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Sensitivity Analysis for Direction of Arrival Estimation Using Capon and Music Algorithms in Mobile Radio Environment

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***Abstract*—**An array antenna system with innovative signal processing can improve the resolution of a source direction of arrival (DoA) estimation. High resolution techniques take the advantage of array antenna structures to better process the incoming waves. They also have the capability to identify the direction of multiple targets. This paper investigates performance of the DOA estimation algorithm namely; Capon and MUSIC on the uniform linear array (ULA). The simulation results show that in Capon and MUSIC algorithm the resolution of the DOA techniques improves as number of snapshots, number of array elements, signal-to-noise ratio and separation angle between the two sources θ increases.

***Keywords*—**Antenna array, Capon, MUSIC, Direction-of-arrival estimation, signal processing, uniform linear arrays.

I. Introduction

IN the last decades, accurate determination of a signal direction of arrival (DOA) has received considerable attention in radar system and communication of military and commercial applications. Wireless communications, radar, radio astronomy, sonar, navigation, tracking of various targets are a few examples of many possible applications. For example, in commercial applications it is necessary to identify the direction of an emergency cell phone call in order to dispatch a rescue team to the proper location. One example of defense applications it is to identify the direction of a possible treads [1].

In wireless mobile communication, the main objective of direction-of-arrival (DOA) estimation is to use the data received at the base-station sensor array to estimate the directions of the signals from the desired mobile users as well as the directions of interference signals. The results of DOA estimation are then used to adjust the weights of the adaptive beamformer so that the radiated power is maximized towards the desired users, and radiation nulls are placed in the directions of interference signals [2].

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There are several methods to estimate the number of incidents plane waves on the antenna arrays and their angle of incidence. The various DOA estimation algorithms are Bartlett, Capon, Min-norm, MUSIC, and ESPRIT. But MUSIC and ESPRIT algorithms are high resolution and accurate methods which are widely used in the design of smart antennas. The popularity of MUSIC is more due to its accuracy and robust method. In this paper, we concentrate the discussion on the application of estimating the DOA of multiple signals. The focuses in this study are on MVDR known also as Capon and MUSIC algorithm. Computer simulation programs using MATLAB were developed to evaluate the direction finding performance of an array processor [3], [5].

II. DOA Estimation Algorithms

The DOA algorithms are classified as quadratic type and subspace type. The Barltett and Capon (Minimum Variance Distortionless Response) are quadratic type algorithms. The both methods are highly dependent on physical size of array aperature, which results in poor resolution and accuracy [4].

Subspace based DOA estimation method is based on the eigen decomposition [5]. The subspace based DOA estimation algorithms MUSIC and ESPRIT provide high resolution, they are more accurate and not limited to physical size of array aperture [2], [5]. Capon and MUSIC algorithm performances is analyzed based on number of snapshots, number of users, user space distribution, number of array elements, and signal to noise ratio.

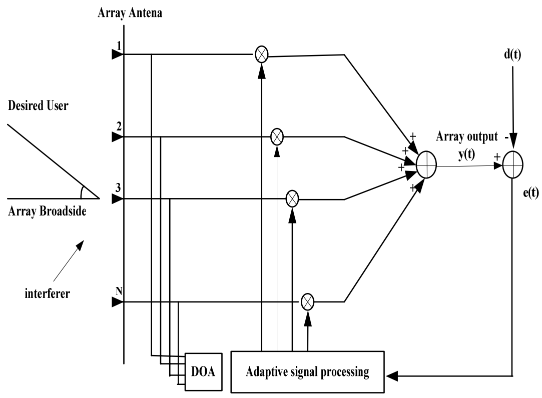

Fig. 1 A functional block diagram of a smart antenna system

The goal of DOA estimation is to use the data received by the array to estimate the direction of arrival of the signal. As shown in Fig.1 it is an antenna system that can modify its beam pattern by means of internal feedback control while it is operating.

