

# Integrated Approaches to Enhance Aggregate Production Planning with Inventory Uncertainty based on Improved Harmony Search Algorithm

P. Luangpaiboon, and P. Aungkulanon

**Abstract**—This work presents a multiple objective linear programming (MOLP) model based on the desirability function approach for solving the aggregate production planning (APP) decision problem upon Masud and Hwang's model. The proposed model minimises total production costs, carrying or backordering costs and rates of change in labor levels. An industrial case demonstrates the feasibility of applying the proposed model to the APP problems with three scenarios of inventory levels. The proposed model yields an efficient compromise solution and the overall levels of DM satisfaction with the multiple combined response levels. There has been a trend to solve complex planning problems using various metaheuristics. Therefore, in this paper, the multi-objective APP problem is solved by hybrid metaheuristics of the hunting search (HuSIHSA) and firefly (FAIHSA) mechanisms on the improved harmony search algorithm. Results obtained from the solution of are then compared. It is observed that the FAIHSA can be used as a successful alternative solution mechanism for solving APP problems over three scenarios. Furthermore, the FAIHSA provides a systematic framework for facilitating the decision-making process, enabling a decision maker interactively to modify the desirability function approach and related model parameters until a good optimal solution is obtained with proper selection of control parameters when compared.

**Keywords**—Aggregate Production Planning, Desirability Function Approach, Improved Harmony Search Algorithm, Hunting Search Algorithm and Firefly Algorithm.

## I. INTRODUCTION

AGGREGATE production planning (APP) is a capacity planning over the medium-time horizon, often from approximately 2 to 18 months in advance, to support forecasted customer demand [1]. The Aggregation refers to the idea of translating forecasted sales demand and production capacity into future manufacturing plans for a family of products and focusing on overall capacity rather than the individual products or services. The proposes of the APP are to generate the near future aggregated production levels for each product type to meet fluctuating or uncertain demand via forecasting and to make decisions and strategies concerning hiring, overtime, layoffs, backorders,

subcontracting and inventory level including appropriate scarce resources so that the planned products and services will be available to meet all customer requirements. There is a decrease in the amount of data used during the planning process of the APP and therefore enables various plans to be modified more frequently. In general, the APP focuses on the determination of optimal production, workforce, and inventory levels over a fixed planning horizon. Thus its objectives are to maximise the net profit, utilisation of production resource, customer service or to minimise of inventory investment, changes in production rate, change in workforce level [2].

The objective of this paper is to investigate the performances of the algorithmic approaches on the multi-objective linear programming model of the APP. A simulation study is based on the data from a Thai firm. It aims to enhance the efficiency of production planning and pay more attention to the harmonious balance between various objectives and uncertainty in inventory. This paper is organised as follows. Section II describes the multi-objective linear programming model of the APP. Sections III, IV and V are briefing about algorithms of bee, hunting search and firefly, respectively. Section VI shows design and analysis of computational experiments for comparing the performance of the hybrid methods. The conclusion is also summarised and it is followed by acknowledgment and references.

## II. MULTI-OBJECTIVE LINEAR PROGRAMMING MODEL (MOLP) FOR AGGREGATE PRODUCTION PLANNING (APP)

In developing APP the criteria to deal with fluctuating demands consist of varying production level, changing the level of workforce and applying inventory. Weighing the relative advantages and disadvantages of each criteria or objective and developing a hybrid policy of multi-objective need be applied. The multi-objective APP problem can be described as follows. Assume that a company manufactures  $N$  types of products to satisfy the market demand over a planning horizon of  $T$ . The problem involves determining the most effective means of satisfying forecasted demand by changing hiring and layoffs, inventory levels, overtime work, subcontracting, back orders and other decision variables. A study of the multi-objective APP decision model follows Masud and Hwang (1980) and specifies three objective functions to minimise total production costs, carrying or backordering costs and rates of change in labor levels [3]. Total regular time production is determined via the sum of a product of the production cost per unit ( $v_{it}$ ) and produced quantity ( $P_{it}$ ) for the  $i$ th product excluding the labor cost in the  $t$ th period including a product of regular

