf{X}) : \mathbf{X} \neq 0,\ \dot{V}(\mathbf{X}) = 0\right\}.$$

In order to make a stable region in the sense of Lyapunov as wide as possible, we define a performance

$$PI = -\gamma. \quad (13)$$

A set of parameters included in the control of Eq.(10) is

$$\bar{\Omega} = \{M, N_{1i}, a_{ij}, b_{ij}, \hat{X}_i, \cdots\} \tag{14}$$

which is suboptimally selected by minimizing $PI$ with the aid of GA[9] as follows.

<**ALGORITHM**>

**step1:Apriori:** Set values $\bar{\Omega}_{apriori}$ appropriately.

**step2:Parameter:** Choose $\Omega \subset \bar{\Omega}$ to be improved and rewrite

$$\Omega = \{N_{1i}, a_i, b_i, \cdots\} = \{\alpha_k : k = 1, \cdot\cdot, K\}.$$

**step3:Coding:**Represent each $\alpha_k$ with a binary bit string of $\widetilde{L}$ bits and then arrange them into one string of $\widetilde{L}K$ bits.

**step4:Initialization:**Randomly generate an initial population of $\widetilde{q}$ strings

$$\{\Omega_p : p = 1, \cdot\cdot, \widetilde{q}\}.$$

**step5:Decoding:**Decode each element $\alpha_k$ of $\Omega_p$ by

$$\alpha_k = (\alpha_{k,max} - \alpha_{k,min})A_k/(2^{\widetilde{L}} - 1) + \alpha_{k,min}$$

where $\alpha_{k,max}$:maximum, $\alpha_{k,min}$:minimum, and $A_k$:decimal values of $\alpha_k$.

**step6:Lyapunov function:** Make $\gamma = \gamma_p$ $(p = 1, \cdot\cdot, \widetilde{q})$ for $\Omega_p$ by using Eq.(11).

**step7:Fitness value calculation:**Calculate

$$PI_p = -\gamma_p$$

by Eq.(13), or fitness $F_p = -PI_p$.

**step8:Reproduction:** Reproduce each of individual strings with the probability of

$$F_p / \sum_{j=1}^{\widetilde{q}} F_j.$$

**step9:Crossover:**Pick up two strings and exchange them at a crossing position by a crossover probability $P_c$.

**step10:Mutation:**Alter a bit of string (0 or1) by a mutation probability $P_m$.

**step11:Repetition:**Repeat step5$\sim$step10 until prespecified $G$-th generation. If unsatisfied, go to step2.

As a result, we have a suboptimal control $u(\mathbf{X})$ for the string with the best performance over all the past generations.

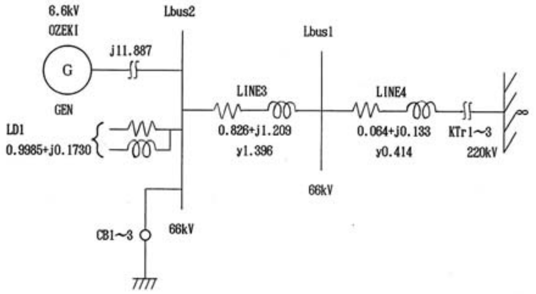


Fig. 3 Diagram of Ozeki-Power-Plant

## IV. Numerical Example

Consider a field excitation control problem of power system. Fig. 3 is a diagram of Ozeki-Power-Plant of Kyushu Electric Power Company in Japan. This system is assumed to be described[7] by

$$\begin{aligned}
&\widetilde{M}\frac{d^2\delta}{dt^2} + \widetilde{D}\frac{d\delta}{dt} + P_e = P_{in} \\
&P_e = E_I^2 Y_{11}\cos\theta_{11} + E_I\widetilde{V}Y_{12}\cos(\theta_{12} - \delta) \\
&E_I + T'_{d0}\frac{dE'_q}{dt} = E_{fd} \\
&E_I = E'_q + (X_d - X'_d)I_d \\
&I_d = -E_I Y_{11}\sin\theta_{11} - \widetilde{V}Y_{12}\sin(\theta_{12} - \delta) \\
&\widetilde{D} = \widetilde{V}^2\Big\{\frac{T''_{d0}(X'_d - X''_d)}{(X'_d + X_e)^2}\sin^2\delta \\
&\qquad\qquad + \frac{T''_{q0}(X_q - X''_q)}{(X_q + X_e)^2}\cos^2\delta\Big\},
\end{aligned}$$