In the design of adaptive array smart antenna for mobile communication the performance of DOA estimation algorithm depends on many parameters such as number of mobile users and their space distribution, the number of array elements and their spacing, the number of signal samples and SNR. [1]

A. Signal Model

Consider a number of plane waves from M narrow-band sources impinging from different angles θ_i , i = 1, 2…, M, impinging into a uniform linear array (ULA) of N equi-spaced sensors, as shown in Fig. 2. At a particular instant of time t, t=1, 2… K, where K is the total number of snapshots taken, the array output will consist of the signal plus noise components. The signal vector x(t) can be defined as [4]:

$$x(t) = \sum_{m=1}^{M} a(\theta_m).s_m(t) \tag{1}$$

where s(t) is an M×1 vector of source waveforms, and for a particular source at direction θ from the array boresight; a(θ) is an N×1 vector referred to as the array response to that source or array steering vector for that direction. It is given by:

$$a(\theta) = [1e^{-j\phi} .... e - j^{-j(N-1)\phi}]^T \tag{2}$$

where T is the transposition operator, and ϕ represents the electrical phase shift from element to element along the array. This can be defined by:

$$\phi = (2\frac{\pi}{\lambda})d\cos(\theta) \tag{3}$$

where d is the element spacing and λ is the wavelength of the received signal. The signal vector x(t) of size N×1 can be written as:

$$x(t) = A(\theta).s(t) \tag{4}$$

where A(θ) =[a(θ_1) … a(θ_M)] is an N×M matrix of steering vectors and S(t) =[s1(t) … sM(t)] is an N×M matrix of source vector. The model described in (4) can never explain the observed data; this is may be due to noise and modeling errors. Therefore to account for these effects, an additive noise term w(t) is included. Hence the array output consists of the signal plus noise components, and it can be defined as:

$$x(t) = A(\theta).s(t) + w(t) \tag{5}$$

where x(t) and w(t) are assumed to be uncorrelated and w(t) is modelled as temporally white and zero-mean complex Gaussian process. (5) can be written in matrix form of size N×K as:

$$X = A.S + W \tag{6}$$

where S=[s(1) … s(K)] is an M×K matrix of source waveforms and W=[w(1) …w(K)] is an N×K matrix of sensor noise. The spatial correlation matrix R of the observed signal vector x(t) can be defined as:

$$R_{xx} = E[x(t).x(t)^H] \tag{7}$$

$$R_{xx} = E[A(\theta).s(t).s(t)^H.A^H(\theta)] + E[w(t)\ .w(t)^H]$$
$$R_{xx} = A(\theta).R_{ss}.A^H(\theta) + R_{ss}$$
$$R_{xx} = A(\theta).R_{ss}.A^H(\theta) + \delta_{n2I} \tag{8}$$

where σn2 is the variance of noise and I is the identity matrix.

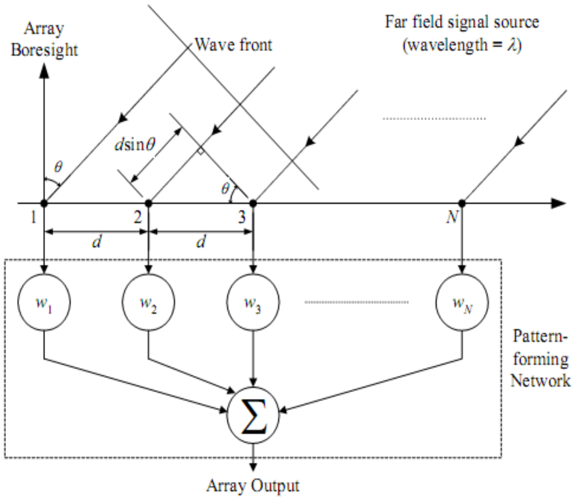

Fig. 2 A plane wave incident on a uniform linear array of N-equi-spaced sensors [4]

For this signal model, the correlation matrix R_{xx} will have M signal eigenvalues, and N-M noise eigenvalues. Let E_s be the matrix constructed of the corresponding M signal eigenvectors E_s=[e_1 e_2…e_M], and En be the matrix containing the remaining N-M noise eigenvectors E_n=[e_{M+1} e_{M+2} … e_N].

