# Region Segmentation based on Gaussian Dirichlet Process Mixture Model and its Application to 3D Geometric Stricture Detection

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*Abstract*—In general, image-based 3D scenes can now be found in many popular vision systems, computer games and virtual reality tours. So, It is important to segment ROI (region of interest) from input scenes as a preprocessing step for geometric stricture detection in 3D scene. In this paper, we propose a method for segmenting ROI based on tensor voting and Dirichlet process mixture model. In particular, to estimate geometric structure information for 3D scene from a single outdoor image, we apply the tensor voting and Dirichlet process mixture model to a image segmentation. The tensor voting is used based on the fact that homogeneous region in an image are usually close together on a smooth region and therefore the tokens corresponding to centers of these regions have high saliency values. The proposed approach is a novel nonparametric Bayesian segmentation method using Gaussian Dirichlet process mixture model to automatically segment various natural scenes. Finally, our method can label regions of the input image into coarse categories: "ground", "sky", and "vertical" for 3D application. The experimental results show that our method successfully segments coarse regions in many complex natural scene images for 3D.

*Keywords*—Region segmentation, tensor voting, image-based 3D, geometric structure, Gaussian Dirichlet process mixture model

## I. INTRODUCTION

IN general, image-based rendering during the past decade has advanced the commercial production of virtual models from photographs a reality. Image based 3D modeling can be found in many popular computer games and virtual reality tours. However, the generation of 3D scene from 2D natural scene remains a complicated and time-consuming process, often requiring special equipment, a large number of photographs, manual interaction, or all of them. So, it has given to the professionals and ignored by the general public. In order to solve problems of 3D modeling, various researches have been performed Image-based 3D modeling methods by [1,2,3,4].

To this work, we consider to dealing with outdoor scenes and assume that a scene is consisted in a single ground plane, piece-wise planar objects sticking out of the ground at right angles, and the sky. First of all, we perform simple feature such as pixel colors and filter responses. So, we find uniform region, called 'superpixels" in the input scene [13]. To fine superpixels in this work, we segment uniform region based on Gaussian Dirichlet process mixture model. The goal of scene segmentation is to classification a given input image into homogeneous regions, or pattern classes. The work can be applied to a multitude of important computer vision applications, ranging from vision guided autonomous robotics,

remote sensing to medical diagnosis, and retrieval in large image database. In general, various segmentation approaches have been proposed. These can be largely categorized into four classes: threshold-based, edge or boundary-based, region-based, and model-based techniques [5-6].

Here, we have mainly an interest with a segmentation method using model-based techniques. One of these methods expresses the probability density for the whole data set as a finite mixture model, in case that the mixtures can be constructed with any types of components, but more commonly multivariate Gaussian densities are used. However, most of these algorithms require the analyst to specify the number of classes based either on a priori knowledge or on an educated guess. It is obvious that the quality of resulting segmentation is largely dependent on the exact estimation of mixture components. Hence, we have to determine the optimal number of clusters before analyzing a given data. To solve this problem, various criteria have been proposed in the literature. These criteria are Akaike's information criterion (AIC), Bayesian information criterion (BIC), minimum description length (MDL), cross validation information criterion (CVIC), and covariance inflation criterion (CIC). Nevertheless, these methods are not able to determine automatically the number of components when we segment a given image into several regions.

To resolve these optional issues, a relatively new tool, Dirichlet process mixture (DPM) models have been proposed in machine learning literature. DPM models have emerged as a nonparametric alternative to finite mixture models with theoretically a countable infinite number of mixture components. Eventually, as part of the model-fitting procedure, the nonparametric Bayesian inference scheme induced by the DPM model yields a posterior distribution on the proper number of model component densities, rather than selecting a fixed number of mixture components. Hence, the obtained nonparametric Bayesian formulation eliminates the need for doing inferences about the number of mixture components required for representing the modeled data.