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time work force cost per employee hour ( $r_t$ ) and number of work force level in the  $t$ th period ( $W_t$ ), a product of hiring cost ( $h_t$ ) and number of hired workers ( $H_t$ ) in the  $t$ th period and a product of layoff cost ( $f_t$ ) and number of laid off workers ( $F_t$ ) in the  $t$ th period. The carrying or backordering cost is from a product of the inventory carrying cost for product  $i$  ( $c_{it}$ ) and inventory level for product  $i$  in  $t$ th period ( $I_{it}$ ). Finally, it is the summation of number of hired ( $H_t$ ) and laid off workers ( $F_t$ ). In this model, there are some related constraints of inventory and labor levels, labor capacity in regular and overtime and non-negativity constraints on decision variables where parameters and constant definitions are the forecasted demand for the  $i$ th product in the  $t$ th period ( $d_{it}$ ), the produced quantity per worker in regular time for the  $i$ th production the  $t$ th period ( $K_{it}$ ), initial inventory level for the  $i$ th product ( $I_{oi}$ ), the minimal inventory level available for the  $i$ th production the  $t$ th period ( $I_{it}^{MIN}$ ), the minimal ( $W_{MIN}$ ) and maximal ( $W_{MAX}$ ) work force levels available in the  $t$ th period. The mathematical programming models of the APP objectives are given as follow.

$$\text{MIN } Z_1 = \sum_{i=1}^N \sum_{t=1}^T (v_{it} P_{it}) + \sum_{t=1}^T (r_t W_t + h_t H_t + f_t F_t) \quad (1)$$

$$\text{MIN } Z_2 = \sum_{i=1}^N \sum_{t=1}^T (c_{it} I_{it}) \quad (2)$$

$$\text{MIN } Z_3 = \sum_{t=1}^T (H_t + F_t) \quad (3)$$

Subject to

$$P_{it} + I_{i,t-1} - I_{it} = d_{it} \quad (4)$$

$$I_{it} \geq I_{it}^{MIN} \quad (5)$$

$$W_t - W_{t-1} - H_t + F_t = 0 \quad (6)$$

$$W_{MIN} \leq W_t \leq W_{MAX} \quad (7)$$

$$P_{it} - K_{it} * W_t \leq 0 \quad (8)$$

$$P_{it}, I_{it}, W_t, H_t, F_t \geq 0 \quad (9)$$

$$W_t, H_t, F_t \in \text{Integer} \quad (10)$$

The objective functions of the APP model are required to be simultaneously optimised by the decision maker (DM) in the framework of overall satisfactory levels. The DM needs to determine a desired achievement degree and importance (or weight) of each of the objective of the multi-objective APP model. Many engineering optimisation problems have been focused on the case with only one goal or objective. However, it is quite common that multiple objectives are of interest. A determination of the optimised decision variables would require simultaneous consideration of all the objectives [4].

Consequently, it is desirable for a decision maker (DM) to determine an overall optimal solution or a best compromise of all desired characteristics simultaneously. This problem is formulated as a multi-objective optimisation model subject to various requirements on problem constraints and decision variables. Suppose that there are  $M$  objectives which are determined by the decision variable vector. This approach

involves transformation of the  $i$ th objective model ( $o_i$ ) to a dimensionless desirability function ( $d_i(o_i)$ ) which combines the DM's important objectives and desires when building the optimisation procedure. In transforming each  $o_i$  to  $d_i$  one-sided and two-sided desirability transformations depend on whether each of the  $M$  objectives has to be the larger-the-better (LTB) or smaller-the-better (STB) or nominal-the-better (NTB). Values from 0 to 1 will be assigned for the possible values of each objective, in which  $d_i$  approaches 1 as the objective approaches its target value. Note that  $d_i = 0$  if the  $i$ th objective lies outside its corresponding acceptable levels. Transformation for objectives are defined as followed.

