

# Further Investigations on Higher Mathematics Scores for Chinese University Students

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**Abstract**—Recently, X. Ge and J. Qian investigated some relations between higher mathematics scores and calculus scores (resp. linear algebra scores, probability statistics scores) for Chinese university students. Based on rough-set theory, they established an information system  $S = (U, C \cup D, V, f)$ . In this information system, higher mathematics score was taken as a decision attribute and calculus score, linear algebra score, probability statistics score were taken as condition attributes. They investigated importance of each condition attribute with respect to decision attribute and strength of each condition attribute supporting decision attribute. In this paper, we give further investigations for this issue. Based on the above information system  $S = (U, C \cup D, V, f)$ , we analyze the decision rules between condition and decision granules. For each  $x \in U$ , we obtain support (resp. strength, certainty factor, coverage factor) of the decision rule  $C \rightarrow_x D$ , where  $C \rightarrow_x D$  is the decision rule induced by  $x$  in  $S = (U, C \cup D, V, f)$ . Results of this paper gives new analysis of on higher mathematics scores for Chinese university students, which can further lead Chinese university students to raise higher mathematics scores in Chinese graduate student entrance examination.

**Keywords**—Rough set, support, strength, certainty factor, coverage factor.

## I. INTRODUCTION

“It is a capital mistake to theorize before one has data”. This is a Sherlock Holmes’ motto, which appeared in the story “A Scandal in Bohemia”. No doubt that the most famous contribution to reasoning from data should be attributed to the renowned Sherlock Holmes, whose mastery of using data in reasoning has been well known world wide for over hundred years. How to extract and analyze useful information hidden data? Z. Pawlak proposed a logic-mathematical method: rough-set theory [8]. This theory has shown to be an effective tool in solving the above question. In recent years, rough-set theory has been widely implemented in the many fields of natural science and societal science [1], [2], [4], [6], [7], [11], [16], [20], [21], [22], [23], [24]. This paper gives an interesting application in education by rough-set theory.

In [5], X. Ge and J. Qian investigated some relations between higher mathematics scores in Chinese graduate student entrance examination and calculus scores (resp. linear algebra scores, probability statistics scores) in subject’s completion examination of Chinese university for Chinese university students. Based on rough-set theory, they select 20 students as a sample to established an information system  $S = (U, C \cup D, V, f)$ . In this information system, higher mathematics score was taken as a decision attribute and

calculus score, linear algebra score, probability statistics score were taken as condition attributes. In their investigations, 20 students were taken as a totality. they investigated importance of each condition attribute with respect to decision attribute and strength of each condition attribute supporting decision attribute. Note that higher mathematics scores, calculus scores, linear algebra scores and probability statistics scores each student obtained are not only represent the population but also individual information. It is natural to consider the following question.

**Question 1.1:** For each student in the above sample, how to characterize degree of calculus scores, linear algebra scores and probability statistics scores implying higher mathematics scores?

To give some answers of Question 1.1, this paper further investigates information system  $S = (U, C \cup D, V, f)$  established by X. Ge and J. Qian in [5]. we analyze the decision rules between condition granules and decision granules in  $S = (U, C \cup D, V, f)$ . For each  $x \in U$ , we obtain support (resp. strength, certainty factor, coverage factor) of the decision rule  $C \rightarrow_x D$ , where  $C \rightarrow_x D$  is the decision rule induced by  $x$  in  $S = (U, C \cup D, V, f)$ . This paper gives new analysis of on higher mathematics scores for Chinese university students, which can further lead Chinese university students to raise higher mathematics scores in Chinese graduate student entrance examination.

## II. PROPAEDEUTICS

In this section, we recall basic concepts for rough set theory and decision rule [8], [9], [10], [11], [12], [13], [17], [18].

**Notation 2.1:** (1) For a set  $B$ ,  $|B|$  denotes the cardinal of  $B$ .

(2) For a family of sets  $\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_k$ ,  $\bigwedge \{\mathcal{F}_i : i = 1, 2, \dots, k\} = \{\bigcap \{F_i : i = 1, 2, \dots, k\} : F_i \in \mathcal{F}_i, i = 1, 2, \dots, k\}$ .

(3) Let  $R$  be an equivalence relation on a set  $U$ .  $U/R$  denotes the family consisting of all equivalence classes with respect to  $R$  and  $[u]$  denotes the equivalence class with respect to  $R$  containing  $u \in U$ .

