

# Exploring the Ambiguity Resolution in Spacecraft Attitude Determination Using GNSS Phase Measurement

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**Abstract**—Attitude Determination (AD) of a spacecraft using the phase measurements of the Global Navigation Satellite System (GNSS) is an active area of research. Various attitude determination algorithms have been developed in yester years for spacecrafts using different sensors but the last two decades have witnessed a phenomenal increase in research related with GPS receivers as a stand-alone sensor for determining the attitude of satellite using the phase measurements of the signals from GNSS. The GNSS-based Attitude determination algorithms have been experimented in many real missions. The problem of AD algorithms using GNSS phase measurements has two important parts; the ambiguity resolution and the determining of attitude. Ambiguity resolution is the widely addressed topic in literature for implementing the AD algorithm using GNSS phase measurements for achieving the accuracy of millimeter level. This paper broadly overviews the different techniques for resolving the integer ambiguities encountered in AD using GNSS phase measurements.

**Keywords**—Attitude Determination, Ambiguity Resolution, GNSS, LAMBDA Method, Satellite.

## I. INTRODUCTION

THE Attitude of a spacecraft is its orientation in space. Attitude determination is the process of computing the orientation of the spacecraft relative to either an inertial reference or some other object of interest, such as the Earth. This typically involves several types of sensors on each spacecraft and sophisticated data processing procedures. The accuracy limit is usually determined by a combination of processing procedures and spacecraft hardware [1].

Attitude determination and control subsystem (ADCS) is one of the crucial subsystems of the satellite. It involves the representation of attitude using Euler angles, direction cosine matrix, Rodriguez parameters or Quaternions. Attitude representation is elaborated in detail along with the significance of each representation by Shuster [2]. Shrivastava and Modi [3] have explained the various environmental forces, torques and their effects on the satellite dynamics.

Determination of attitude includes attitude sensors like sun sensor, magnetometer, horizon sensor, Earth sensor, Star

Sensors, Inertial Measurement units (IMU), gyroscope, GPS receiver etc and then applying point to point or recursive attitude determination algorithms to predict the attitude using satellite kinematics and dynamics models along with the environmental models.

Attitude control and stabilization is obtained by applying active or passive techniques and using linear, non linear, adaptive or fuzzy control scheme and using the actuators like magnetic coils, momentum wheels, reaction wheels, permanent magnets, thrusters, jets or control moment gyros. A detailed description and comparison of sensors and actuators is given in [1].

In the past four decades, various attitude determination algorithms for non GNSS-Based AD have been developed and tested. The original concept based on least square estimation was given by Wahba [4]. The first category is point to point estimation algorithms like Triad [1], QUaternion ESTimator QUEST [5], Singular Value Decomposition (SVD) [6] and Fast Optimal Attitude Matrix (FOAM) [7]. The recursive attitude determination algorithms were reviewed by Crassidis et al. [8] in detail and these algorithms include REQUEST [9], OPTIMAL-REQUEST [10], Minimum Model Error (MME) [11], Euler-q [12], Optimal-REQUEST [13], Adaptive Optimal-REQUEST [14] and Kalman Filtering algorithm [15], [16].

The paradigm of Global Navigation Satellite System (GNSS) includes the constellation of navigational satellites in the Medium Earth Orbit (MEO). Currently, GPS is the only fully functional navigation system but the future has promises for the upgraded GLONASS by Russia, Galileo by European Union, Beidou by China and regional navigation systems like GAGAN by India and QZSS by Japan. The origin, evolution and future of GNS are described in detail by Parkinson [17].

The application areas of GNSS are diverse and imagination dependent. They encompass PVT information, aviation and aerospace industry, emergency and rescue, location based services, mining, agriculture and civil engineering, environmental monitoring and hazard management, transportation, e-banking, commerce, geodesy, and many more.

One of the application areas of GNSS is the attitude determination of satellites that is the focus of this contribution especially a problem of resolving the integer ambiguities while using the phase measurements of the GNSS signal for determining the attitude of the satellite and in particular,

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single-frequency, single-epoch GNSS-based attitude determination. Section II elaborates the method of attitude determination using GNSS phase measurements and identifies the ambiguity resolution problem while Section III describes GNSS-based AD algorithms and Section IV presents GNSS based attitude determination models. Section V overviews the methods of resolving the integer ambiguities and paper concludes with some recommendations and conclusions.

