

A CFD Study of Turbulent Convective Heat Transfer Enhancement in Circular Pipeflow

Perumal Kumar, Rajamohan Ganesan

Abstract—Addition of milli or micro sized particles to the heat transfer fluid is one of the many techniques employed for improving heat transfer rate. Though this looks simple, this method has practical problems such as high pressure loss, clogging and erosion of the material of construction. These problems can be overcome by using nanofluids, which is a dispersion of nanosized particles in a base fluid. Nanoparticles increase the thermal conductivity of the base fluid manifold which in turn increases the heat transfer rate. Nanoparticles also increase the viscosity of the basefluid resulting in higher pressure drop for the nanofluid compared to the base fluid. So it is imperative that the Reynolds number (Re) and the volume fraction have to be optimum for better thermal hydraulic effectiveness. In this work, the heat transfer enhancement using aluminium oxide nanofluid using low and high volume fraction nanofluids in turbulent pipe flow with constant wall temperature has been studied by computational fluid dynamic modeling of the nanofluid flow adopting the single phase approach. Nanofluid, up till a volume fraction of 1% is found to be an effective heat transfer enhancement technique. The Nusselt number (Nu) and friction factor predictions for the low volume fractions (i.e. 0.02%, 0.1 and 0.5%) agree very well with the experimental values of Sundar and Sharma (2010). While, predictions for the high volume fraction nanofluids (i.e. 1%, 4% and 6%) are found to have reasonable agreement with both experimental and numerical results available in the literature. So the computationally inexpensive single phase approach can be used for heat transfer and pressure drop prediction of new nanofluids.

Keywords—Heat transfer intensification, nanofluid, CFD, friction factor

I. INTRODUCTION

MANY industrial processes involve heat transfer, which is accomplished using heat transfer fluids such as water, ethylene glycol and engine oil. Thermal properties of these fluids determine the thermal efficiency as well as the size of the equipments. Hence, many different techniques are being employed to improve the thermal properties of these fluids, especially the thermal conductivity.

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Addition of milli or micro sized solid particles is one of the very old techniques of heat transfer enhancement. Industrially, this technique is not attractive because of the inherent problems such as sedimentation, increased pressure drop, fouling and erosion of the flow channel. These problems can be overcome with nanofluids, which is a dispersion of nanosized particles in a base fluid. The nanosized particles increase the thermal conductivity of the base fluid which in turn increases the heat transfer rate. This property has attracted the attention of researchers in the past decade, though the mechanism is not fully understood yet.

A lot of work has been done recently on the forced convective heat transfer of nanofluids in pipe flow. Wen and Ding [1] studied the convective heat transfer in the entrance region under laminar regime using aluminium oxide nanofluid in a circular tube with constant heat flux. Migration of nanoparticles and the subsequent disturbance of the boundary layer were attributed to the enhancement in heat transfer rate. Zeinali Heris et al [2] compared the heat transfer enhancement by copper and aluminium oxide nanofluids in laminar pipe flow under constant wall temperature conditions and found the aluminium oxide nanofluid better than the copper oxide nanofluid. Hwang et al [3] measured pressure drop and heat transfer coefficient in fully developed laminar pipe flow using constant heat flux conditions. Based on the experimental results they showed that the experimental friction factor was in good agreement with the theoretical predictions using the Darcy equation. Whereas, the Shah equation for heat transfer coefficient prediction under constant heat flux conditions in the laminar regime was found to be inadequate for nanofluids. The enhancement in heat transfer coefficient was found to exceed the enhancement in thermal conductivity by a large margin. The flattened velocity profile caused by the particle migration to the centerline of pipe was proposed to be the possible mechanism for convective heat transfer enhancement.

Kim et al [4] conducted experiments with aluminium oxide and amorphous carbonic nanofluids in the laminar and turbulent regimes and concluded that the mechanism for heat transfer enhancement was different for the two regimes. The delaying and disturbance of the thermal boundary layer was attributed to the heat transfer enhancement in the laminar regime. Whereas, in the turbulent regime, increase in thermal conductivity was responsible for heat transfer enhancement.