where $\delta$: phase angle, $\dot{\delta}$: rotor speed, $\widetilde{M}$: inertia coefficient, $\widetilde{D}(\delta)$: damping coefficient, $P_{in}$: mechanical input power, $P_e(\delta)$: generator output power, $\widetilde{V}$: reference bus voltage, $E_I$: open circuit voltage, $E_{fd}$: field excitation voltage, $X_d$: direct axis synchronous reactance, $X'_d$: direct axis transient reactance, $X_e$: external impedance, $Y_{11}\angle\theta_{11}$: self-admittance of the network, $Y_{12}\angle\theta_{12}$: mutual admittance of the network, and $I_d(\delta)$: direct axis current of the machine. Put $x$=$[x[1], x[2], x[3]]^T$=$[E_I - \hat{E}_I, \delta - \hat{\delta}_0, \dot{\delta}]^T$ and $u = E_{fd} - \hat{E}_{fd}$, so that

$$\begin{bmatrix} \dot{x}[1] \\ \dot{x}[2] \\ \dot{x}[3] \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} + \begin{bmatrix} g_1(x) \\ 0 \\ 0 \end{bmatrix} u \tag{15}$$

where

$$\begin{aligned}
f_1(x) &= -\frac{1}{kT_{d0}}(x[1] + \hat{E}_I - \hat{E}_{fd}) \\
&\quad + \frac{(X_d - X'_d)\widetilde{V}Y_{12}}{k}X_3\cos(\theta_{12} - x[2] - \hat{\delta}_0) \\
f_2(x) &= x[3] \\
f_3(x) &= -\frac{\widetilde{V}Y_{12}}{\widetilde{M}}(x[1] + \hat{E}_I)\cos(\theta_{12} - x[2] - \hat{\delta}_0) \\
&\quad - \frac{Y_{11}\cos\theta_{11}}{\widetilde{M}}(x[1] + \hat{E}_I)^2 - \frac{\widetilde{D}}{\widetilde{M}}x[3] + \frac{P_0}{\widetilde{M}} \\
g_1(x) &= \frac{1}{kT_{d0}}, \quad k = 1 + (X_d - X'_d)Y_{11}\sin\theta_{11}.
\end{aligned}$$

Parameters are

| | | | |
|---|---|---|---|
| $\widetilde{M} =$ | $0.016095[pu]$ | $T_{d0} =$ | $5.09907[sec]$ |
| $\widetilde{V} =$ | $1.0[pu]$ | $P_0 =$ | $1.2[pu]$ |
| $X_d =$ | $0.875[pu]$ | $X'_d =$ | $0.422[pu]$ |
| $Y_{11} =$ | $1.04276[pu]$ | $Y_{12} =$ | $1.03084[pu]$ |
| $\theta_{11} =$ | $-1.56495[pu]$ | $\theta_{12} =$ | $1.56189[pu]$ |
| $X_e =$ | $1.15[pu]$ | $X''_d =$ | $0.238[pu]$ |
| $X_q =$ | $0.6[pu]$ | $X''_q =$ | $0.3[pu]$ |
| $T''_{d0} =$ | $0.0299[pu]$ | $T''_{q0} =$ | $0.02616[pu]$ |
| $\hat{E}_I =$ | $1.52243[pu]$ | $\hat{\delta}_0 =$ | $48.57°$ |
| $\hat{\dot{\delta}}_0 =$ | $0.0[deg/sec]$ | $\hat{E}_{fd} =$ | $1.52243[pu]$. |

TABLE I
PERFORMANCES

| Method | $x^T(0)$ : initial point | | | | |
|---|---|---|---|---|---|
| | $[0, 0.4, 0]$ | $[0, 0.5, 0]$ | $[0, 1.0, -5]$ | $[0, 1.2, 0]$ | $[0, 1.3, 0]$ |
| LOC | 0.95375 | × | × | × | × |
| AACC(Hamil) | 0.94224 | 1.23581 | 7.19167 | 1.90626 | × |
| AACC(Lyap) | 0.93033 | 1.23630 | 7.39742 | 1.85533 | 2.87179 |

× : very large value

Set $\mathbf{X} = [x^T, x[4]]^T = [x[1], x[2], x[3], x[4]]^T$, $n = 3$, $\hat{X}_0 = \hat{\delta}_0 = 48.57°$, $C(x)$=$x[2]$, $L = 1$, $\mathbf{Q}$=diag$(1, 1, 1, 1)$, $R$=1, $\sigma_i = 0.33294(0 \le i \le M)$ and $x[4](0) = 1$. Experiments are carried out for the new control(AACC), and the ordinary linear optimal control(LOC)[2].