$$R_{xx} = \sum_{m=1}^{M} \lambda_k e_k e_k^H = E_s.A_s.E_s^H + \delta_n^2.E_s.E_s^H \tag{9}$$

In real array measurements, the covariance matrices are unknown and they can be estimated from a finite amount of measurement called snapshots. Therefore the natural estimate of the correlation matrix or the sample covariance matrix is given by:

$$\hat{R}_{xx} = \sum_{k=1}^{k} x(k).x(k)^H$$

Or

$$\hat{R}_{xx} = \frac{1}{k}\sum_{k=0}^{k-1} x(k).x(k)^H \quad (10)$$

B. DOA Estimation using Capon Algorithm

The Capon algorithm involves estimating the noise subspace from the correlation matrix on which the M array steering vectors are projected. These steering vectors are also known as direction vectors and they represent the response of an ideal array to the signal sources. The signal sources can be derived from the direction vectors which are as orthogonal to the noise subspace [5], [6].

The algorithm starts by constructing a real-life signal model described in the previous section. The peaks in the Capon angular spectrum occur whenever the steering vector E(φ) is orthogonal to the noise subspace. This technique minimizes the contribution of the undesired interferences by minimizing the output power while maintaining the gain along the look direction to be constant, usually unity. That is,

$$\min E[|y(\theta)|^2] = \min w^H R_{xx} w, w^H A(\theta_0) = 1 \quad (11)$$

Using Lagrange multiplier, the weight vector that solves (1) can be shown to be:

$$W_{cap} = \frac{R_{xx}^{-1} A(\theta)}{A^H(\theta) R_{xx}^{-1} A(\theta)} \quad (12)$$

The output power of the array as a function of the DOA estimation using Capon beam forming method is given by Capon spatial spectrum (P_{cap}) as:

$$P_{cap} = \frac{1}{A^H(\theta) R_{xx}^{-1} A(\theta)} \quad (13)$$

The angles of arrival are estimated by detecting the peaks in this angular spectrum.

C. DOA Estimation Using MUSIC Algorithm

MUSIC is an acronym which stands for Multiple Signal classification. It is high resolution subspace DOA technique which gives the estimation of number of signals arrived, hence their direction of arrival. The algorithm is based on exploiting the eigenstructure of input covariance matrix. The incident signals are somewhat correlated creating non diagonal signal correlation matrix. The algorithm is used to describe experimental and theoretical techniques involved in determining the parameters of multiple wave fronts arriving at an antenna array from measurements made on the signal received at the array elements [7]. MUSIC deals with the decomposition of covariance matrix into two orthogonal matrices, i.e., signal-subspace and noise-subspace. Estimation of DOA is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated. This makes the covariance matrix diagonal. The steps of the algorithm are summarized as follows:

Step 1: Collect input samples X_k, k = 0… N-1 and estimate the input covariance matrix

$$\hat{R}_{xx} = \frac{1}{k}\sum_{k=0}^{k-1} x_k . x_k^H \quad (14)$$

Step 2: Perform eigen decomposition on $\hat{R}_{XX}$

$$\hat{R}_{xx} E = E A \quad (15)$$

where $\Lambda = \text{diag}\{\lambda_0, \lambda_1, \ldots, \lambda_{M-1}\}, \lambda_0 \geq \lambda_1 \geq \ldots \geq \lambda_{M-1}$ are the eigenvalues and E =[q_0 q_1…..q_{M-1}] are the corresponding eigenvectors of $\hat{R}_{XX}$

Step 3: Estimate the number of signals $\hat{L}$ from the multiplicity K, of the smallest eigenvalue λ_{min} as equation:

$\hat{L}$ = M – K

Step 4: Compute the MUSIC spectrum by the following Eq.

$$\hat{P}_{MUSIC\,(\theta)} = \frac{A^H(\theta) A(\theta)}{A^H(\theta) E_n E_n^H A(\theta)} \quad (16)$$

Step 5: Find the $\hat{L}$ largest peaks of $\hat{P}$MUSIC (θ) to obtain estimates of the Direction -Of- Arrival.