Rea et al [5] investigated the laminar convective heat transfer and pressure loss for alumina – water and zirconia – water nanofluids in a uniformly heated vertical tube. Heat transfer enhancement was observed to be higher in the entrance region than in the fully developed region. The agreement between the experimental and predicted Nusselt numbers was found to be good. This prompted the authors to conclude that nanofluids behave like homogeneous mixtures and the enhancement in heat transfer was only due to the improved mixture properties with respect to that of water. Ben Mansour et al [6] experimentally investigated the thermally developing laminar mixed convection flow of water and Al_2O_3 mixture inside an inclined tube with a uniform wall heat flux. They observed that a higher particle volume concentration clearly induces a decrease of the Nusselt number for the horizontal inclination. On the other hand, for the vertical one, the Nusselt number remains nearly constant with an increase of particle volume concentration from 0 to 4%. The apparent contradictory behavior observed between experimental data and analytical/numerical results regarding the heat transfer enhancement of nanofluids prompted them to raise serious concerns regarding the applicability of using the single phase and homogeneous fluid model for nanofluids under natural convection effect. Nassan et al [7] compared the performance of Al_2O_3 – water and CuO – water nanofluids in a square duct. They also suggested further theoretical and experimental investigations to understand the heat transfer characteristics of nanofluids in noncircular ducts like triangular ducts, rectangular ducts with different aspect ratios and other possible noncircular ducts with different nanofluids.

Sundar and Sharma [8] conducted forced convective heat transfer experiments in the turbulent regime with pipes employing twisted tape inserts with and without Al_2O_3 – water nanofluid. The increase in pressure drop for nanofluids was found to be negligible, whereas considerable increase in heat transfer coefficient was observed both with and without pipe inserts. They also developed generalized correlations for the estimation of Nusselt number and friction factor for pipes with and without inserts. Farajollahi et al [9] compared the heat transfer enhancement of Al_2O_3 – water and TiO_2 - water nanofluids in a shell and tube heat exchanger. They observed different optimum volume concentration for both the nanofluids in which the heat transfer characteristics showed maximum enhancement. The nanoparticle with less mean diameter (TiO_2 nanoparticle) had a lower optimum volume concentration. Comparison of the experimental data with that predicted by Xuan and Li [10] correlation was found to have good agreement. Vajjha et al [11] used a mixture of ethylene glycol and water as base fluids to compare the heat transfer enhancement by aluminium oxide, copper oxide and silicon dioxide nanoparticles and developed generalized correlations for the prediction of Nusselt number and friction factor for turbulent pipe flow under constant heat flux conditions.

In annular duct flow, Nasiri et al [12] observed that the heat transfer performance of aluminium oxide and titanium oxide nanofluids to be similar when the concentration was same in the turbulent regime under constant wall temperature

conditions. Recently, Sonawane et al [13] observed good heat transfer enhancement with aviation turbine fuel – aluminium oxide nanofluid even at low concentrations.

Earlier numerical investigations on forced convective heat transfer considered nanofluids as a homogeneous fluid and adopted a single phase approach to predict heat transfer enhancement [14, 15]. More recently, the two phase approach has been used by some researchers, but the opinion about these two approaches is varied. Bianco et al [16] observed only a maximum of 11% difference between single and two phase results for the laminar regime. So they opined that single phase approach is good enough to test new nanofluids as it requires information about the particle and the base fluid with no reference to the mixture. Other researchers [17, 18] found the mixture model to be working better than the single phase model in the turbulent regime. Akbari et al [19] for the first time compared three different two phase models and the single phase model in the laminar regime. Single and two phase models were found to be predicting identical hydrodynamic fields but very different thermal ones.

A closer look at all the experimental and numerical works reveals that most of the forced convective heat transfer studies in pipe flow have been done with constant wall flux boundary condition. So in this work, a systematic computational fluid dynamic investigation with constant wall temperature boundary condition has been carried out adopting the single phase approach in the turbulent regime and the results are compared with the experimental and numerical results available in the literature.

II. CFD MODELLING

A. Geometry Creation and Grid Independence Study

A circular pipe of diameter 0.017 m diameter and length 10 m was used as the geometry. Grid independence study was carried out to find out the optimum grid size without compromising the accuracy of results. Different mesh sizes were tested in order to examine the effect of number of cell volumes on the Nusselt number (Fig.1).

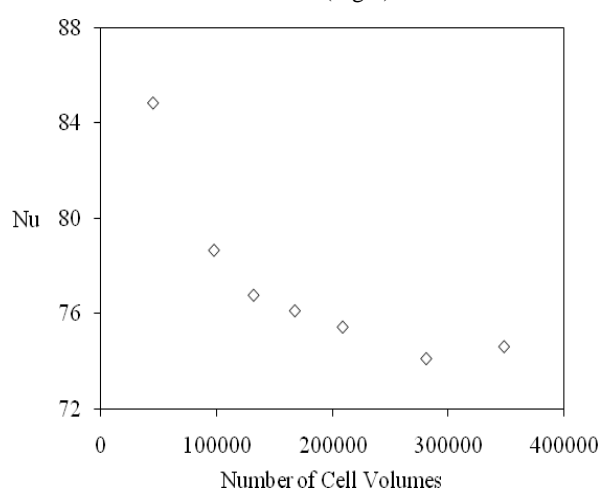


Fig. 1 Grid independence study

It can be observed that the Nusselt number for water decreases gradually and becomes constant beyond a certain number of cell volumes. Beyond this, any further increase in the number of cell volumes only increases the computational time, without any significant improvement in the Nusselt number. Similar trend was also observed with the nanofluids. So this "optimum" mesh size was selected for further study with both water and the nanofluids.