(4) Let  $\mathcal{R}$  be a family of equivalence relations on  $U$ . Then  $\bigwedge \{U/R : R \in \mathcal{R}\}$  is a partition of  $U$  and is denoted by  $U/\mathcal{R}$ . The equivalence relation induced by  $U/\mathcal{R}$  is also denoted by  $\mathcal{R}$ .

**Definition 2.2:**  $S = (U, C \cup D, V, f)$  is called an information system.

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(1)  $U$ , a nonempty finite set, is called the universe of discourse.

(2)  $A = C \cup D$  is a finite set of attributes, where  $C$  and  $D$  are disjoint nonempty sets of condition attributes and decision attributes respectively.

(3)  $f: U \times A \longrightarrow V$  is an information function.

(4)  $V = \bigcup \{V_\alpha : \alpha \in A\}$ , where  $V_\alpha = \{f(u, \alpha) : u \in U\}$ .

**Remark 2.3:** An information system  $S = (U, C \cup D, V, f)$  can be expressed a data table, which is called decision table, whose columns are labeled by elements of  $A$ , rows are labeled by elements of  $U$ , and  $f(u, \alpha)$  lies in the cross of the row labeled by  $u$  and the column labeled by  $\alpha$ .

**Notation 2.4:** Let  $S = (U, C \cup D, V, f)$  be an information system.

(1) For  $a \in C \cup D$ , we define an equivalence relation  $\sim$  on  $U$  as follows:

$$u_i \sim u_j \iff f(u_i, a) = f(u_j, a).$$

$U/a$  denotes the family consisting of all equivalence classes with respect to  $\sim$ .

(2) For  $B \subset C \cup D$ ,  $\bigwedge \{U/b : b \in B\}$  is a partition of  $U$ , which is denoted  $U/B$ . The equivalence relation induced by  $U/B$  is also denoted by  $B$ .

**Definition 2.5:** Let  $S = (U, C \cup D, V, f)$  be an information system and  $B \subset C \cup D$ .

(1) An equivalence class of the partition  $U/B$  containing  $x \in U$  is denoted by  $B(x)$  and called  $B$ -granule induced by  $x$ .

(2)  $C(x)$ ,  $D(x)$  and  $(C \cup D)(x)$  are called the condition granule, the decision granule and the condition-decision granule induced by  $x$ , respectively.

**Definition 2.6:** Let  $S = (U, C \cup D, V, f)$  be an information system and  $x \in U$ . A sequence  $f(x, c_1), f(x, c_2), \dots, f(x, c_n), f(x, d_1), f(x, d_2), \dots, f(x, d_m)$  is called a decision rule induced by  $x$  in  $S = (U, C \cup D, V, f)$ , where  $\{c_1, c_2, \dots, c_n\} = C$  and  $\{d_1, d_2, \dots, d_m\} = D$ .

**Remark 2.7:** Let  $S = (U, C \cup D, V, f)$  be an information system and  $x \in U$ . The decision rule induced by  $x$  in  $S = (U, C \cup D, V, f)$  is denoted by  $f(x, c_1), f(x, c_2), \dots, f(x, c_n) \longrightarrow f(x, d_1), f(x, d_2), \dots, f(x, d_m)$  or in short  $C \longrightarrow_x D$ .

**Definition 2.8:** Let  $S = (U, C \cup D, V, f)$  be an information system and  $x \in U$ .

$$\text{Put } (C(x)) = \frac{|C(x)|}{|U|} \text{ and } (D(x)) = \frac{|D(x)|}{|U|}.$$

(1) The number  $\text{supp}_x(C, D)$  is called a support of the decision rule  $C \longrightarrow_x D$ , where  $\text{supp}_x(C, D) = |C(x) \cap D(x)|$ .

(2) The number  ${}_x(C, D)$  is called the strength of the decision rule  $C \longrightarrow_x D$ , where  ${}_x(C, D) = \frac{\text{supp}_x(C, D)}{|U|}$ .

(3) The number  $\text{cer}_x(C, D)$  is called a certainty factor of the decision rule  $C \longrightarrow_x D$ , where  $\text{cer}_x(C, D) = \frac{{}_x(C, D)}{(C(x))}$ .

(4) The number  $\text{cov}_x(C, D)$  is called a coverage factor of the decision rule  $C \longrightarrow_x D$ , where  $\text{cov}_x(C, D) = \frac{{}_x(C, D)}{(D(x))}$ .