III. Simulation Results and Performance Evaluation

The Capon and MUSIC techniques for DOA estimations are simulated using MATLAB tool. The performance of the algorithms has been analyzed by considering the effect of changing a number of parameters related to the signal environment as well as the sensor array. A uniform linear array with M elements has been considered in our simulation experiments and all inputs were made fixed when the effects of changing a parameter value was investigated. In these simulations, it is considered a linear array antenna formed by 10 elements that are evenly spaced with the distance of λ/2. The noise is considered to be additive, having the 0.1 variance value. The simulation has been run for two signals coming from different angles Θ_1=200 and Θ_2=-200 for different value of snapshots, SNR, and array elements. These two signals are considered to have equal amplitudes.

A. Capon Algorithm

The Capon estimation technique can be simulated by evaluating the inverse of the autocorrelation matrix R_{xx}. Of course carrying out this matrix inversion problem is a time consuming process, a fact that makes this technique computationally inefficient, and hence, less popular. In order to illustrate the effect of each of the input parameters on the performance of the algorithm, the other parameters should be kept constant.

Case 1: Capon Spectrum for Varying the Values of SNR

To consider the influence of the SNR, the number of array elements and the horizontal angular resolution or separation between the users must be kept constant. In Fig. 3, below, ten array elements where used to estimate the DOA of two users located at -20 and 20 degrees using the algorithm. The simulation was carried out three times for different values of the SNR (15, 25, and 35), it is clear that the SNR slightly effects the performance of the algorithm. As the SNR increase, the noise level at the output of the array decreases proportionally, the performance improves.

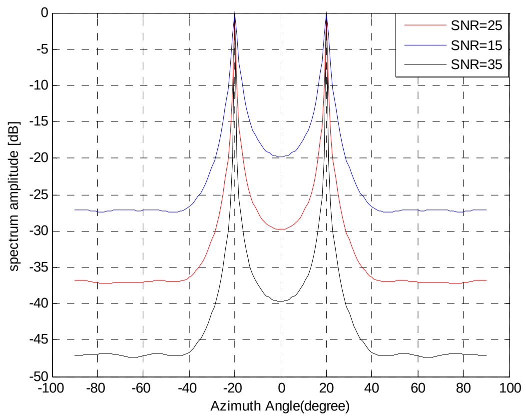

Fig. 3 The effect of varying the SNR on the Capon

Case 2: Capon Spectrum for Varying Number of Array Elements

Fig. 4 shows the Capon angular spectrum using N=6, 8, and 12 element array. The beam width decreases as the number of array elements increase leading to better resolution capabilities. The Capon algorithm performs better since it produces an angular spectrum with a sharp peak and a lower noise floor. It is evident that using more elements improves the resolution of the algorithm in detecting the incoming signals. This achieved, however at the expense of computational efficiency and hardware complexity of the antenna array.

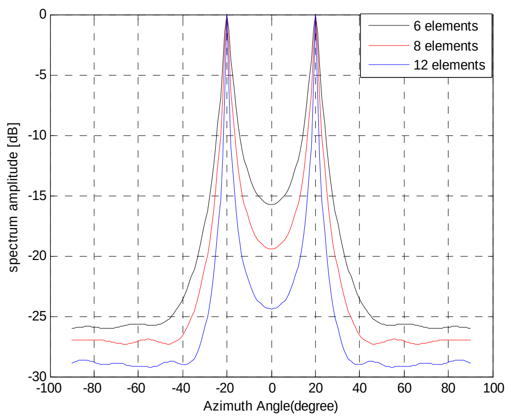

Fig. 4 The effect of varying the number of array elements

Case 3: Capon Spectrum for Varying Angular Separation

The performance of the algorithm degrades when there are many mobile users, because the spatial correlation between the incoming signals makes it difficult for the algorithm to resolve them successfully. Fig. 5 proves the fact the capon has better resolution, the capons technique can resolve two signals with an angular separation of 10 degrees, however, it starts to fail as the users come closer. Complete failure to resolve the signals occurs when the users come as close as 5 degrees with an SNR of 15 dB. It is evident that using more elements improves the resolution of the algorithm in detecting the incoming signals. The performance improves significantly as the user moves away from each other, for which the capon angular spectrum has sharper peaks and lower noise floor.