1) AACC(Lyap):
$M$=1, $\hat{X}_1 = 80°$, $D_0 = (-\infty, a - \hat{\delta}_0]$, $D_1$=$[a - \hat{\delta}_0, \infty)$. The parameters are suboptimally selected along the algorithm of section III. $\Omega$=$\{\eta_i, N_{1i}, N_2, a\}$,$G$=100, $\widetilde{q}$=100, $\widetilde{L}$=8, $P_c$=0.8, $P_m$=0.03.
It results that $\eta_i$=1.840000, $N_{10}$=3.864706, $N_{11}$=0.896884, $N_2$=9.752941 and $a$=$58.431373°$.

2) AACC(Hamil):
The parameters are suboptimally selected by minimizing the Hamiltonian[8]. $\Omega$=$\{N_{1i}, a\}$. It results that $N_{10}$=2.517647, $N_{11}$=1.035294 and $a$=$74.215686°$.

Table1 shows performances by the AACC(Lyap), the AACC(Hamil) and the LOC. The cost function of Table1 is

$$\widetilde{J} = \frac{1}{2} \int_0^{25} \left( \mathbf{X}^T \mathbf{Q} \mathbf{X} + u^T \mathbf{R} u \right) dt.$$

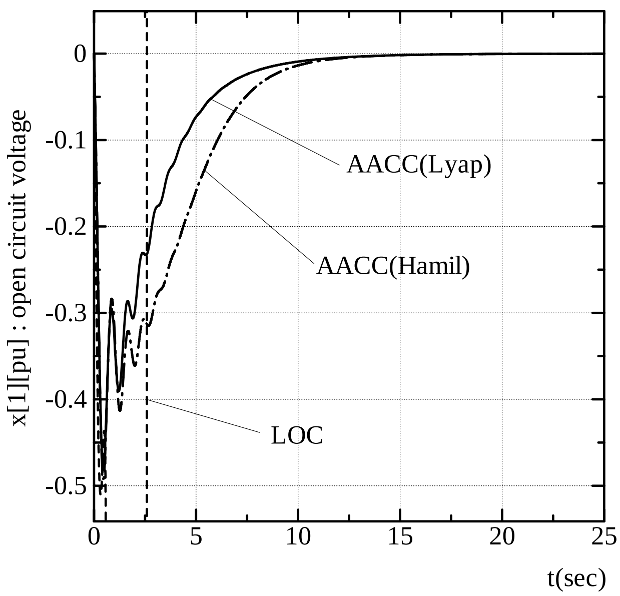


Fig. 4 Responses of LOC, AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.2, 0])$

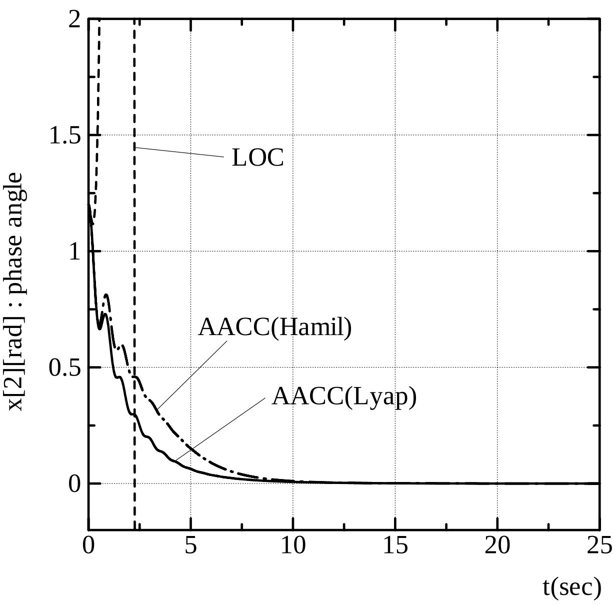


Fig. 5 Responses of LOC, AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.2, 0])$

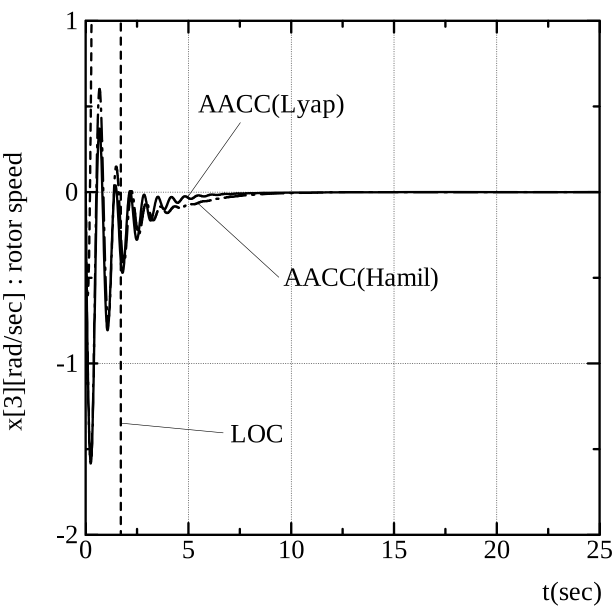


Fig. 6 Responses of LOC, AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.2, 0])$