TABLE I  
DECISION TABLE

$U$	$c_1$	$c_2$	$c_3$	$d$
$u_1$	$c_{11}$	$c_{12}$	$c_{13}$	$d_1$
$u_2$	$c_{11}$	$c_{22}$	$c_{13}$	$d_1$
$u_3$	$c_{11}$	$c_{12}$	$c_{23}$	$d_1$
$u_4$	$c_{11}$	$c_{22}$	$c_{13}$	$d_1$
$u_5$	$c_{11}$	$c_{12}$	$c_{23}$	$d_1$
$u_6$	$c_{21}$	$c_{12}$	$c_{13}$	$d_2$
$u_7$	$c_{31}$	$c_{12}$	$c_{13}$	$d_2$
$u_8$	$c_{31}$	$c_{12}$	$c_{13}$	$d_3$
$u_9$	$c_{31}$	$c_{32}$	$c_{33}$	$d_3$
$u_{10}$	$c_{31}$	$c_{32}$	$c_{13}$	$d_3$
$u_{11}$	$c_{31}$	$c_{12}$	$c_{33}$	$d_2$
$u_{12}$	$c_{31}$	$c_{12}$	$c_{33}$	$d_3$
$u_{13}$	$c_{11}$	$c_{32}$	$c_{13}$	$d_1$
$u_{14}$	$c_{11}$	$c_{12}$	$c_{33}$	$d_1$
$u_{15}$	$c_{11}$	$c_{22}$	$c_{13}$	$d_1$
$u_{16}$	$c_{21}$	$c_{12}$	$c_{13}$	$d_1$
$u_{17}$	$c_{21}$	$c_{12}$	$c_{13}$	$d_2$
$u_{18}$	$c_{21}$	$c_{22}$	$c_{13}$	$d_1$
$u_{19}$	$c_{21}$	$c_{12}$	$c_{23}$	$d_1$
$u_{20}$	$c_{21}$	$c_{22}$	$c_{23}$	$d_1$

**Remark 2.9:** For an information system  $S = (U, C \cup D, V, f)$ , it is clear that  $\text{supp}_x(C, D) = |(C \cup D)(x)|$  for each  $x \in U$ .

**Remark 2.10:** Let  $S = (U, C \cup D, V, f)$  be an information system and  $x \in U$ .

(1) If  $\text{cer}_x(C, D) = 1$ , then  $C \longrightarrow_x D$  is called a certain decision rule.

(2) If  $0 < \text{cer}_x(C, D) < 1$ , then  $C \longrightarrow_x D$  is called an uncertain decision rule.

### III. DECISION TABLE

Throughout this section and next section, information system  $S = (U, C \cup D, V, f)$  is expressed as Table I (Decision Table), which is established by X. Ge and J. Qian in [5]. Here  $U = \{u_1, u_2, \dots, u_{20}\}$ ,  $C = \{c_1, c_2, c_3\}$ ,  $D = \{d\}$ ,  $f$  and  $V$  are given as Definition 2.2 and Remark 2.3.

Now we give some explanations for Table I (see [5]).

**Remark 3.1:**  $U$  is the sample of 20 students.

**Remark 3.2:**  $c_1, c_2, c_3$  are three condition attributes in the information system, i.e.,  $c_1, c_2, c_3$  denote linear algebra score, calculus score and probability statistics score respectively.  $d$  is the decision attribute in the information system, i.e.,  $d$  denotes higher mathematics score.

**Remark 3.3:** Calculus scores for 20 students come from Yancheng Teachers University subject's completion examination.

(1)  $c_{11}$  indicates score lower than 60.

(2)  $c_{21}$  indicates score between 60 and 80.

(3)  $c_{31}$  indicates score between 81 and 100.

**Remark 3.4:** Linear algebra scores for 20 students come from Yancheng Teachers University subject's completion examination.

(1)  $c_{12}$  indicates score lower than 60.

- (2)  $c_{22}$  indicates score between 60 and 80.  
 (3)  $c_{32}$  indicates score between 81 and 100.

*Remark 3.5:* probability statistics scores for 20 students come from Yancheng Teachers University subject's completion examination.

- (1)  $c_{13}$  indicates score lower than 60.  
 (2)  $c_{23}$  indicates score between 60 and 80.  
 (3)  $c_{33}$  indicates score between 81 and 100.

*Remark 3.6:* Higher mathematics scores for 20 students come from higher mathematics examination simulated Chinese graduate student entrance examination.