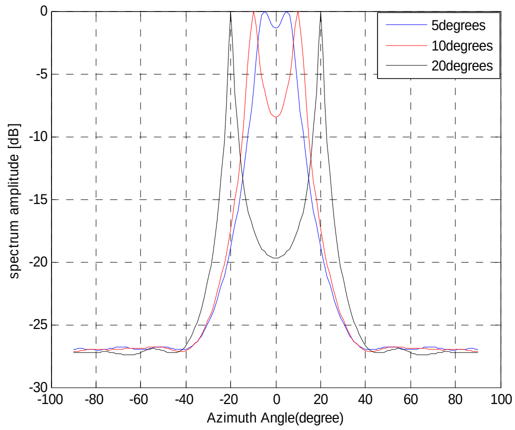

Fig. 5 The effect of varying angular separation

Case 4: Capon Spectrum for Varying Number of Signal Snapshots

Fig. 6 shows Capon spectrum with K=100, 200 and 1000 with keeping the other parameter constant. We can observe that from the figure that by increasing the number of the snapshots peaks becomes sharper and the noise floor becomes better.

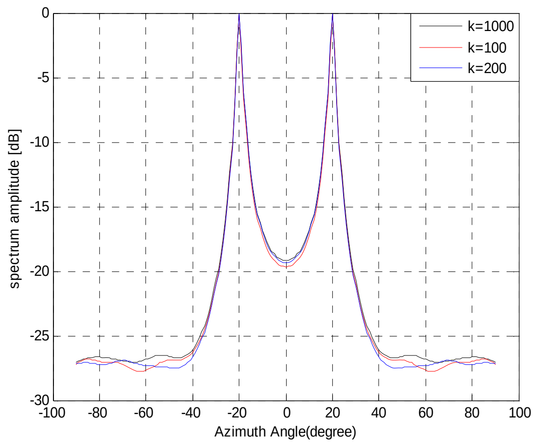

Fig. 6 The effect of varying the number of snapshot

Case 5: Capon Spectrum for Varying Element Array Spacing

Fig. 7 shows the Capon spectrum for an element spacing of d=0.25λ and d=0.5λ, respectively. When the elements of the antenna array are placed too close to each other, mutual coupling effects dominate resulting in inaccuracies in the estimated angles of arrival, as shown in Fig. 7 for which d=0.25λ.

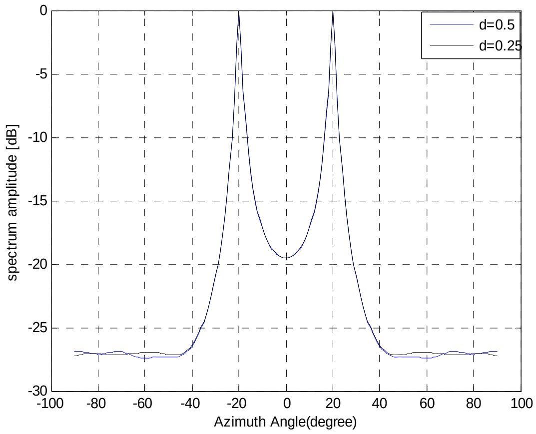

Fig. 7 The effect of element spacing d on Capon

Mutual coupling effects for closely spaced elements must, therefore, be taken into account when designing the sensor array. To overcome this problem, the spacing between the elements of the antenna array must be increased resulting in a better resolution of the estimated peaks, as shown in Fig.7 for which d=0.5λ.