B. Governing Equations

Steady state simulations were carried out by solving mass, momentum and energy conservation equations, which are expressed as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0 \quad (1)$$

$$\frac{\partial (\rho \bar{u})}{\partial t} + \nabla \cdot (\rho \bar{u} \bar{u}) = \rho g - \nabla P + \nabla \cdot (\bar{\tau}) \quad (2)$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\bar{u} (\rho E + P)) = \nabla \cdot \left(K_{eff} \nabla T \right) \quad (3)$$

where ρ is the density, u is the velocity, p is the pressure, τ is the viscous stress tensor, E is the energy and K_{eff} is the effective thermal conductivity. The standard k - ϵ turbulence model was used to model turbulence. The k and ϵ equations are as follows:

$$\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \rho \epsilon \quad (4)$$

$$\frac{\partial}{\partial t} (\rho \epsilon) + \frac{\partial}{\partial x_i} (\rho \epsilon u_i) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] + C_{1\epsilon} \frac{\epsilon}{k} G_k - C_{2\epsilon} \rho \frac{\epsilon^2}{k} \quad (5)$$

where G_k represents the generation of turbulence kinetic energy due to the mean velocity gradients and $C_{1\epsilon}$, $C_{2\epsilon}$, σ_k and σ_ϵ are the standard k - ϵ model constants. The turbulent viscosity, μ_t is computed as follow:

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon} \quad (6)$$

Where k , ϵ and C_μ are turbulent kinetic energy, turbulent kinetic energy dissipation rate and turbulent viscosity constant, respectively.

C. Nanofluids thermophysical properties

Thermo physical properties of the Al_2O_3 nanofluid such as viscosity (μ), thermal conductivity (K), density (ρ) and specific heat (C) were estimated using the following empirical correlations developed by Pak and Cho [20]. It should be

mentioned that the particle size of the Al_2O_3 nanoparticles considered in this study is 47 nm.

$$\mu_{nf} = \mu_w (1 + 39.11\phi + 533.9\phi^2) \quad (7)$$

$$K_{nf} = K_w (1 + 7.47\phi) \quad (8)$$

$$\rho_{nf} = \phi \rho_p + (1 - \phi) \rho_w \quad (9)$$

$$C_{nf} = \phi C_p + (1 - \phi) C_w \quad (10)$$

where ϕ is particle volume fraction and the subscripts w and nf refer to water and the nanofluid respectively. In the calculations, the density of alumina and water were taken as 3970 kg/m^3 and 1000 kg/m^3 and that of specific heat of alumina and water as 880 $J/kg K$ and 4178 $J/kg K$ respectively.

D. Boundary conditions

Low (0.02%, 0.15 and 0.5%) and high volume fraction nanofluids (1%, 4% and 6%) at 315 K were used as the working fluids. For comparison purposes, water was also employed as working fluid. The numerical studies were carried out with uniform velocity profile at the inlet of the horizontal pipeline. The direction of the flow was defined normal to the boundary. Turbulent intensity, I and the hydraulic diameter, D_h were specified for an initial guess of turbulent quantities (k and ϵ). The turbulent intensity was estimated for each case based on the formula $I = 0.16(Re)^{-1/8}$ and was set at 5% from calculations. Outflow boundary condition was used at the outlet boundary. The wall of the pipe was assumed to be perfectly smooth with zero roughness height. A constant wall temperature of 289 K was used at the wall boundary.

E. Numerical Solution Strategy

The commercial CFD solver FLUENT 6.3.26 was employed to solve the governing equations with a segregated solver. The second-order upwind scheme was used for discretization of convection, energy, turbulent kinetic energy and turbulent kinetic energy dissipation rate terms. The SIMPLE algorithm was used to resolve the coupling between velocity and pressure fields. The convergence criterion is based on the residual value of calculated variables such as mass, velocity components, turbulent kinetic energy (k), turbulent kinetic energy dissipation rate (ϵ), and energy. In the present calculations, the initial residual values were set to 10^{-4} for all variables, except energy for which 10^{-6} is used. The under-relaxation factors used for the stability of the converged solutions are set at their default values. The numerical simulation was decided as converged when the sum of normalized residuals for each conservation equation and variables was less than the set residual values. However, when the residual for the continuity equation reached a minimum plateau before the value of 10^{-4} , the mass balance is monitored on the flux report and was used as a secondary indicator of convergence when the net imbalance is less than 1% of the inlet flux through the domain boundary.