$$d_i(o_i) = \begin{cases} 0 & \text{if } o_i \leq O_i^{MIN} \text{ or } o_i > O_i^{MAX}, \\ \left[ \frac{o_i - O_i^{MIN}}{T_i^{MIN} - O_i^{MIN}} \right]^{P1_i} & \text{if } O_i^{MIN} < o_i \leq T_i^{MIN}, \\ \left[ \frac{O_i^{MAX} - o_i}{O_i^{MAX} - T_i^{MAX}} \right]^{P2_i} & \text{if } T_i^{MAX} < o_i \leq O_i^{MAX}, \\ 1 & \text{if } T_i^{MIN} < o_i \leq T_i^{MAX} \end{cases} \quad (11)$$

$$d_i(o_i) = \begin{cases} 0 & \text{if } o_i \leq O_i^{MIN}, \\ \left[ \frac{o_i - O_i^{MIN}}{O_i^{MAX} - O_i^{MIN}} \right]^{P3_i} & \text{if } O_i^{MIN} < o_i \leq O_i^{MAX}, \\ 1 & \text{if } o_i \geq O_i^{MAX} \end{cases} \quad (12)$$

; where  $o_i$  is the  $i$ th objective model,  $\Omega$  is the experimental region,  $T_i^{MIN}$  and  $T_i^{MAX}$  are the lower and upper targets of the  $i$ th response, respectively.  $O_i^{MIN}$  and  $O_i^{MAX}$  are the minimal and maximal acceptable values of the  $i$ th objective, respectively. The power coefficients of  $P1_i$ ,  $P2_i$  and  $P3_i$  are the parameters that determine the shape of  $d_i(o_i(x))$ . Especially, if  $P1_i$  or  $P2_i$  equals one, the shape is linear. If  $P1_i$  or  $P2_i$  is larger than 1, it is a convex and if  $P1_i$  or  $P2_i$  is less than 1, it is a concave. It should be noted that, if  $T_i^{MIN}$  equals  $T_i^{MAX}$ , the trapezoidal desirability function for the nominal-the-best reduces to a triangular one. Each objective ( $o_i$ ) is transformed to a desirability value of  $d_i(o_i)$ . The individual optimisation value of  $d_i(o_i)$ , whose value are scaled between zero and one, increases as the "desirability" of the corresponding response is improved [5]. All the desirabilities are then aggregated using the geometric mean to provide the overall assessment of the desirability of the combined response levels or  $D = \text{MAX}(d_1(o_1), d_2(o_2), \dots, d_M(o_M))$ .

$D$  provides a value less than or equal to the lowest individual optimisation desirability value and will increase as the balance of the properties is more favorable. A multi-objective optimisation problem is formally defined as