B. Music Algorithm

In this algorithm, a study has been also made between the minimum angles of separation $\theta_{\min}$ that can be resolved for different incoming signals from sources and how is its dependence on the SNR of the incoming signals, the number of the array elements M and the number of collected data samples N, assuming a ULA with isotropic sources. Two input signals from two different sources are incident at different angles on a uniform linear of M elements under equal spacing d = λ / 2. The performance of the MUSIC algorithm have been studied, by varying the angle of separation between the signals under several SNR conditions and different snapshots N, to find the minimum resolvable angle which can be detectable by detecting the two peaks measured.

Case 1: The Effect of Varying the SNR

The effect of changing the SNR is shown in Fig. 8, with three different values (6, 12, and 20) dB. It is clear that as SNR increases, the ratio between the output peaks and the noise level at the output of the array increases proportionally.

Case 2: The Effect of Varying the Horizontal Angle Separation

The effect of varying horizontal angle separation on MUSIC algorithm, sharper peaks increases as the angle separation between signals increases as shown in Fig. 9, since MUSIC is a high resolution technique; it is capable to resolve the signal from two users.

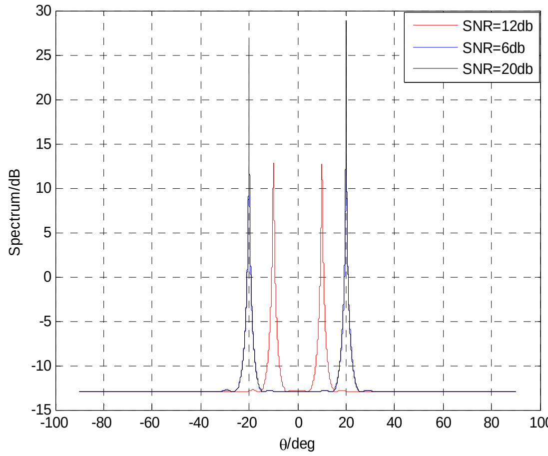

Fig. 8 The effect of varying the SNR on MUSIC

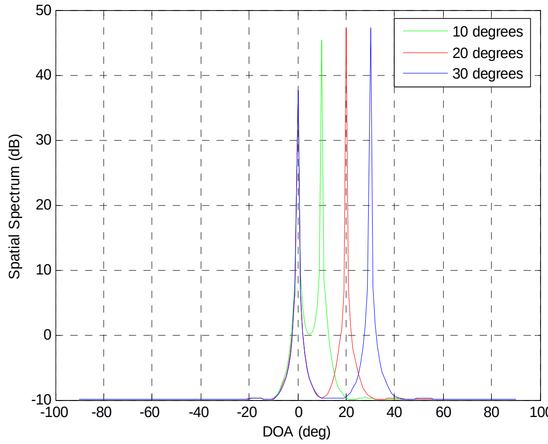

Fig. 9 The effect of varying the angular separation on the MUSIC

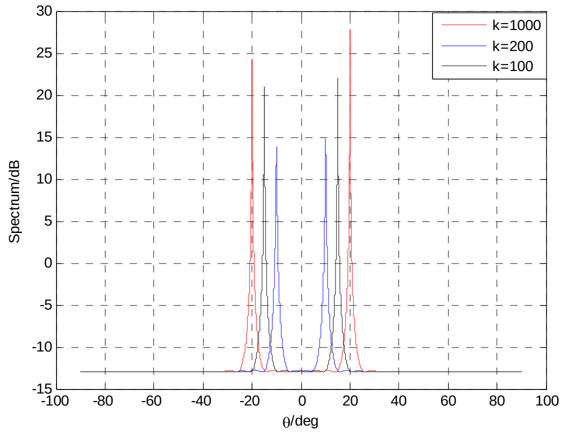

Fig. 10 The effect of varying the number of snapshot on the MUSIC algorithm

Case 3: The Effect of Varying Number of Snapshots

Fig. 10 shows MUSIC spectrum as a function of the number of snapshots with K=100, K=200 and K=1000 and keeping all parameter constant. We can notice that by the increasing the number of snapshots peaks in the MUSIC spectrum become further sharper for higher number of snapshots e.g. with K=1000.