- (1)  $d_1$  indicates score lower than 90.  
 (2)  $d_2$  indicates score between 90 and 120.  
 (3)  $d_3$  indicates score between 120 and 150.

*Proposition 3.7:* The following are some related partitions of  $U$ .

- (1)  $U/c_1 = \{\{u_1, u_2, u_3, u_4, u_5, u_{13}, u_{14}, u_{15}\}, \{u_6, u_{16}, u_{17}, u_{18}, u_{19}, u_{20}\}, \{u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}\}$ .  
 (2)  $U/c_2 = \{\{u_1, u_3, u_5, u_6, u_7, u_8, u_{11}, u_{12}, u_{14}, u_{16}, u_{17}, u_{19}\}, \{u_2, u_4, u_{15}, u_{18}, u_{20}\}, \{u_9, u_{10}, u_{13}\}\}$ .  
 (3)  $U/c_3 = \{\{u_1, u_2, u_4, u_6, u_7, u_8, u_{10}, u_{13}, u_{15}, u_{16}, u_{17}, u_{18}\}, \{u_3, u_5, u_{19}, u_{20}\}, \{u_9, u_{11}, u_{12}, u_{14}\}\}$ .  
 (4)  $U/C = \{\{u_1\}, \{u_2, u_4, u_{15}\}, \{u_3, u_5\}, \{u_6, u_{16}, u_{17}\}, \{u_7, u_8\}, \{u_9\}, \{u_{10}\}, \{u_{11}, u_{12}\}, \{u_{13}\}, \{u_{14}\}, \{u_{18}\}, \{u_{19}\}, \{u_{20}\}\}$ .  
 (5)  $U/D = \{\{u_1, u_2, u_3, u_4, u_5, u_{13}, u_{14}, u_{15}, u_{16}, u_{18}, u_{19}, u_{20}\}, \{u_6, u_7, u_{11}, u_{17}\}, \{u_8, u_9, u_{10}, u_{12}\}\}$ .  
 (6)  $U/(C \cup D) = \{\{u_1\}, \{u_2, u_4, u_{15}\}, \{u_3, u_5\}, \{u_6, u_{17}\}, \{u_7\}, \{u_8\}, \{u_9\}, \{u_{10}\}, \{u_{11}\}, \{u_{12}\}, \{u_{13}\}, \{u_{14}\}, \{u_{16}\}, \{u_{18}\}, \{u_{19}\}, \{u_{20}\}\}$ .

#### IV. DECISION RULES

At first, we give condition granules, decision granules and condition-decision granules of  $S = (U, C \cup D, V, f)$ , respectively. The following three propositions can be obtained by Proposition 3.7 immediately.

*Proposition 4.1:* For information system  $S = (U, C \cup D, V, f)$  and  $x \in U$ , we have the following condition granules.

- (1)  $C(x) = \{x\}$  for each  $x \in \{u_1, u_9, u_{10}, u_{13}, u_{14}, u_{18}, u_{19}, u_{20}\}$ .  
 (2)  $C(x) = \{u_2, u_4, u_{15}\}$  for each  $x \in \{u_2, u_4, u_{15}\}$ .  
 (3)  $C(x) = \{u_3, u_5\}$  for each  $x \in \{u_3, u_5\}$ .  
 (4)  $C(x) = \{u_6, u_{16}, u_{17}\}$  for each  $x \in \{u_6, u_{16}, u_{17}\}$ .  
 (5)  $C(x) = \{u_7, u_8\}$  for each  $x \in \{u_7, u_8\}$ .  
 (6)  $C(x) = \{u_{11}, u_{12}\}$  for each  $x \in \{u_{11}, u_{12}\}$ .

*Proposition 4.2:* For information system  $S = (U, C \cup D, V, f)$  and  $x \in U$ , we have the following decision granules.

- (1)  $D(x) = \{u_1, u_2, u_3, u_4, u_5, u_{13}, u_{14}, u_{15}, u_{16}, u_{18}, u_{19}, u_{20}\}$  for each  $x \in \{u_1, u_2, u_3, u_4, u_5, u_{13}, u_{14}, u_{15}, u_{16}, u_{18}, u_{19}, u_{20}\}$ .  
 (2)  $D(x) = \{u_6, u_7, u_{11}, u_{17}\}$  for each  $x \in \{u_6, u_7, u_{11}, u_{17}\}$ .

- (3)  $D(x) = \{u_8, u_9, u_{10}, u_{12}\}$  for each  $x \in \{u_8, u_9, u_{10}, u_{12}\}$ .

