Image Enhancement using α-Trimmed Mean ε-Filters

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Abstract—Image enhancement is the most important challenging preprocessing for almost all applications of Image Processing. By now, various methods such as Median filter, α-trimmed mean filter, etc. have been suggested. It was proved that the α-trimmed mean filter is the modification of median and mean filters. On the other hand, ε-filters have shown excellent performance in suppressing noise. In spite of their simplicity, they achieve good results. However, conventional ε-filter is based on moving average. In this paper, we suggested a new ε-filter which utilizes α-trimmed mean. We argue that this new method gives better outcomes compared to previous ones and the experimental results confirmed this claim.

Keywords—Image Enhancement - Median Filter - ε-Filter – α-trimmed mean filter.

I. INTRODUCTION

IMULSE noise, which is often introduced into images during acquisition and transmission, has undesirable affects on different image processing purposes. There have been much more methods for removing impulse noise [1]. Generally, the noise removal methods can be categorized as linear and nonlinear methods. Moving Average filter is an example of linear methods and median, min and max filters are the examples of nonlinear methods. However, among all kinds of nonlinear methods for impulse noise, the median filter is used widely due to its effective noise suppression capability and high computational efficiency. Some improved median filters have been proposed to tackle median filter's problems [2,3,4,5,6,7,8].

On the other hand, ε-filter is a nonlinear filter introduced in speech processing for the first time. A conventional ε-filter is a nonlinear filter in the time domain, which can reduce noise while preserving the original signal [9,10,11]. Despite its simple design, a conventional ε-filter, which is based on weighted moving average, has some desirable features for noise reduction. It does not need an advance noise estimate as well as it is easy to design and calculation costs are small. However, due to drawbacks of moving average filter, the median ε-filter was introduced for impulsive noise removal [12]. The calculation cost of median ε-filter is small since it requires only switching operation. Moreover, its biggest advantage is that it prevents the non-corrupted pixels with noise from damaging. However, in the absence of noise it acts better than the normal median filter while its behavior in the presence of noise is like the normal median filter.

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On the other side of the coin, we already know that the moving average filter suppresses additive white Gaussian noise better than the median filter, while the median filter is better at preserving edges and rejecting impulses [13]. The best choice taking advantages of both moving average and median filter was proposed by [14], called the a-trimmed mean filter. The α-trimmed mean filter rejects the smaller and the larger observation data depending on the value of α.

In this paper, we take advantages of both median and moving average filters in the calculation of ε-filter. This goal is achieved by using α-trimmed mean filter in computing ε-filter instead of exerting moving average or median filter. The new proposed method, called α-Trimmed Mean ε-Filter, is utilized to enhance images. The outcomes, which are compared to previous ε-filters, demonstrate the good improvement.

This paper is organized as follows. In section 2, the basic concept of conventional ε-filter is provided. Then, we explain the α-Trimmed Mean ε-Filters in section 3. Section 4 is devoted to experimental results. Section 5 deals with conclusion.

II. CONVENTIONAL ε-FILTER

The conventional ε-filter was proposed as Eq. (1).

$$y[m,n] = x[m,n] + \sum_{l=M_1}^{M_2} \sum_{k=N_1}^{N_2} a_{l,k} \; F\big(x[m+l,n+k] - x[m,n]\big)$$

(1)

where $x[m,n], y[m,n]$ are the gray scale value of $(m,n)^{th}$ pixel in the input and output image, respectively. In addition, $a_{l,k}$ is the ε-filter's coefficient, fulfilling the following constrain.

$$\sum_{l=M_1}^{M_2} \sum_{k=N_1}^{N_2} a_{l,k} = 1$$

(2)

Also, $F(x)$ is a nonlinear function described as follows.

$$|F(x)| \le \varepsilon \; ; \; -\infty \le x \le \infty$$

(3)

where ε is a constant.