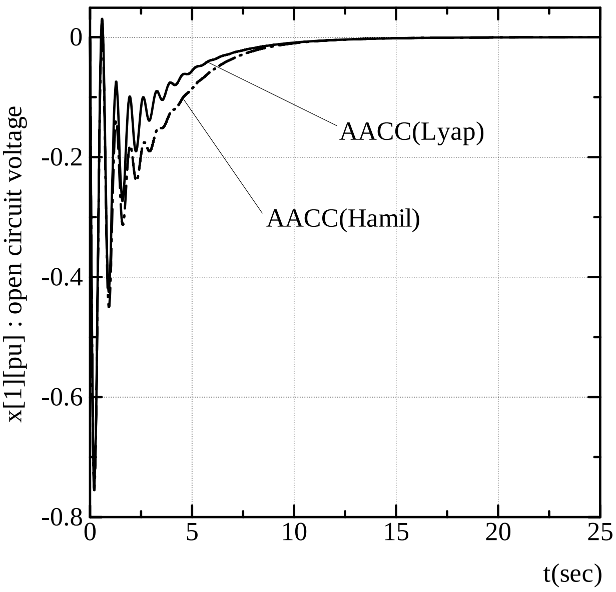


Fig. 7 Responses of AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.0, -5])$

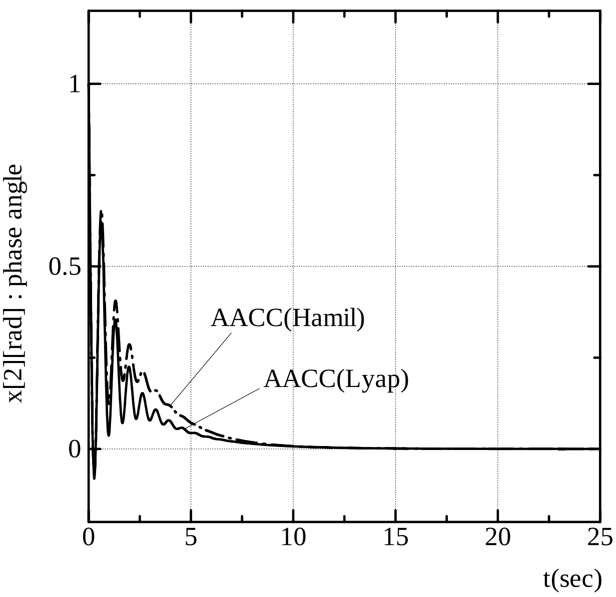


Fig. 8 Responses of AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.0, -5])$

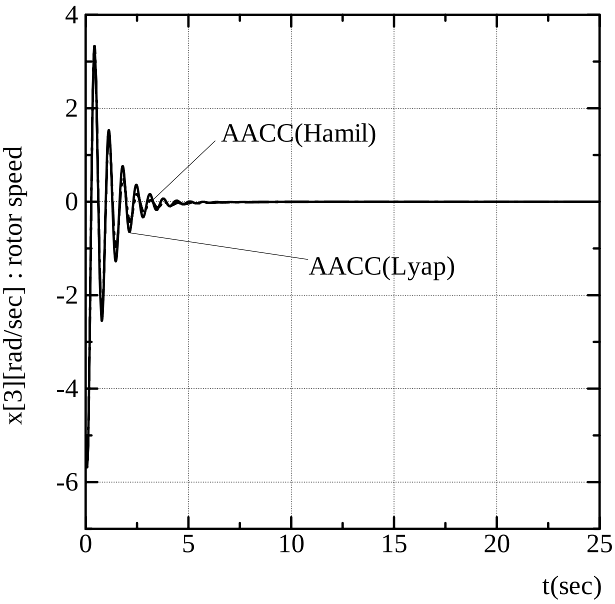


Fig. 9 Responses of AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.0, -5])$