## II. ATTITUDE DETERMINATION USING GNSS PHASE MEASUREMENTS

Attitude determination (AD) of the satellite using GNSS measurements involve two types of measurements namely the code measurements and the phase measurements. The accuracy and precision achieved by the phase measurements is far better than the code or pseudo range based measurements. The basic principle underline the attitude determination using phase measurements is the Interferometric Principle as given by [18], that is, with a pair of GPS antenna, a user can determine phase difference between like signals of that antenna pair and this gives the attitude of the platform. With two antennae/single baseline one is able to estimate the pointing direction, namely the compass solution, while configuration of three or more non-collinear antennae allows the user to estimate the full attitude of a platform [19]. Although the accuracy of a stand-alone GNSS attitude system might not be comparable with the one obtainable with other modern attitude sensors, a GNSS-based system presents several advantages: it is inherently driftless, minor maintenance is required and it is not as expensive as other high-precision systems, such as INS and Star Trackers [20].

Attitude Determination of spacecraft can be done by using space borne GPS receivers or the commercial off-the-shelf (COTS) GPS receivers. GNSS-based AD is dedicated or non-dedicated. In dedicated AD system, a single exclusively GPS receiver is used while in non-dedicated AD system, a set of independent, general purpose independent receivers are used for the attitude determination of the satellite [21].

Bauer et al. [22] presented an overview of the Space borne GPS receivers and the GPS flight experiments. In literature, various GNSS Phase based AD missions have been discussed namely GADACS [23], SPARTAN [24], RADCAL [25], REX II [26], Gravity Probe B [27], UNISAT [28], Gyrostat [29], UOSAT-12 [30] and ALSAT-1 [31]. Bauer et al. [22] and Chu et al. [32] have reviewed the Technology experiments of Attitude Determination with GPS.

Many GPS receiver manufacturing companies like Trimble Navigation [33], [34], Texas Instruments [35], [36], Ashtech [37]-[39], Adroit Systems [40], and others are developing GPS receivers with multiple antennas for attitude determination.

## III. GNSS-BASED ATTITUDE DETERMINATION ALGORITHMS

AD algorithms using GNSS Phase measurements are expressed by Bar-Itzhack [41]. Three-axis AD by Kalman

filtering of GPS signals are presented in [42]. Crassidis et al. [43] presented another Algorithm named as attitude-lean-losing-estimator (ALLEGRO) using GPS recursive operations. Park et al. [44] describes a set of numerical gradient-based optimization algorithms for solving the GPS-based AD problem. Madsen et al. [45] presented a new algorithm that utilizes signal to noise ratio measurements from canted antennas to produce three-axis attitude solutions. Three new algorithms were developed by Nadler et al. [46] for solving the attitude estimation problem, a discrete Newton-Raphson-based algorithm, a continuous Newton-Raphson algorithm, and an algorithm that is based on the Eigen problem structure of the nonlinear equations, which are related to the minimization of the quartic cost function. Chun et al. [47] formulated the Attitude determination problem as an optimization problem. Li et al. [48] present an algorithm based on iteration method to resolve the Wahba's problem through the small-angle approach. Kuang et al. [49] proved that QUEST provides the good solutions of attitude for the gyrostat satellite by using the GPS signals. Li et al. [50] cited the AMES algorithm for attitude determination using single and double difference phase measurements. Tsai [51] formulated the problem of AD as a constrained total least-square (CTLs) problem.

## IV. GNSS-BASED ATTITUDE DETERMINATION MODELS

An overview of GNSS models and their applications in various fields are given in textbooks such as [52]–[57]. GNSS models have two main categories; non-positioning or geometry-free models and the positioning or geometry-based models. Different GNSS models can also be distinguished based on the differencing that is applied. By differencing, we mean to take the differences between observations from different receivers and/or different satellites. It is often applied in order to eliminate some of the parameters from the observation equations [58]. The Single Difference and double difference Methods are explained in [59]. Unconstrained baselines are baselines for which a-priori information about the length is not available and constrained baselines are baselines for which the length is known and constant [60].