Noise with small magnitude is suppressed as the original signal is preserved by this method. Basic function for $F(x)$ is written as follows.

$$F(x) = \begin{cases} x & ; \; |x| \le \varepsilon \\ 0 & ; Otherwise \end{cases}$$

(4)

This function ignores points which are far from x more than ε and smoothes those close to x. Therefore, the ε-filter

reduces noise while preserving the rash attack and decay of the speech signal.

Thereafter, it is modified and median ε-filter was suggested [12]. Consider a $N \times N$ window centered around $(m,n)^{th}$ pixel, where N is an odd integer. We also define the set of pixels within the window as follow.

$$\Theta_{(m,n)} = \left\{ \begin{array}{l} x(i,j) \mid m - \dfrac{(N-1)}{2} \leq i \leq m + \dfrac{(N-1)}{2} \\ , n - \dfrac{(N-1)}{2} \leq j \leq n + \dfrac{(N-1)}{2} \end{array} \right\} \quad (5)$$

Considering the set $\Theta_{(m,n)}$, an intermediate set called $\Psi_{(m,n)}$ is formed as bellow.

$$\Psi_{(m,n)} = \left\{ \begin{array}{l} v(i,j) = x(i,j) + \\ F\big(x(m,n) - x(i,j)\big) \mid x(i,j) \in \Theta_{(m,n)} \end{array} \right\} \quad (6)$$

Indeed, a new subset of pixel images is constructed according to Eq. (6). Then, the median of this new pixel set will be calculated and the outcome will be placed in $(m,n)^{th}$ pixel of the output image.

The pixels of intermediate set $\Psi_{(m,n)}$ takes the below form.

$$v(i,j) = \begin{cases} x(m,n) & ; |x(i,j) - x(m,n)| \leq \varepsilon \\ x(i,j) & ; |x(i,j) - x(m,n)| > \varepsilon \end{cases} \quad (7)$$

Therefore, the median ε-filter is defined as Eq. (8).

$$y(m,n) = Med\left\{ v(i,j) \mid v(i,j) \in \Psi_{(m,n)} \right\} \quad (8)$$

where *Med* represents the median function.

In fact, in this filter it is assumed that the difference between the central pixel and noise is much bigger than the difference between the central pixel and another pixel of the image such as edges' pixels. So that, the above median ε-filter finds difference between the gray scales of central pixel (m,n) and other pixels within the window (i,j); if this difference is less than a certain threshold (ε), it means that $(i,j)^{th}$ pixel is the image's pixel and should be replaced with the value of central pixel (m,n) in the intermediate set $\Psi_{(m,n)}$. Consequently, while the median of the intermediate set is being calculated, edges' points and other details of the image won't affect the outcome of median filter, so that the image won't be damaged. On the other hand, if this difference is greater than the threshold, it symbolizes the presence of noise. Therefore, the value of noise should not be replaced with the value of central pixel and the noise will be eliminated right after passing through the median filter.

III. α-TRIMMED MEAN ε-FILTERS

In order to calculate the α-trimmed mean, the data should be sorted low to high and summed the central part of the ordered array. The number of data values which are dropped from the average is controlled by the trimming parameter α which assumes values between 0 and 0.5. The trimmed mean for a *N*-sample signal can be written as a function of the trimming parameter alpha:

$$y(m,n) = \frac{1}{N - 2[\alpha N]} \sum_{j=[\alpha N]+1}^{N-[\alpha N]} X_{(j)} \quad (9)$$

where $[.]$ is the greatest integer function and $X_{(j)}; j = 1,2,..,N$ are the order statistics of input's samples $x(m,n)$, which are sorted as $X_{(1)} \leq X_{(2)} \leq ..... \leq X_{(N)}$.

In the case that α is zero, none of the data samples will be rejected and the summation will be computed over the entire samples. On the other hand, if α is opted 0.5, all samples, except for the middle one, are rejected and the α-trimmed mean will turn into median filter. Consequently, the α-trimmed mean filter can be used as a compromise between the median filter and the moving average filter.