*Proposition 4.3:* For information system  $S = (U, C \cup D, V, f)$  and  $x \in U$ , we have the following condition-decision granules.

- (1)  $(C \cup D)(x) = \{x\}$  for each  $x \in \{u_1, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{16}, u_{18}, u_{19}, u_{20}\}$ .  
 (2)  $(C \cup D)(x) = \{u_2, u_4, u_{15}\}$  for each  $x \in \{u_2, u_4, u_{15}\}$ .  
 (3)  $(C \cup D)(x) = \{u_3, u_5\}$  for each  $x \in \{u_3, u_5\}$ .  
 (4)  $(C \cup D)(x) = \{u_6, u_{17}\}$  for each  $x \in \{u_6, u_{17}\}$ .

Now we give characterizations of decision rules for information system  $S = (U, C \cup D, V, f)$  by a support, the strength, a certainty factor and a coverage factor of the decision rule  $C \rightarrow_x D$  for each  $x \in U$ .

By Proposition 4.3 and Remark 2.9, we have a support of the decision rule  $C \rightarrow_x D$  for each  $x \in U$ .

*Theorem 4.4:* The following hold for information system  $S = (U, C \cup D, V, f)$  and  $x \in U$ .

- (1)  $\text{supp}_x(C, D) = 1$  for each  $x \in \{u_1, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{16}, u_{18}, u_{19}, u_{20}\}$ .  
 (2)  $\text{supp}_x(C, D) = 3$  for each  $x \in \{u_2, u_4, u_{15}\}$ .  
 (3)  $\text{supp}_x(C, D) = 2$  for each  $x \in \{u_3, u_5, u_6, u_{17}\}$ .

By Theorem 4.4, we have the strength of the decision rule  $C \rightarrow_x D$  for each  $x \in U$ .

*Theorem 4.5:* The following hold for information system  $S = (U, C \cup D, V, f)$  and  $x \in U$ .

- (1)  $x(C, D) = \frac{\text{supp}_x(C, D)}{|U|} = 1/20 = 0.050$  for each  $x \in \{u_1, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{16}, u_{18}, u_{19}, u_{20}\}$ .  
 (2)  $x(C, D) = \frac{\text{supp}_x(C, D)}{|U|} = 3/20 = 0.150$  for each  $x \in \{u_2, u_4, u_{15}\}$ .  
 (3)  $x(C, D) = \frac{\text{supp}_x(C, D)}{|U|} = 2/20 = 0.100$  for each  $x \in \{u_3, u_5, u_6, u_{17}\}$ .

The following lemma can be obtained from Definition 2.8 immediately.

*Lemma 4.6:* The following hold for information system  $S = (U, C \cup D, V, f)$  and  $x \in U$ .

- (1)  $\text{cer}_x(C, D) = \frac{\text{supp}_x(C, D)}{|C(x)|}$ .  
 (2)  $\text{cov}_x(C, D) = \frac{\text{supp}_x(C, D)}{|D(x)|}$ .

By Lemma 4.6, theorem 4.4, Proposition 4.1 and Proposition 4.2, we have a certainty factor and a coverage factor of the decision rule  $C \rightarrow_x D$  for each  $x \in U$ .

*Theorem 4.7:* The following hold for information system  $S = (U, C \cup D, V, f)$  and  $x \in U$ .

- (1)  $\text{cer}_x(C, D) = \frac{\text{supp}_x(C, D)}{|C(x)|} = \frac{1}{1} = 1.000$  for each  $x \in \{u_1, u_9, u_{10}, u_{13}, u_{14}, u_{18}, u_{19}, u_{20}\}$ .  
 (2)  $\text{cer}_x(C, D) = \frac{\text{supp}_x(C, D)}{|C(x)|} = \frac{1}{2} = 0.500$  for each  $x \in \{u_7, u_8, u_{11}, u_{12}\}$ .

$$(3) \text{cer}_x(C, D) = \frac{\text{supp}_x(C, D)}{|C(x)|} = \frac{3}{3} = 1.000 \text{ for each } x \in \{u_2, u_4, u_{15}\}.$$

$$(4) \text{cer}_x(C, D) = \frac{\text{supp}_x(C, D)}{|C(x)|} = \frac{2}{2} = 1.000 \text{ for each } x \in \{u_3, u_5\}.$$

$$(5) \text{cer}_x(C, D) = \frac{\text{supp}_x(C, D)}{|C(x)|} = \frac{2}{3} = 0.667 \text{ for each } x \in \{u_6, u_{17}\}.$$

$$(6) \text{cer}_x(C, D) = \frac{\text{supp}_x(C, D)}{|C(x)|} = \frac{1}{3} = 0.333 \text{ for } x = u_{16}.$$

