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Combining Ant Colony Optimization and Dynamic Programming for Solving a Dynamic Facility Layout Problem

A. Udomsakdigool and S. Bangsaranthip

Abstract**—**This paper presents an algorithm which combining ant colony optimization in the dynamic programming for solving a dynamic facility layout problem. The problem is separated into 2 phases, static and dynamic phase. In static phase, ant colony optimization is used to find the best ranked of layouts for each period. Then the dynamic programming (DP) procedure is performed in the dynamic phase to evaluate the layout set during multi-period planning horizon. The proposed algorithm is tested over many problems with size ranging from 9 to 49 departments, 2 and 4 periods. The experimental results show that the proposed method is an alternative way for the plant layout designer to determine the layouts during multi-period planning horizon.

Keywords**—**Ant colony optimization, Dynamic programming, Dynamic facility layout planning, Metaheuristic

I. Introduction

THE facility layout problem is the arrangement of departments within a facility with respect to some objectives such as the material handling cost. In an environment where material handling flows is fixed during the planning horizon, a static layout analysis would be sufficient. The solution procedure can be formulated as a quadratic assignment problem. In today's market based and dynamic environment, such flows can change quickly due to changes in the design of an existing product, the addition or deletion of a product, replacement of existing production equipment, shorter product life cycles and changes in the production quantities and associated production schedule (Shore and Tompkins [1]). This problem is known as the dynamic facility layout problem (DFLP). The DFLP involves the design of facility layouts based on a multi-period planning horizon. During this horizon, the material handling flows between pairs of departments in the layout may change. If this changes warrant it, layout re-arrangements may be planned in one or more periods. The analysis is based on the trade off between the increased flow cost of inefficient layouts and added rearrangement costs. However, dynamic layout analysis may not be justified in every situation. When the cost of layout rearrangement is negligible, dynamic layout analysis is not necessary.

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Due to the high complexity of computational of the DFLP; there are many efficient methods which can find good solutions in an acceptable time have been widely studies. Rosenblatt [2] was the first to present solution techniques for the DFLP; he developed an optimal solution methodology, identified bounding procedures, and established heuristic techniques. Urban [3] developed a steepest descent pairwise exchange technique similar to CRAFT. Conway and Venkataramanan [4] used a genetic algorithm to solve the DFLP, and Kaku and Mazzola [5] used a tabu search heuristic. Balakrishnan *et al*. [6] improved the pairwise exchange heuristic by presenting a backward-pass pairwise exchange heuristic with forecast windows. Baykasoglu and Gindy [7] presented a simulated annealing (SA) heuristic, and Balakrishnan *et al*. [8] presented a hybrid genetic algorithm for the DFLP. Dunker *et al*. [9] combined evolutionary computation and dynamic programming for solving the DFLP. McKendall *et al*.[10] developed 2 versions of heuristics based on simulated annealing heuristics for the DFLP. Recently Balakrishnan and Cheng [11] developed a steepest descent pairwise exchange heuristic in solving the rolling horizon problem. A good survey on the DFLP can be found in Balakrishnan and Cheng [12]. They gave detailed explanations about some of the available algorithms on DLP along with their comparisons.

In recent year, the newly metaheuristic, Ant Colony Optimization (ACO) has been receiving the extensive attention due to its successful applications to many combinatorial optimization problems including facility layout problem. Baykasoglu *et al*. [13] presented an ant colony algorithm for solving budget constrained and unconstrained DFLP. McKandall [14] developed hybrid ant systems for the DFLP.

In this paper the ant colony algorithm is combined in DP procedure for solving the dynamic facility plant layout problem. The remainder of this paper is organized as follows. In Section 2 we introduce a mathematical model for the DFLP. Section 3 presents the proposed algorithm. The numerical experiments is illustrated Section 4. The last section expresses the conclusions.

II. The Dynamic Facility Layout Problem

In the SFLP a group of departments are arranged into a layout in order to minimize the total cost of flow between departments under the assumption of material flows between departments is constant over time. The DFLP extends the SFLP by assuming that the material flows can change over time. This in turn might necessitate layout rearrangement during the planning horizon. The costs associated with this model are the material handling costs for each period in the planning horizon typically based on the well known quadratic assignment problem (QAP) formulation, and any rearrangement costs involved in changing the layout between periods. The rearrangement cost includes the fixed costs of move such as the cost of lost production and the variable costs such as the cost to move the machine unit distance. Thus the objective function in a DFLP is defined as the minimization of total cost of flow costs and rearrangement cost for a series of static layout problems. The total cost, C, of assigning n departments to n locations over a planning horizon of T time periods is expressed as follows:

$$C=\sum_{t=1}^{T}\left[\sum_{\substack{i=1\\ i\neq k}}^{n}\sum_{\substack{j=1\\ j\neq l}}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}F_{ikt}D_{jl}X_{ijt}X_{klt}+\sum_{i=1}^{n}S_{it}Y_{it}+R_tZ_t\right] \quad (1)$$

where F_{ikt} is the workflow cost from department i to department k in time period t, D_{jl} is the distance from location j to location l, S_{it} is the variable rearrangement cost of moving department i at the beginning of period t, and R_t is the fixed rearrangement cost associated with making any layout changes at the beginning of period t. The decision variables are

$$\sum_{i=1}^{n}X_{ijt}=\begin{cases}1 & \text{if department } i \text{ is placed at location } j \text{ in period } t\\ 0 & \text{otherwise}\end{cases}$$

$$\sum_{i=1}^{n}Y_{it}=\begin{cases}1 & \text{if department } i \text{ is moved at the beginning}\\ & \text{of period } t\\ 0 & \text{otherwise}\end{cases}$$

$$Z_t=\begin{cases}1 & \text{if any rearrangement is made at the beginning}\\ & \text{of period } t\\ 0 & \text{otherwise}\end{cases}$$

The optimal solution methodology is developed using Rosenblatt [25] dynamic programming as the following recursive relationship:

$$C_{tm}^{*}=\min\left\{C_{t-1}^{*}+R_{km}\right\}+Q_t^m \quad (2)$$

where R_{km} is the rearrangement cost from layout A_k to layout A_m ($R_{kk}=0$), Q_{tm} is the material handling cost for layout A_m in period t and C^*_{tm} is the minimum total cost for all periods up to t, where layout A_m is being used in period t ($C^*_{01}=0$, assuming a given initial layout). The rearrangement costs may consist of fixed cost that results from the disruption, or possible shutdown of its operations and/or variable costs depending only on those departments being moved.

With this model, an n location T period problem, the total number of possible solutions to the general dynamic facility layout problem is $(n!)^T$. Complete enumeration of all these possibilities to identify feasible solution and the optimal one is not practical due to the computational complexity. To restrict the state space of the model, any layout for a given period does not need to be considered such as the layout with high material handling cost.

In this paper, the problem is separated into static and dynamic phase. In static phase, the best ranked solutions for each period are considered. In this phase ant colony optimization is employed for finding the best ranked solution. Then the DP procedure is performed in the dynamic phase.

III. The Proposed Algorithm

A. Ant Colony Algorithm

Ant colony algorithms are becoming popular approaches for solving combinatorial optimization problems in the literature. A comprehensive review on ant algorithms can be found in Dorigo and Stützle [15]. Basically in ant algorithm a finite-size colony of artificial ants searches for good-quality solutions of the static facility layout (SFLP). The concept of the ant algorithm is to have a population of artificial ants that iteratively constructs solution for to the SFLP, ants assign the department to the location according to the 2-step proportional transition rule until all departments are assigned. The material handling cost is computed and a pheromone update rule is applied. When ants repeat the solution procedure for a number of iteration the solution will be emerge. The brief description is shown in Figure 1 with the details described as follows.

Step 1 Initialization: Firstly all parameters in ant colony algorithm are initialized. The algorithm is terminated when reach 2000 iterations. The number of ants is set equal to the number of departments. The exploration/exploitation weight, q_0 is 0.5. The pheromone information weight, α is 0.5. The number of best group of solution, S_{bg} equal to 20% of the number of ants. The random number, q is uniformly distributed in [0, 1]. The pheromone value, τ on each path is initialized with random values drawn from the interval (0.1, 0.25). The lower bound of pheromone value is set to a small positive constant (0.001) to prevent the algorithm from converging to a solution.

Step 2 Solution constructions: In iteration t, at a construction step the ant^k assigns the department i to location j by applying the 2 step probability transitional rule. Firstly it takes a random number q. If $q \le q_0$, the department i is chosen according to equation (3).Otherwise the department i is selected according to equation (4). Ants repeat this process until all departments are assigned to all locations. The solution will be kept in the Tabu^k. After that ant^k calculates the material handling cost, C^k as equation (1) exclude the rearrangement cost.

$$d_{ij}^{k}(t)=\begin{cases}\arg\left\{\max[\alpha\tau_{ij}^{k}(t)+(1-\alpha)\eta_{ij}^{k}]\right\} & \text{if } q\le q_0\\ \quad j\in L_i^k & \\ L \quad \text{otherwise} & \end{cases} \quad (3)$$

where i is the department which assign to location j. L_i is the list of locations that operation i can be located. L is a location selected from the random proportional transition rule defined as equation (4).