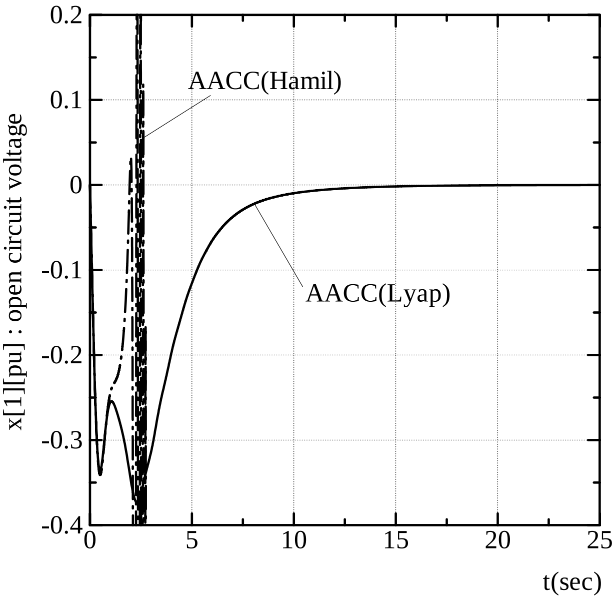


Fig.10 Responses of AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.3, 0])$

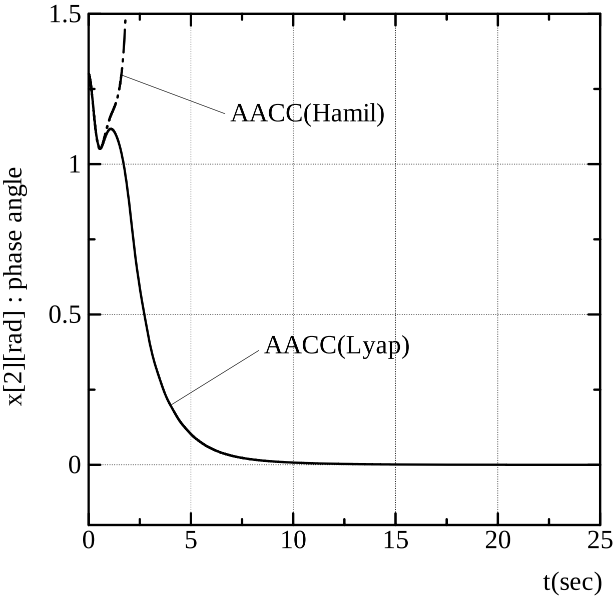


Fig.11 Responses of AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.3, 0])$

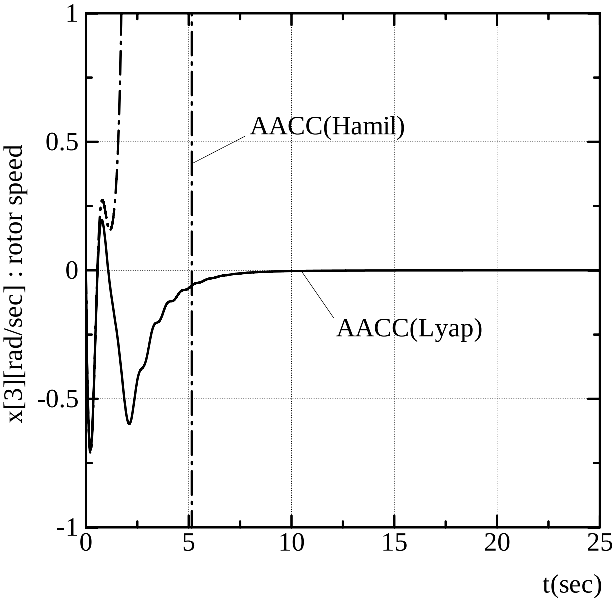


Fig.12 Responses of AACC(Hamil), AACC(Lyap) $(x^T(0) = [0, 1.3, 0])$

Figs. 4, 5 and 6 show the responses in case of $x^T(0) = [0, 1.2, 0]$. Figs. 7, 8 and 9 show the responses in case of $x^T(0) = [0, 1.0, -5]$. Figs. 10, 11 and 12 show the responses in case of $x^T(0) = [0, 1.3, 0]$. These results indicate that the stable region of AACC(Lyap) is better than the AACC(Hamil) and LOC.

## V. Conclusions

We have studied an augmented automatic choosing control designed by Lyapunov functions using the gradient optimization automatic choosing functions for nonlinear systems. This approach was applied to a field excitation control problem of power system to demonstrate the splendidness of the AACC. Simulation results have shown that this controller could improve performance remarkably well.

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