Maximise  $D$

Subject to

$$d_i(o_i) \geq D, i = 1, 2, \dots, M.$$

$$Ax \leq b, x \geq 0$$

$$D \in (0, 1) \quad (13)$$

### III. IMPROVED HARMONY SEARCH ALGORITHM (IHSA)

Metaheuristics have several advantages over traditional calculus-based optimisation algorithms. The former is simple in concept, few in parameters, derivative information and mathematical requirements, and easy in implementation whereas the latter generally requires certain mathematical properties such as differentiability and continuity including convexity. These performances make metaheuristics powerful alternative algorithms that can be easily adopted for various types of engineering optimisation problems. The harmony search algorithm (HSA), recently developed by Geem et al. (2001), is inspired by the process in a musical performance where a musician continues to improvise the pitches of their instruments in order to obtain an improved state of musically pleasing harmony determined by an aesthetic quality standard [6]. In a similar way, the optimisation process seeks to find a global optimum (a perfect state) as determined by an objective function. In other words, a determination of the aesthetic quality via the pitch of each musical instrument is analogous to finding the optimality in an optimisation process via an assignment of the set of values to each decision variable. For improvisation a skilled musician has three possible scenarios of playing any famous piece of music exactly from memory, playing or adjusting slightly something similar to the aforementioned piece or composing new or random notes [7]-[9]. These three scenarios of quantitative optimisation process is then formalized the corresponding algorithmic mechanisms of memory consideration, pitch adjustment and random selection, respectively. With some troubles in performing local search for numerical applications the modification procedures of the improved HSA or IHSA are stated in order to improve the fine-tuning characteristic of the HSA. The steps in the procedure of the IHSA are as follows. IHSA parameters are initialized which consist of harmony memory size (HMS), number of solution vectors in harmony memory), harmony memory consideration rate ( $P_{HMC}$ ), minimal ( $P_{PAR}^{MIN}$ ) and maximal ( $P_{PAR}^{MAX}$ ) pitch adjustment rate, minimal ( $BW^{MIN}$ ) and maximal ( $BW^{MAX}$ ) bandwidth and the termination criteria (the maximum number of improvisations or iterations, NI).

The HMS harmony memory (HM) matrix is randomly initialised using a uniform distribution. In an improvisation, a new harmony vector is generated based on three scenarios. In the memory consideration scenario, the new value of each decision variable is chosen from any of the values in the specified HM range for that decision variable. However, there is a probability of  $P_{HMC}$  in using this scenario for improvising a new harmony. The  $P_{HMC}$ , which is a value between 0 and 1, is the rate of choosing one value from the historical values stored in the HM for each decision variable, while  $(1-P_{HMC})$  is the rate of randomly selecting

one value from the possible range of values for that decision variable. Without a violation on the boundaries, each component obtained by memory consideration is examined to determine whether it should be further modified or mutated using the changeable level of pitch adjustment rate of  $P_{PAR}(Iter)$  (14) and distance bandwidth of  $BW(Iter)$  (15) depending on the iteration (Iter) for fine-tuning of optimised solution vectors and convergence rate of algorithm toward the optimum. If the newly generated harmony vector, shifting to neighboring values within a certain range, has a better objective function than the worst harmony in HM, the new harmony is updated in the HM and the existing worst harmony is excluded from the HM. The iterative procedures are terminated when the stopping criteria (maximum number of improvisations, NI) is satisfied. The pseudo code is used to briefly explain to all the procedures of the IHSA shown in Fig. 1.

$$P_{PAR}(Iter) = P_{PAR}^{MIN} + \frac{(P_{PAR}^{MAX} - P_{PAR}^{MIN})}{NI} Iter \quad (14)$$

$$BW(Iter) = BW^{MAX} \exp \left[ \ln \left( \frac{BW^{MIN}}{BW^{MAX}} \right) \frac{Iter}{NI} \right] \quad (15)$$

#### Procedure IHSA Meta-heuristic()

**Begin;**

Define algorithm parameters;

Initialise the HMS harmony memories randomly within the bounds;

Evaluate the objective functions for all HMS;

**For**  $j = 1$  to  $IM$

Randomly select a position of  $[1, 2, \dots, HMS]$  to improvise;

Generate a random number in the range  $[0, 1]$  or  $RN1$ ;

Check  $RN1$  with  $P_{HMC}$ ;

**If**  $RN1 < P_{HMC}$  better, then pick the component from memory;

Generate a random number in the range  $[0, 1]$  or  $RN2$ ;

**If**  $RN2 < P_{PAR}$  better, then adjust the harmony by  $BW(Iter)$ ;

Generate a random number in the range  $[0, 1]$  or  $RN3$ ;

**If**  $RN3 > 0.5$

Pitch Adjustment Harmony vector increase via  $P_{PAR}(Iter)$ ;

**Else**

Pitch Adjustment Harmony vector decrease via  $P_{PAR}(Iter)$ ;

**End if;**

**Else**

Do nothing;