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\alpha\tau_{ij}^{k}(t)+(1-\alpha)\eta_{ij}^{k}]}{\sum_{j\in L_i^k}[\alpha\tau_{ij}^{k}(t)+(1-\alpha)\eta_{ij}^{k}]} \\ 0 \quad \text{otherwise} \end{cases} \tag{4}$$

where p_{ij} is a probability of assigning department i to location j, τ_{ij} is the pheromone trail of assigning department i to location j, η_{ij} is the heuristic information of assigning department i to location j. The η_{ij} is calculated as equation (5).

$$n_{ij} = \frac{1}{\sum_{l=1}^{A}\sum_{k=1}^{S} f_{ik} d_{jl} X_{ij} X_{kl}} \tag{5}$$

where A is the set of departments which already assigned, S is the set of locations which assign to departments in A.

Step 3 Local improvement: ant^k explores the best solution from a neighborhood solution by swapping procedure. Two different departments are randomly selected, swap their locations, and calculated the material handling cost. If a better solution has been found accept the swap and update the Tabu^k. If the swapping is not satisfied try another swapping until no swapping can be done.

Step 4 Pheromone updating: After all the ants complete their solutions, the best solution in iteration, S_i is compared with the best solution found since the start of algorithm, S_s and the best one is used to update its pheromone matrix. The rule of pheromone updating is defined as equation (6).

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \rho\vartriangle\tau_{ij}(t) \tag{6}$$

where $\vartriangle\tau_{ij}(t) = \begin{cases} 1 & \text{if } (i,j)\in \text{Tabu}^k \text{ of } S_s \\ 0 & \text{otherwise} \end{cases}$

B. The Dynamic Procedure

The output from ant colony procedure is the best ranked static solutions of each period. Then dynamic procedure is applied to find the set of layout while considering all period during a multi-period planning horizon. The recursive cost function as illustrate in equation (2) is calculated. The best set of layout is the set that minimize the total cost including material handing cost and rearrangement cost.

Input: A problem instance of the DFLP
/*Ant colony Procedure for SFLP*/
/ Step 1: Initialization */*
*/*Set Parameters value*/*
Set number of iteration, $N_t = t_{\max}$
Set number of ants, $N_a = a$
Set Exploration/Exploitation weight, $q_0 = q_{0c}$
Set Pheromone information weight, $\alpha = \alpha_c$
Set Pheromone evaporating rate, $\rho = \rho_c$
Set Number of best group solution $S_{bg} = b$
*/*initialize pheromone value*/*
For each edge (i,j) **do**
    Set an initial pheromone value $\tau_{ij}(t_0) = \tau_0$
End for
*/*Main loop*/*
/ Step 2: Solution Construction */*
Set Best solution since start algorithm, $S_s(t_0) = \phi$
Set Best solution in iteration, $S_i(t_0) = \phi$
For $t = 1$ to t_{max} **do**
    For $k = 1$ to a **do**
        Set Tabu list , $Tabu^k = \phi$
        Set Department list, D^k= all department
        Set Lacation , L^k= all location
    End for
    For $k = 1$ to a **do**
    / Build the solution for each ant */*
    For ant $k = 1$ to a **do**
        Ant builds a tour step by step until $D^k = \phi$, $L^k = \phi$ by apply the following steps:
            Ant randomly choose q number, q = rand$(0,1)$
            Choose the location j from L^k according to equation (3) If $q \le q_0$, Otherwise equation (4)
            Keep department i and location j in Tabk^k and delete department i from D^k ,location j from L^k
            Compute the material handling *cost* C^k according to equation (1) where the rearranged costs are ignored
    End for
    / Step 3: local improvement, option */*
    For ant $k = 1$ to a **do**
        Apply local improvement by swapping two different departments.
        If an improve is found **then**
            Update solution in Tabu^k
    End for
    For ant $k = 1$ to a **do**
        Select best solution of ants in iteration t, $S_i(t)$
    End for
    Update the $S_i(t)$
    Update the *best solution since start algorithm,* $S_s(t)$
    Update the best group of solutions, $S_g(t)$
    / Step 5: Update pheromone trial */*
    For each edge (i,j) in Tabu^k of $S_s(t)$**do**
        Update pheromone trials according to the equation (4)
    End for
End for
Output of Ant Procedure: Best ranked of static solutions
/*Dynamic Procedure for DPLP*/
Calculate the recursive cost function according to the equation (2) **Output of Program**: Best set of layout during time horizon for the dynamic problem