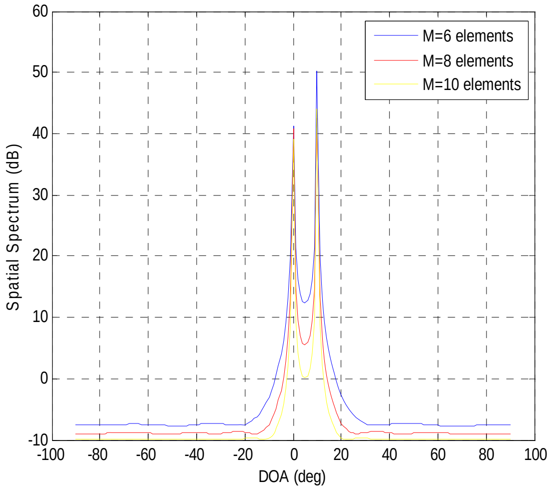

Fig. 11 The effect of varying the number of sensors in MUSIC

Case 4: The Effect of Varying the Number of Array Sensor

The effect of increasing the number of array sensor on the performance of the MUSIC algorithm can be shown in Fig. 11. Small reduction in beamwidths and the noise level is observed. The spacing between the elements of the sensor array must be increased resulting in a better resolution of the estimated peaks, as shown in Fig. 13 for which d=0.5λ.

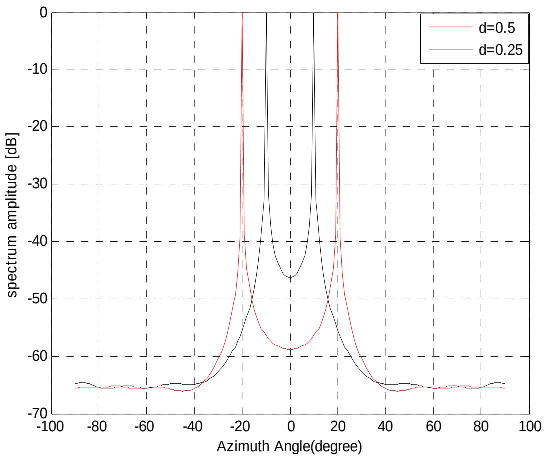

Fig. 12 The effect of element spacing d on the MUSIC

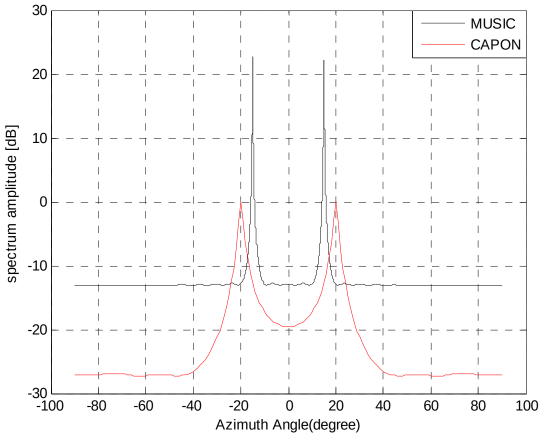

Fig.13 Comparison of Capon and MUSIC

IV. Conclusion

A number of numerical experiments were conducted to investigate the effect of various parameters on the performance of the Capon and MUSIC algorithms and its ability to resolve incoming signals accurately and efficiently. The developed simulation tool can be used to improve and accelerate the design of wireless networks. It can also be used for computer-aided learning of modern communication systems utilizing adaptive antenna arrays. The simulation results of MUSIC show that the performance improves with more elements in the array, with large snapshots of signals and greater angular separation between the signals. These improvements are seen in form of the sharper peaks in the MUSIC. Clearly MUSIC is more stable and accurate and provides high resolution and this adds new possibility of user separation through SDMA and can be widely used in the design of smart antenna system. MUSIC presented the best behavior MUSIC requires more computational resources.

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