**Theorem 4.8:** The following hold for information system  $S = (U, C \cup D, V, f)$  and  $x \in U$ .

$$(1) \text{cov}_x(C, D) = \frac{\text{supp}_x(C, D)}{|D(x)|} = \frac{1}{12} = 0.083 \text{ for each } x \in \{u_1, u_{13}, u_{14}, u_{16}, u_{18}, u_{19}, u_{20}\}.$$

$$(2) \text{cov}_x(C, D) = \frac{\text{supp}_x(C, D)}{|D(x)|} = \frac{1}{4} = 0.250 \text{ for each } x \in \{u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}.$$

$$(3) \text{cov}_x(C, D) = \frac{\text{supp}_x(C, D)}{|D(x)|} = \frac{3}{12} = 0.250 \text{ for each } x \in \{u_2, u_4, u_{15}\}.$$

$$(4) \text{cov}_x(C, D) = \frac{\text{supp}_x(C, D)}{|D(x)|} = \frac{2}{12} = 0.167 \text{ for each } x \in \{u_3, u_5\}.$$

$$(5) \text{cov}_x(C, D) = \frac{\text{supp}_x(C, D)}{|D(x)|} = \frac{2}{4} = 0.500 \text{ for each } x \in \{u_6, u_{17}\}.$$

**Remark 4.9:** (1) If  $x \in \{u_1, u_2, u_3, u_4, u_5, u_9, u_{10}, u_{13}, u_{14}, u_{15}, u_{18}, u_{19}, u_{20}\}$ , then  $C \rightarrow_x D$  is a certain decision rule in  $S = (U, C \cup D, V, f)$ .

(2) If  $x \in \{u_6, u_7, u_8, u_{11}, u_{12}, u_{16}, u_{17}\}$ , then  $C \rightarrow_x D$  is an uncertain decision rule in  $S = (U, C \cup D, V, f)$ .

Theorem 4.4, Theorem 4.5, Theorem 4.7 and Theorem 4.8 can be expressed as Table II (characterizations of Decision Rules), which give characterization of decision rules for information system  $S = (U, C \cup D, V, f)$ .

TABLE II  
CHARACTERIZATIONS OF DECISION RULES

$U$	support	strength	certainty	coverage
$u_1$	1	0.050	1.000	0.083
$u_2$	3	0.150	1.000	0.250
$u_3$	2	0.100	1.000	0.167
$u_4$	3	0.150	1.000	0.250
$u_5$	2	0.100	1.000	0.167
$u_6$	2	0.100	0.667	0.500
$u_7$	1	0.050	0.500	0.250
$u_8$	1	0.050	0.500	0.250
$u_9$	1	0.050	1.000	0.250
$u_{10}$	1	0.050	1.000	0.250
$u_{11}$	1	0.050	0.500	0.250
$u_{12}$	1	0.050	0.500	0.250
$u_{13}$	1	0.050	1.000	0.083
$u_{14}$	1	0.050	1.000	0.083
$u_{15}$	3	0.150	1.000	0.250
$u_{16}$	1	0.050	0.333	0.083
$u_{17}$	2	0.100	0.667	0.500
$u_{18}$	1	0.050	1.000	0.083
$u_{19}$	1	0.050	1.000	0.083
$u_{20}$	1	0.050	1.000	0.083

## V. POSTSCRIPT

(1) Although decision rules obtained in this paper deal with single student of the sample of 20 students, they also reflect overall picture of this sample in some aspects. Since this sample is selected at random, these results are still interesting. As stated earlier, this paper gives new analysis of on higher mathematics scores for Chinese university students, which can further lead Chinese university students to raise higher mathematics scores in Chinese graduate student entrance examination.

(2) The investigation in this paper is based on partitions of the finite universe  $U$  of discourse, but by using these partitions we are not able to solve neighboring question in numerical representations for some factor attributes. For example, if a higher mathematics score of some student is 90, then  $d_1$  or  $d_2$  is it? In recent years, the rough Set theory has been developed from partitions of the universe of discourse to covers of the universe of discourse (see [19], [25], for example), which may provide a satisfactory solution for this neighboring question. Further exploratory might be performed towards this direction.

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