**End if;**

**Else**

Pick a new random value in the allowed range;

**End if;**

Replace the newly evaluated harmony if better;

**End for;**

**End;**

**End procedure;**

Fig. 1 Pseudo Code of the IHSA Meta-heuristic

### IV. HUNTING SEARCH ALGORITHM (HuS)

HuS is inspired by a model of group hunting of animals when searching for food [10]. The HuS mechanisms come from a process in cooperating to enclose a prey and catch it. Each position distance of real hunter and prey place specify a hunter chance to finally catch that prey. It is one among various metaheuristic to search for the global optimum. By analogy with the real hunting process, the iterative optimisation processes seek a global solution as determined by an objective function. Each step of the HuS replaces the current solution by a random neighbourhood solution,

chosen with a preset probability that depends both on the difference between the corresponding function values assigned to each decision variable. Compared to real group hunting when solving engineering optimisation problems each hunter is replaced with a candidate solution of the problem or an artificial hunter. However, there are some differences when compared. In group hunting of animals, real hunters can see or sense the smell of the prey. The hunter can then determine his prey position at least. In contrast to this, optimisation problems have no indication of the optimal solution. In group hunting of animals, the prey is dynamic but in artificial hunting problems, the optimal solution is static. Finally, in group hunting of animals the prey must correct the position from time to time to escape the hunters. However, in artificial hunting problems the optimal solution does not change its position during the iterative searching process.

From the difficulties on both real and artificial group hunting, the latter needs to resemble the dynamics of the real hunting process. Moreover, it is assumed that the leader has found the optimal solution and other hunting members move towards it. As a result, artificial hunters or candidate solutions move towards the current leader or the current best candidate solution via the maximal movement toward the leader (MML) operator. From the literatures, the preferable levels of MML are between 0.05, when applying with large number of iterations, and 0.4, when applying with small number of iterations. If any of them finds a new solution better than the current leader, it becomes the new leader. When real hunting creatures gradually move toward their prey, they correct their position with a consideration on both the position of other hunting members and the position of the prey. Therefore, in the HuS, when other artificial hunters move toward the previous leader, the artificial hunters correct their position based on the position of other members by applying the parameter of hunting group consideration rate (HGCR). For the real animals, if the prey can flee from the ring, members of the group or hunters organise to enclose the prey again. In the HuS, artificial hunters take the above capability, so they can seek the onslaught ring. If the candidate solutions are too close to each other, the group is reorganised to find the optimal solution in the next iteration. The search process terminates when the maximal iterations of hunting the prey or searching the optimum or the function for the leader and the worst hunter in the group after reorganising meet the preset values.

## V. FIREFLY ALGORITHM (FA)

The firefly algorithm (FA) is a metaheuristic nature-inspired, optimization algorithm [11]. It is based on the social flashing behavior of fireflies, or lighting bugs. Particularly, although the FA has many similarities with other metaheuristics which are based on the so-called swarm intelligence, it is much simpler on both concept and implementation. Its main advantage is the fact that it uses mainly real random numbers, and it is based on the global communication among the fireflies, and as a result, it seems more effective in multi-objective optimisation such as the APP problems in our case. The primary purpose for a firefly's flash is to act as a signal system to attract other fireflies. Now this can idealise some of the flashing

characteristics of fireflies so as to consequently develop firefly-inspired algorithms. For simplicity in describing the FA, there are three particular idealised rules which are based on some of the major flashing characteristics of real fireflies. On the first rule, each firefly attracts all the other fireflies with weaker flashes and they will move towards more attractive and brighter ones regardless their sex. Secondly, the degree of attractiveness of a firefly is proportional to its brightness which decreases as the distance from the other firefly increases due to the fact that the air absorbs light. The less bright one will move towards the brighter one. If there is not a brighter or more attractive firefly than a particular one, it will then move randomly. Finally, no firefly can attract the brightest firefly and it moves randomly. The brightness or light intensity of a firefly is determined by the value of the objective function of a given problem.