Fig. 1 Procedure of the proposed algorithm

IV. Computational Study

A. Problem Instance

The computational study is carried out with the problem size ranging from 9 departments to 49 departments for 2 and 4

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periods. The tested problem assumes equal department sizes and deterministic material flow. The material flow data and cost move data are generated from the random number as follows

The density of material flow between each pair of departments is divided into three groups: high, medium and low. The number of forward flow is equal to the random number from 100 to 120, 50 to 60 and 10 to 20 for high, medium and low density respectively. The number of reverse flows (such as reworks) between those pair of departments is 10% of the number of forward flow. The material flows are changed for each period.

The cost move is a random number drawn from 1 to 5. The variable rearrangement cost of each department is around 0.01% to 0.05% of total cost flow of the initial solution which generate from ant algorithm while the fixed rearrangement cost is 0.01%.

B. Results

The proposed algorithm is coded in C and tested on an Intel(R) Core(TM) 2 Duo 2 GHz 3GB RAM Window platform. Firstly the problems with 2 periods are tested then the 4 period problems are tested. Each problem is tested for ten trials and kept the best one. The results are shown in Table I. The example of layout obtained from the proposed algorithm is illustrated in Fig. 1. From Fig.1 each layout is solved independently using ant colony algorithm. They are changed 3 times during the 4 periods. None of these layouts are statically optimal in their respective periods. The total 4 period cost of this plan is $439177.60.

TABLE I THE COMPUTATIONAL RESULTS

2 period problem			
Dimension	No. of departments	Cost ($)	Time (sec)
3x3	9	15036.46	1
4x4	16	82646.79	2
5x5	25	215917.56	5
6x6	36	590080.42	12
7x7	49	1304515.35	32
5x2	10	22508.03	1
6x3	18	111892.80	2
7x4	28	322999.27	6
4 period problem			
Dimension	No. of departments	Cost	Time (sec)
3x3	9	32627.04	<1
4x4	16	162738.89	2
5x5	25	439177.60	9
6x6	36	1212149.34	26
7x7	49	2692189.32	68
5x2	10	45967.84	1
6x3	18	219026.08	4
7x4	28	647594.10	13

20	8	5	4	18
25	24	11	8	20
23	21	5	17	22
16	14	12	19	9
18	13	3	15	10
7	1	2	6	4

(a) 1st period

1	16	25	10	4
25	5	1	24	23
6	22	21	20	11
18	2	14	19	3
15	4	17	8	16
9	13	12	10	7

(b) 2nd period

25	2	6	3	23
8	7	24	9	11
17	22	18	13	21
20	19	15	14	16
5	4	12	1	10

(c) 3rd period

25	16	17	9	3
1	7	12	24	23
21	2	22	11	4
20	18	19	14	15
5	10	8	13	6

(d) 4th period

Fig. 1 The layouts of 25 departments, 4 period problem

When the layout is rearranged, the savings in material handling cost would have been more than offset by the increased layout rearrangement costs as the example shown in Table II. For 2-period problem cost can be saved up to 10% on new arrangement.

TABLE II THE COST SAVING WHEN REARRANGE LAYOUT FOR 2-PERIOD PROBLEM

Dimension	No. of departments	Re-Layout	Do not change	Save	%
3x3	9	15036.46	16424.03	1387.57	9.23
4x4	16	82646.79	84447.51	1800.72	2.18
5x5	25	215917.56	222012.60	6095.04	2.82
6x6	36	590080.42	611580.26	21499.84	3.64
7x7	49	1304515.35	1338497.27	33981.92	2.60
5x2	10	22508.03	24149.66	1641.63	7.29
6x3	18	111892.80	119850.05	7957.25	7.11
7x4	28	322999.27	339500.14	16500.87	5.11

V. CONCLUSION

This paper presents a heuristic algorithm for the DFLP as a quadratic assignment problem (QAP). The algorithm is based on ant colony algorithm and DP procedure. The ant colony algorithm is used to solve the SFLP and then the DP procedure is performed in order to find the layouts during the multi-period time horizon. The proposed algorithm is tested on many problems. The results shown that the solution can be found in a less time. The layouts obtained can save the excess material handling more than the costs of rearrangements. This algorithm is an alternative method for solving the DFLP. For further study the topics should be included the case of unequal department areas and multiple floors. Also the multiple objective cases can be modeled and solved.

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