F. Validation of Numerical Results

Numerical results were made credible by comparing them with data from correlations available in the literature. Nusselt number for the base fluid (water) in the turbulent regime was compared with that of Gnielinski [21] correlation (Fig. 2). Similarly, Blasius correlation [22] was used for friction factor comparison in the turbulent regime (Fig. 3). The experimental results of Sundar and Sharma [8] was also used for comparison. It can be seen from Fig. 2 that Numerical Nu are in very good agreement with the correlation values. Friction factor comparison is in the acceptable limit, though not as good as the Nu comparison.

low and high volume fractions respectively. It can be observed that presence of even a small amount of nanoparticles (0.02%) in a base fluid increases the Nusselt number by about 0.5%. As the volume fraction increases, Nusselt number increases by 1% and 6% for the other two cases (Fig 4b). This increase in heat transfer rate is attributed to the increase in the thermal conductivity of the base fluid. The increase in Nusselt number becomes more accentuated as the Reynolds number increases and this is considered significant as the pressure drop does not increase as steeply as the Nusselt number [8]. This observation suggests that the use of low volume fraction nanofluids is a very effective method of heat transfer enhancement.

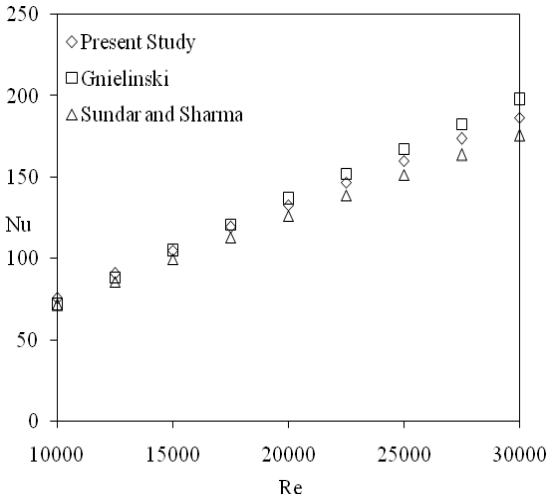


Fig. 2 Comparison of Nusselt number of water with Gnielinski correlation and the experimental values of Sundar and Sharma [8]

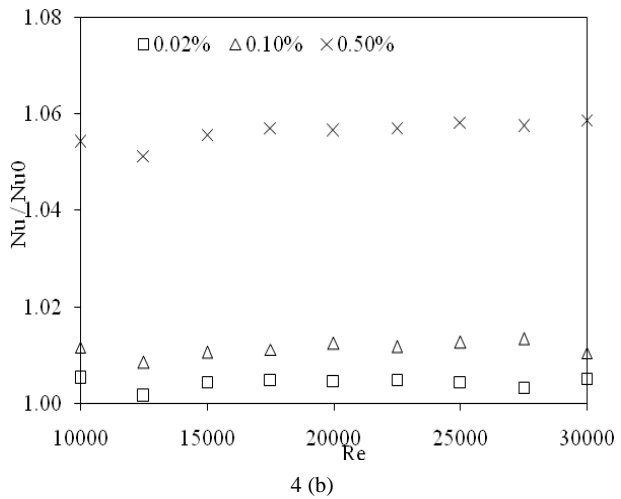
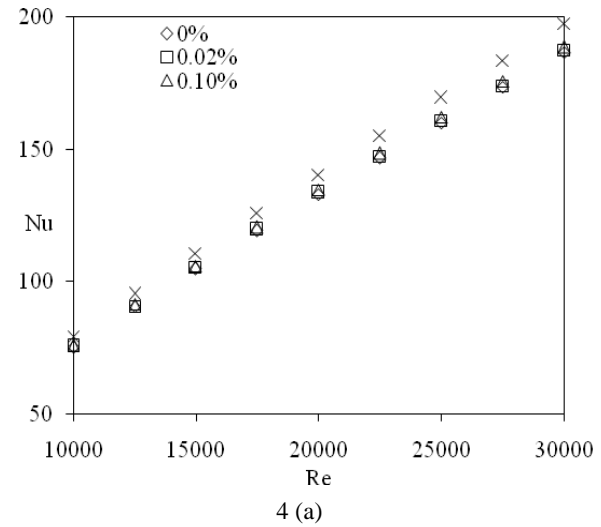


Fig. 4 Effect of volume fraction of Al₂O₃ nanofluid on (a) average Nusselt number (b) Nusselt number ratio for low volume fraction nanofluid (Nu₀ is the Nusselt number for the base fluid)