The brightness of a firefly is affected or determined by the landscape of the objective function. In the FA there are two important issues of the variation of light intensity and the formulation of the attractiveness. For simplicity, it is assumed that the attractiveness of a firefly is determined by its brightness which in turn is associated with the encoded objective function of the optimisation problems. On the attractiveness of the FA the main form of attractiveness function can be any monotonically decreasing functions depending on the distance ( $r_{ij}$ ) between the  $i$ th and  $j$ th of two fireflies, the attractiveness at the source ( $\beta_0$ ) and a fixed light absorption coefficient ( $\gamma$ ) including randomisation parameter ( $\alpha$ ). For most cases in the implementation,  $\beta_0 = 1$  and  $\alpha = [0, 1]$ . The parameter  $\gamma$  characterises the variation of the attractiveness, and its value is crucially important in determining the speed of the convergence and how the FA behaves. In most applications, it typically varies from 0.01 to 100.

## VI. COMPUTATIONAL RESULTS AND ANALYSES

In this work with the computational procedures previously described, a computer simulation program of the desirability function approach of the APP model was implemented in a Visual C#2008 computer program. A comparison of the conventional procedures of IHSA is determined in this section. The hybridisations of IHSA are also stated to combine their advantages and avoid disadvantages. The first variant on this research, called HuSIHSA, replace the pitch adjusting rate with the MML in each iteration for the HMS improvement. The second variant on this research, called FAHSA, replace the pitch adjusting rate with the attractiveness function in each iteration for the HMS improvement. As can be seen, algorithm parameters have an effect on the solution quality. Therefore, their setting should be done carefully in accordance with the guidelines given in the literature that  $HMS$ ,  $P_{HMC}$ ,  $P_{PAR}^{MIN}$ ,  $P_{PAR}^{MAX}$ ,  $BW^{MIN}$ ,  $BW^{MAX}$ ,  $MML$ ,  $\beta_0$ ,  $\alpha$  and  $\gamma$  are set at 30, 0.90, 0.35, 0.99, 0.00001, 4, 1, [0, 1] and 0.01, respectively. Minimal total production cost, carrying or backordering costs and rates of change in labor levels scenarios are calculated from all previous data in the harmony memory with 350 iterations and 120 replicates. The comparisons are made for three scenarios based on

inventory levels with the uniform distribution of  $U(0, 250)$  or S1 and  $U(250, 500)$  or S2 including the minimal level of 500 or S3. Using the DFA of the APP an aim is to simultaneously minimise all three objectives over a 6-month period. At the S1, S2 and S3 scenarios, all objectives seemed to be better at 0.649, 0.767 and 0.772 on the overall levels of decision making desirability (D), respectively (Tables I-III).

The FAIHSA seemed to provide the better level of decision making desirability over all scenarios. When the IHSA and its two variants of HuSIHSA and FAIHSA were compared, the FAIHSA seems to be better in terms of speed of convergence. The basic idea is the attractiveness or absorption coefficient operators which guarantee a quick convergence of the algorithm to the optimal solution. The HuSIHSA seemed to obtain the better satisfaction level only when resources are limited. So, carrying or backordering units are necessary. When the performances of the IHSA and its variants of FAIHSA and HuSIHSA were compared, the FAIHSA seems to be better in terms of the mean desirability level of 0.7001 categorised by each objective function (Fig. 2 and 3), but it is not statistically significant at 95% confidence interval (Fig. 4 and Table IV). The optimal solutions on all scenarios are shown in Table V. The numerical example shows that from all the scenarios the DM can choose various levels of production planning over the feasible ranges to be fitted to the process.