However, as it was stated in the introduction, ε-filter gave good results in image processing. Thereafter, in order to improve it, median ε-filter was suggested [12]. On the other hand, we have already pointed out the upsides of α-trimmed mean filter. It can take advantages of both average and median filters. So, we applied α-trimmed mean on the outcome of ε-filter and introduced a new class of order statistics filters, called α-trimmed mean ε-filter which is defined as eq. (10).

$$y(m,n) = \frac{1}{N - 2[\alpha N]} \sum_{k=[\alpha N]+1}^{N-[\alpha N]} V_{(k)} \quad (10)$$

where $V_{(k)}; k = 1,2,..,N$ are the order statistics of the pixels of intermediate set $v(i,j)$ calculated using Eq. (7), which are sorted as $V_{(1)} \leq V_{(2)} \leq ..... \leq V_{(N)}$.

Note that for spatial filtering, a processing window is slid horizontally and vertically over the entire image in a way that the processing window is centered at a particular image pixel at each time. Then, the pixels inside window are projected into intermediate set by eq. (6).

IV. EXPERIMENTAL RESULTS

In this section, we present results obtained by processing a test image using our proposed method and we compare our method with median filter, median ε-filter and α-trimmed mean filter in order to investigate its effectiveness. So, the tests are carried out on the Lena's picture. In order to compare the results of different methods, we choose a 9×9 square window for all experiments.

(a)   **(b)**

(c)   **(d)**

(e)   **(f)**

Fig. 1 Experimental Results on Lena's Picture; (a) the original image; (b) the noisy image corrupted by salt and pepper noise; (c) the outcome of Median filter; (d) the outcome of α-trimmed mean filter; (e) the outcome of Median ε-filter; (f) the outcome of α-trimmed mean ε-filter

Moreover, Peak signal to Noise Ratio (PSNR) is chosen as the comparison criterion, which is defined for a $M \times N$ image as Eq. (11).

$$PSNR = 10\log\left(\frac{255^2}{\frac{1}{M \times N}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(X(i,j) - \hat{X}(i,j)\right)^2}\right) \quad (11)$$

where X is the original reference image and $\hat{X}$ is the outcome of noise reduction method. Obviously, the greater the value of PSNR, the more quality of the noise reduction method is.

The salt and pepper noise is added to the Lena picture. The density of salt and pepper noise is 0.06, that means 3932 pixels in the Lena picture with the size 256×256 are noisy pixels. Moreover, we choose $\alpha = 0.4$ for both α-trimmed mean filter

and α-trimmed mean ε-filter; and $\varepsilon = 0.4$ for both median ε-filter and α-trimmed mean ε-filter. Afterwards, the noisy image is passed through the four noise reduction methods and PSNR is calculated for each outcome. The original image, noisy image, and the outcomes of four noise reduction methods are shown in Fig.

As it can be obviously seen in Fig. 1 (c) and (d), the outcomes of median and **α**-trimmed mean filters are completely blurred. Nevertheless, the results of Median ε-filter (Fig. 1 (e)) and **α**-trimmed mean **ε**-filter (Fig. 1 (f)) are much better, even though both are very close from the visual point of view. However, the visual comparison is not very accurate which is why we used PSNR in order to compare various methods. The PSNR values are presented in Table .

TABLE I
$PSNR$ VALUES FOR DIFFERENT METHODS OF THE EXPERIMENT

	Median Filter	α-trimmed mean filter	Median ε-Filter	α-trimmed mean ε-filter
PSNR	25.20	25.20	26.92	27.12

As it can be clearly seen from TABLE , α-trimmed mean ε-filter gives the best result among the four examined algorithms.

V. CONCLUSION

In this paper, we introduced a new image enhancement method. Conventional ε-filters would use moving average to calculate the enhanced image, so that they have suffered from drawbacks of moving average filters. Just the same, α-trimmed mean filter was shown to take advantages of both median and α-trimmed mean filters. With the use of α, it can become close to either median or α-trimmed mean filter. We employed α-trimmed mean filter in the calculation of ε-filter. Therefore, we except that conventional ε-filter improve drastically. Experiments justify our claim.

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