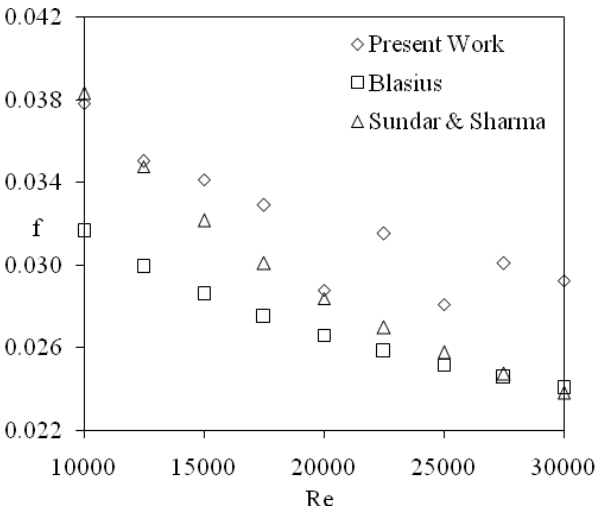
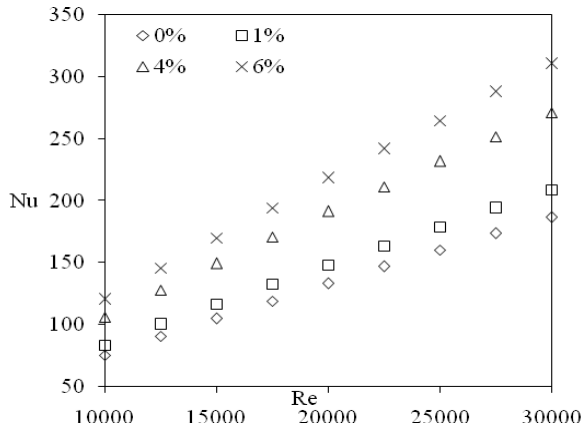


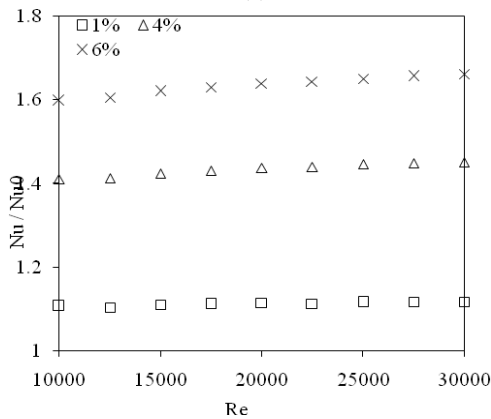
Fig. 3 Comparison of friction factor of water with Blasius correlation and the experimental values of Sundar and Sharma [8]

III. RESULTS AND DISCUSSION

The effect of nanofluid volume fraction on heat transfer enhancement is shown in Fig.4 (a, b) and Fig. 5 (a, b) for the



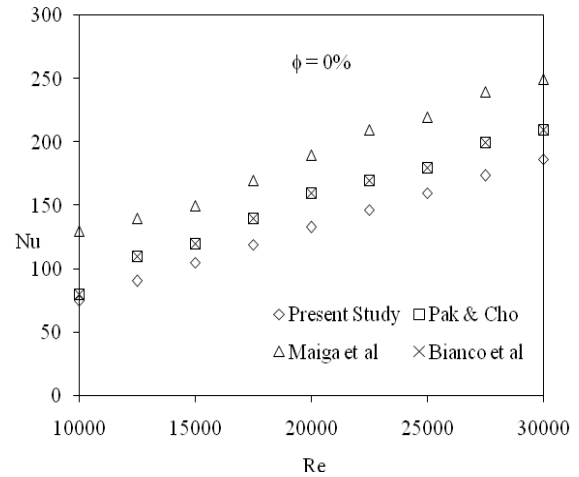
5 (a)



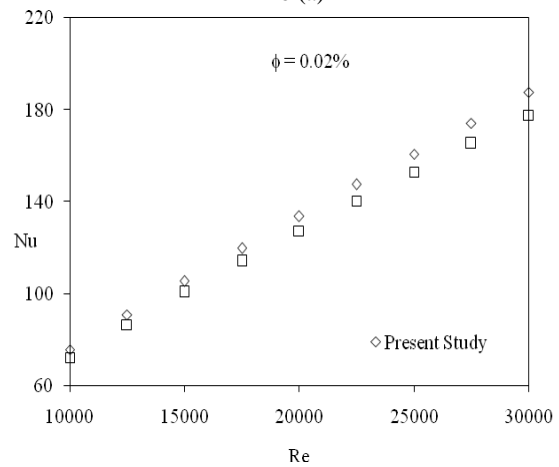
5 (b)