TABLE I  
EXPERIMENTAL RESULTS CATEGORISED BY THE ALGORITHMS ON THE INVENTORY LEVELS OF  $U(0, 250)$

| Algorithm | $d_1$ | $d_2$ | $d_3$ | D     | $Z_1$    | $Z_2$   |
|-----------|-------|-------|-------|-------|----------|---------|
| FAIHSA    | 0.684 | 0.309 | 1.000 | 0.596 | 32316316 | 3438219 |
| HuSIHSA   | 0.740 | 0.369 | 1.000 | 0.649 | 32259942 | 3426228 |
| IHSA      | 0.727 | 0.319 | 1.000 | 0.614 | 32272694 | 3436194 |

TABLE II  
EXPERIMENTAL RESULTS CATEGORISED BY THE ALGORITHMS ON THE INVENTORY LEVELS OF  $U(250, 500)$

| Algorithm | $d_1$ | $d_2$ | $d_3$ | D     | $Z_1$    | $Z_2$   |
|-----------|-------|-------|-------|-------|----------|---------|
| FAIHSA    | 0.600 | 0.752 | 1.000 | 0.767 | 32400419 | 4074380 |
| HuSIHSA   | 0.600 | 0.752 | 1.000 | 0.767 | 32400419 | 4074380 |
| IHSA      | 0.727 | 0.618 | 1.000 | 0.766 | 32272694 | 4114456 |

TABLE III  
EXPERIMENTAL RESULTS CATEGORISED BY THE ALGORITHMS ON THE MINIMAL INVENTORY OF 500

| Algorithm | $d_1$ | $d_2$ | $d_3$ | D     | $Z_1$    | $Z_2$   |
|-----------|-------|-------|-------|-------|----------|---------|
| FAIHSA    | 0.533 | 0.861 | 1.000 | 0.772 | 32466510 | 4384752 |
| HuSIHSA   | 0.338 | 0.835 | 1.000 | 0.656 | 32661571 | 4391344 |
| IHSA      | 0.338 | 0.835 | 1.000 | 0.656 | 32661571 | 4391344 |

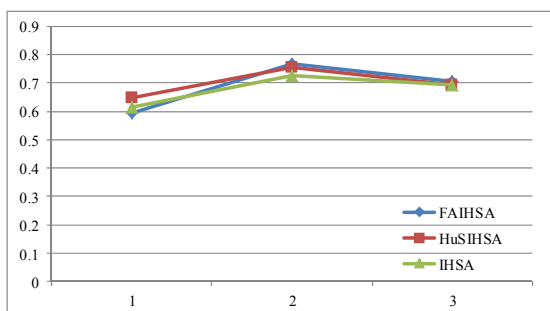


Fig. 2 Desirability Comparison among all Algorithms Categorised by Total Production Costs

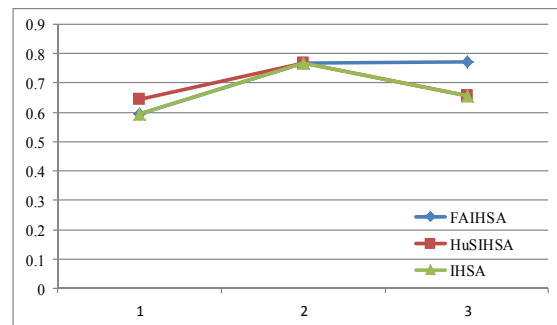


Fig. 3 Desirability Comparison among all Algorithms Categorised by Carrying or Backordering Cost

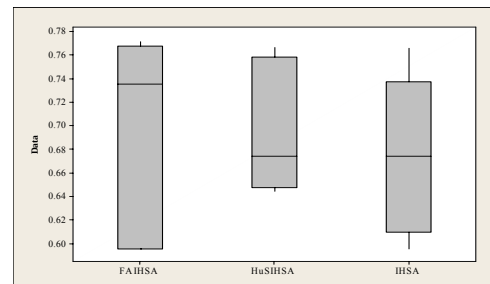


Fig. 4 Desirability Comparison among all Algorithms

TABLE IV  
ONE-WAY ANOVA: DESIRABILITY VERSUS THREE ALGORITHMS

| Source of Variation | Degree of Freedom | Sum of Square | Mean Square | F     | P-Value |
|---------------------|-------------------|---------------|-------------|-------|---------|
| Algorithm           | 0.002             | 2             | 0.001       | 0.207 | 0.815   |
| Residual            | 0.072             | 15            | 0.004       |       |         |
| Total               | 0.074             | 17            |             |       |         |