In principle all the GNSS baseline models can be cast in the following frame of linearized observation equations, by a Gauss-Markov model [61]

$$E(y) = Aa + Bb; \quad D(y) = Q_{yy} \quad (1)$$

where  $y$  is the given GNSS data vector of order  $m$ ,  $a$  and  $b$  are the unknown parameter vectors of order  $n$  and  $p$  respectively.  $E(\cdot)$  and  $D(\cdot)$  denote the expectation and dispersion operator, and  $A$  and  $B$  are the given design matrices that link the data vector to the unknown parameters. Matrix  $A$  contains the carrier wavelengths and the geometry matrix  $B$  contains the receiver-satellite unit line-of-sight vectors. The variance matrix of  $y$  is given by the positive definite matrix  $Q_{yy}$ .

The model given by (1) is termed as the unconstrained model and its Integer Least Squares (ILS) solution is found in

[61]. For attitude determination applications, usually one can benefit from the knowledge of the additional constraint on the baseline vector length and the Integer Least-Squares minimization problem can be reformulated as a Quadratically Constrained Integer Least-Squares (QC-ILS) problem [61].

## V. METHODS FOR AMBIGUITY RESOLUTION

The carrier phase measurements are inherently corrupted by unknown integer numbers and these numbers must be found to take full advantage of the carrier phase measurements. This problem is referred as the Ambiguity Resolution (AR).

Many algorithms have been proposed for ambiguity resolution. One category of these algorithms is referred to as a *SERACH* Method, independent of receiver –satellite geometry and the other one is referred as *MOTION-BASED* Method, which makes use of the information contained in the motion of the platform or the GPS satellites, dependent on receiver – satellite geometry.

In literature, different ambiguity resolution methods have been discussed [62] like Least-Squares Ambiguity Search Technique (LSAST) [63], Fast Ambiguity Resolution Approach (FARA) [64], Modified Cholesky decomposition [65], most widely used Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) [66], Null method [67], Fast Ambiguity Search Filter (FASF) [68], Three Carrier Ambiguity Resolution (TCAR) [69], Integrated TCAR [70], Optimal Method for Estimating GPS Ambiguities (OMEGA) [71], Cascade Integer Resolution (CIR) [71]. A comparison of LAMBDA with CIR, TCAR, ITCAR and the Null-method is presented in [72]-[74].

The estimation process consists of three steps. First a standard least-squares adjustment is applied in order to arrive at the so-called float solution. All unknown parameters are estimated as real-valued. In the second step, the integer constraint on the ambiguities is considered. This means that the float ambiguities are mapped to integer values. Different choices of the map are possible. The float ambiguities can simply be rounded to the nearest integer values or conditionally rounded so that the correlation between the ambiguities is taken into account. The optimal choice is to use the integer least-squares estimator, which maximizes the probability of correct integer estimation. Finally, after fixing the ambiguities to their integer values, the remaining unknown parameters are adjusted by virtue of their correlation with the ambiguities [62].

The integer estimators mostly used in practice [75] are Integer least squares (ILS) [76], Integer Bootstrapping (IB) [77], Integer Rounding (IR) [78] and Best Integer Equivariant estimator (BIE) [62].

The LAMBDA method is currently the standard method for solving unconstrained GNSS ambiguity resolution problems. For nonlinearly constrained ambiguity resolution problems, the single baseline constrained LAMBDA method [79] was presented and the newly proposed The Multivariate

Constrained LAMBDA method [21], [80], [81] algorithm solves for the integer ambiguities and body attitude in an integral manner.

## VI. CONCLUSION AND RECOMMENDATIONS

Attitude determination of a spacecraft using GNSS phase based measurement for the millimeter level accuracy has been overviewed while addressing the inherent problem of resolving the ambiguities due to the phase measurements. GNSS models for ambiguity resolution are presented and different existing and previous ambiguity resolution algorithms and methods are mentioned. Different GNSS-based attitude determination algorithms are also presented to describe the basic principle of GNSS based attitude determination. The latest Baseline Constrained LAMBDA version of the LAMBDA method for resolving the ambiguity in the GNSS-based AD of satellites is also presented. This broad overview can be helpful for the researchers in the field of attitude determination of satellites using GNSS phase measurements.

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