Fig. 5 Effect of volume fraction of Al_2O_3 nanofluid on (a) average Nusselt number (b) Nusselt number ratio for high volume fraction nanofluid

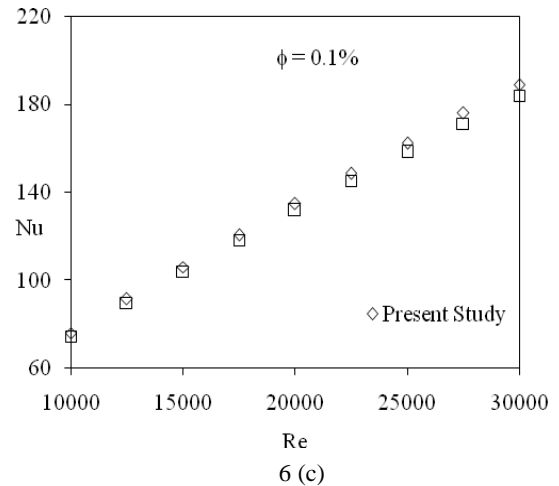
At higher volume fractions, increase in Nusselt number is 10%, 40% and 60% for the three volume fraction studied. But increase in pressure drop becomes significant and it is important to find the optimum volume fraction for each application. According to Bianco et al [18], the wall shear stress, which is an indicator of pressure drop for the 1% nanofluid is only about 10% more than the base fluid, whereas for the 4% and 6% nanofluids the increase is about 200% and 300% respectively. So it is evident that nanofluids are useful only up till a volume fraction of 1%. It should be noted here that the above mentioned authors considered 38 nm particles in their study and used the correlation of Maiga et al [14] for the estimation of viscosity.



6 (a)



6 (b)



6 (c)

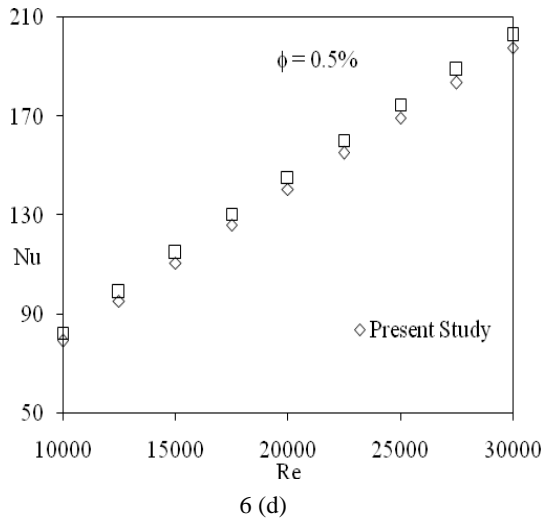


Fig. 6 Comparison of average Nusselt number with correlations available in the literature for (a) $\phi = 0\%$ (b) $\phi = 0.02\%$ (c) $\phi = 0.1\%$ (d) $\phi = 0.5\%$

Fig.6 (a, b, c, d) and Fig.7 (a, b, c) show the comparison of Nusselt number from the present study with the values of other researchers. It can be easily observed that the single phase model predictions agree very well with the experimental values of Sundar and Sharma [8] for the three volume fraction considered in this study. The comparison is good even at the volume fraction of 1% (Fig. 7a), which is outside the experimental range of the above authors. But for the 4% and 6% nanofluids the experimental correlation values are found to be bigger than the numerical values of the present study. This trend suggests that the experimental correlation is valid only for volume fractions less than 1%. Interestingly, the values of the present work particularly for the 1% and 4% agree within 10–20% of other researchers results. This difference in results is considered acceptable as reported by Buongiorno [23] and can be attributed to the difference in correlations used for the estimation of the thermophysical properties of the nanofluids.