TABLE V  
OPTIMAL SOLUTIONS OF THE APP MODEL ON THREE SCENARIOS OF INVENTORY LEVELS

| Product | $W_1$ | S1 | S2 | S3 |
|---------|-------|----|----|----|
| 1       | $w_1$ | 0  | 0  | 0  |
|         | $w_2$ | 47 | 47 | 49 |
|         | $w_3$ | 56 | 56 | 57 |
|         | $w_4$ | 68 | 68 | 68 |
|         | $w_5$ | 68 | 68 | 68 |
|         | $w_6$ | 68 | 68 | 68 |
| 2       | $w_1$ | 0  | 0  | 0  |
|         | $w_2$ | 0  | 0  | 0  |
|         | $w_3$ | 0  | 0  | 0  |
|         | $w_4$ | 24 | 26 | 25 |
|         | $w_5$ | 51 | 50 | 52 |
|         | $w_6$ | 51 | 53 | 51 |
| 3       | $w_1$ | 0  | 0  | 0  |
|         | $w_2$ | 25 | 25 | 25 |
|         | $w_3$ | 45 | 41 | 41 |
|         | $w_4$ | 30 | 31 | 31 |
|         | $w_5$ | 6  | 5  | 6  |
|         | $w_6$ | 36 | 35 | 33 |

## VII. CONCLUSIONS AND DISCUSSIONS

Aggregate Production Planning (APP) deals with matching capacity and forecasted demand and varying customer orders over the medium term of a 2±18-month planning horizon, approximately. The main idea of APP is to set overall production capacity levels of detailed materials and capacity resources for a family of products to meet fluctuating or uncertain forecasted sales demands in the near future, such that APP also determines the appropriate

resources to be used. This study develops a MOLP model of the 3-product 6-period APP decision problem in a desirability approach. The proposed model aims to minimise total production costs, carrying and backordering costs, and the rates of changes in labor levels with reference to labor levels, capacity, warehouse space and the time value of money and various levels of inventory levels. The proposed model yields a compromise solution and the DM's overall levels of desirability [12]. Here, the proposed metaheuristic of improved harmony search algorithm (IHSA) and its hybridisations on the FA (FAIHSA) and HuS (HuSIHSA) mechanisms are explained in details by solving the tested problems in three scenarios. The results obtained from the numerical examples have shown that the FAIHSA generates reasonably well solutions to the present APP formulation.

Therefore, it is concluded that the FAIHSA can be considered as a promising candidate solution technique for solving multi-objective APP problems. There is no doubt that the FA is a very powerful novel population-based method for solving constrained optimisation problems. The idea behind this is that the social behavior and especially the flashing light of fireflies can be easily formulated and associated with the objective function of a given optimisation problem. The experimental results of the FAIHSA showed the efficiency and effectiveness of the firefly mechanism for solving the particular optimisation problem. The FAIHSA achieved good results comparable to those achieved by other stochastic nature-inspired algorithms. Moreover, the fact that the FAIHSA is very simple in concept and easy to implement clearly implies that it could also be effectively applied to other multi-objective optimisation problems and especially to other NP-hard combinatorial optimisation problems. However, from the simulation results, it seems that the proper selection of algorithm parameters is of importance for the convergence of the algorithm as this heavily depends on the nature of the tested problem. Moreover, a refinement and improvement of the initial candidate solution seem very promising and beneficial to further enhance the algorithm's performance, while it might also be possible to hybridise the algorithm together with other metaheuristics for better performances.

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