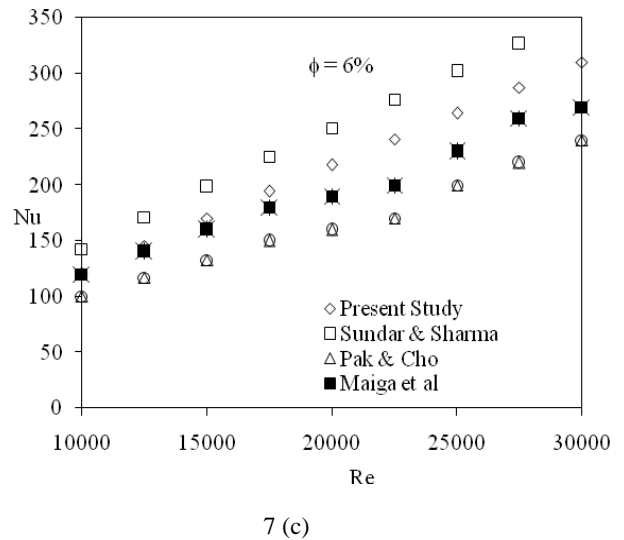
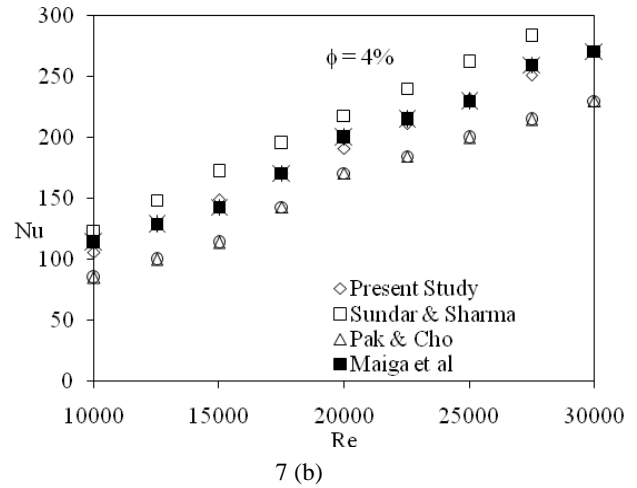
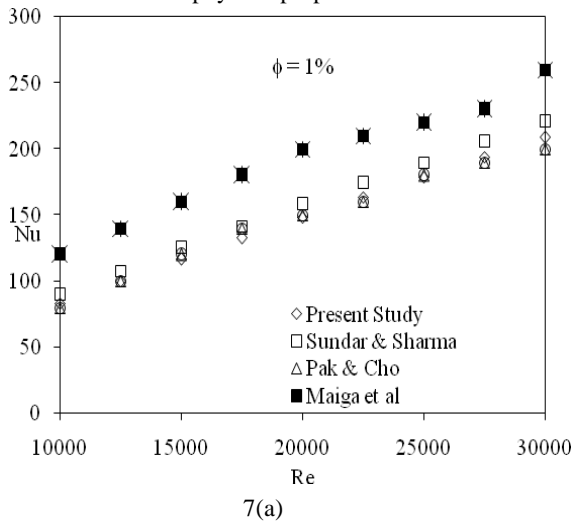
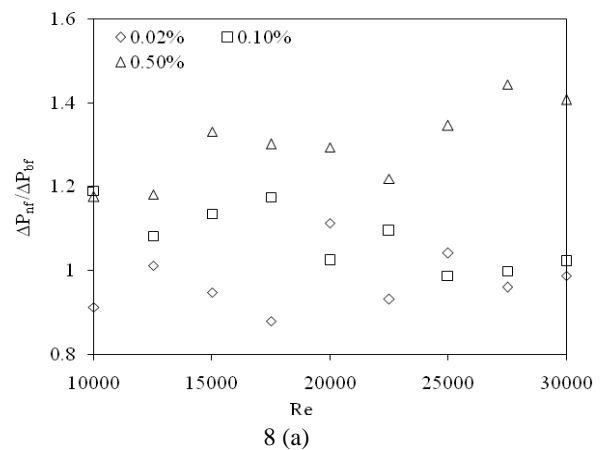


Fig. 7 Comparison of average Nusselt number with the values of Bianco et al [18] S – Single phase model predictions, M – multiphase model predictions, Maiga et al [14], Pak and Cho [20] and Sundar and Sharma [8] for (a) $\phi = 1\%$ (b) $\phi = 4\%$ (c) $\phi = 6\%$



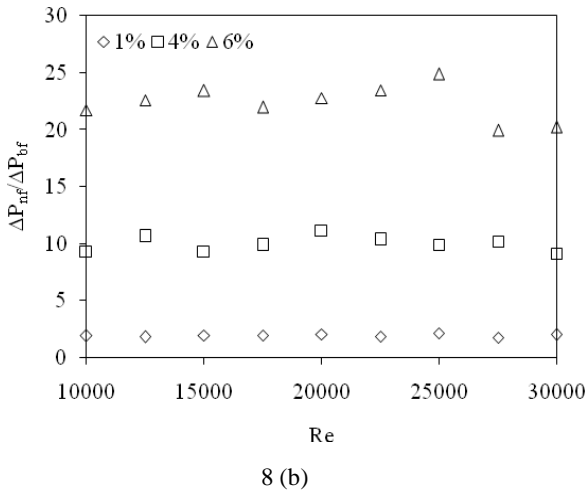
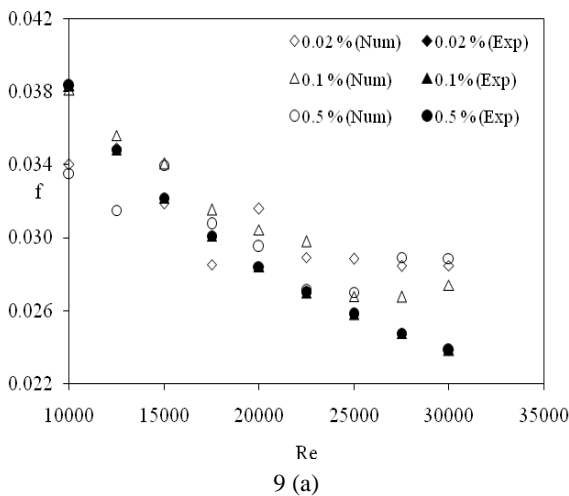


Fig. 8 Effect of volume fraction of Al_2O_3 nanofluid on pressure drop ratio (a) $\phi = 0.02\%$, 0.1% and 0.5% (b) $\phi = 1\%$, 4% and 6% (bf refers to the base fluid)

Pressure drop ratio for the low and high volume fractions are shown in Fig. 8 (a, b). It is evident that the pressure drop increases dramatically as the volume fraction increases, particularly above 1%. This can be attributed to the increase in the viscosity of nanofluid with increase in the volume fraction of the nanoparticles. Moreover, it can be observed from the results of Maiga et al [14], Lee et al [24] and Sundar and Sharma [8] that viscosity of the nanofluid increases with increasing particle size of the nanoparticles as well. Even for a volume fraction of 1%, viscosity of nanofluid made of 47 nm particles is about 33% more than the viscosity of nanofluid prepared from 35 nm particles. This difference rises to about 130% and 180% for the 4% and 6% nanofluids, respectively. This explains the exponential increase in the pressure drop for 4% and 6% nanofluids.

The friction factor values for both the low and high volume fraction nanofluids are reported in Fig. 9 (a, b). It can be observed that the numerical friction factor values are within 10 – 15% of the experimental values of Sundar and Sharma [8].



9 (a)

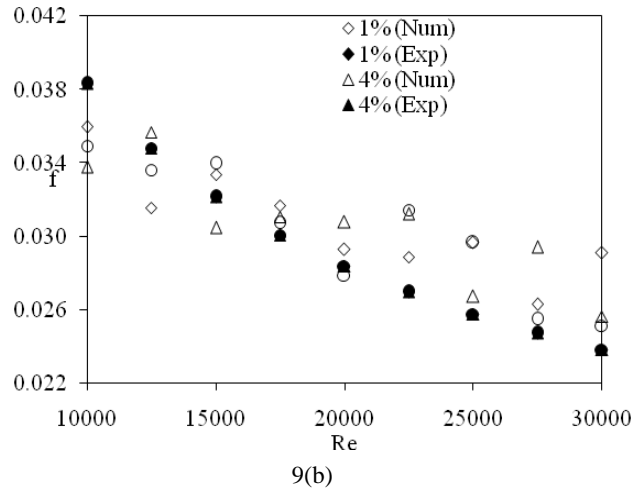


Fig. 9 Comparison of friction factor values with the experimental correlation values of Sundar and Sharma [8] for (a) $\phi = 0.02\%$, 0.1% and 0.5% (b) $\phi = 1\%$, 4% and 6%

In contrast to the Nusselt number, friction factor values are better predicted by the experimental correlation of these authors. This is due to the fact that the friction factor is only dependent on Reynolds number. So the experimental correlation will be able to predict friction factor even for higher volume fraction nanofluids with reasonable accuracy. Moreover, single phase models are found to be predicting similar hydrodynamic fields as the multiphase models. But, convective heat transfer coefficient is better predicted by the multiphase models both in the laminar and turbulent regimes [16, 17, 18, and 19]. According to Bianco et al [16] heat transfer prediction by the computationally inexpensive single phase model is acceptable, especially for testing new nanofluids as it requires only data of the particles and the base fluid. Moreover, it is highly likely that the accuracy of the single phase model prediction could be improved by using temperature dependent properties of the nanofluid.

IV. CONCLUSIONS

Heat transfer enhancement in turbulent pipe flow by Al_2O_3 nanofluid has been investigated numerically using the single phase approach for constant wall temperature boundary condition for the first time. Both the experimental values and the numerical predictions show that heat transfer enhancement increases as the volume fraction of the nanoparticles increases. But the rate of enhancement remains constant with increasing Reynolds number. Pressure drop increases dramatically for volume fraction bigger than 1%. So for better thermal hydraulic effectiveness, it is important to operate at an optimum volume fraction. The single phase model predicts convective heat transfer coefficient with reasonable accuracy and considered good enough to test new nanofluids as it is computationally inexpensive compared to the multiphase approach.

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