Under this motivation, we propose a novel nonparametric Bayesian segmentation method using Gaussian Dirichlet process mixture model, to automatically segment various color images. This method incorporates both Dirichlet process mixture model as the prior distribution for mixture components and the multivariate Gaussian distribution as the likelihood function of observed data. We have also described an efficient variational Bayesian inference algorithm newly proposed recently to learning the proposed model. And we apply it to a series of color images, demonstrating its advantages over existing methodologies.

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## II. DIRICHLET PROCESS MIXTURE BASED ON GAUSSIAN

### A. Dirichlet Process

The Dirichlet process, denoted as $DP(\alpha, G_0)$, is a random measure on measures and is parameterized by the innovation parameter $\alpha$ and a base distribution $G_0$ [7,8]. That is, for any finite measurable partition $(A_1, A_2, \cdots, A_r)$ of a measurable space $\Theta$, the random vector $(G(A_1), G(A_2), \cdots, G(A_r))$ is distributed as a finite-dimensional Dirichlet distribution with parameters $(\alpha G_0(A_1), \alpha G_0(A_2), \cdots, \alpha G_0(A_r))$:

$(G(A_1), G(A_2), \cdots, G(A_r)) \sim (\alpha G_0(A_1), \alpha G_0(A_2), \cdots, \alpha G_0(A_r))$.

A first interpretation on the Dirichlet process is provided by the Polya urn scheme due to Blackwell and MacQueen (1973). The Polya urn scheme shows that not only are draws from the Dirichlet process discrete, but also that they exhibit a clustering property. Assume we randomly draw a sample distribution G from a $DP(\alpha, G_0)$, and subsequently, we independently draw N random variables $\varphi_1, \varphi_2, \cdots, \varphi_N$ from G

$$G \mid (\alpha, G_0) \sim DP(\alpha, G_0) \qquad (1)$$
$$\varphi_n \mid G \sim G, \quad n = 1, \cdots, N.$$

Integrating out G, the joint distribution of the variables $\varphi_1, \varphi_2, \cdots, \varphi_N$ can be shown to exhibit a cluster effect. Specifically, given the first $N-1$ samples of G, $\varphi_1, \varphi_2, \cdots, \varphi_{N-1}$, it can be shown that a new sample $\varphi_N$ is either drawn from the base distribution $G_0$ with probability $\frac{\alpha}{\alpha + N - 1}$ or selected from the existing draws, according to a multinomial allocation, with probabilities proportional to the number of the previous draws with same allocation. Let $\{\varphi_1^*, \varphi_2^*, \cdots, \varphi_K^*\}$ denote the distinct values of $\varphi_1, \varphi_2, \cdots, \varphi_{N-1}$, and let $\{n_1, n_2, \cdots, n_K\}$ be the number of values in $\varphi_1, \varphi_2, \cdots, \varphi_{N-1}$ that equal to $\varphi_1^*, \varphi_2^*, \cdots, \varphi_K^*$. Then, the conditional distribution of $\varphi_N$ given $\varphi_1, \varphi_2, \cdots, \varphi_{N-1}$ follows a Polya urn scheme and has the following form:

$$p(\varphi_N \mid \{\varphi_n, n = 1, \cdots, N - 1\}, \alpha, G_0) = \frac{\alpha}{\alpha + N - 1} G_0 + \sum_{k=1}^{K} \frac{n_k}{\alpha + N - 1} \delta_{\varphi_k^*} \qquad (2)$$

where $\delta_{\varphi_k^*}$ denotes the distribution concentrated at a single point $\varphi_k^*$. These results illustrate two key properties of the DP scheme. First, the innovation parameter $\alpha$ plays a key role in determining the number of distinct parameter values. A larger $\alpha$ induces a higher tendency of drawing new parameters from the base distribution $G_0$; indeed, as $\alpha \to \infty$, we get $G \to G_0$. On the contrary, as $\alpha \to 0$, all $\varphi_1, \varphi_2, \cdots, \varphi_N$ tend to cluster to a single random variable. Second, the more often a parameter is shared, the more likely it will be shared in the future.

Another characterization of the unconditional distribution of the random variable G drawn from $DP(\alpha, G_0)$ is provided by the stick-breaking construction due to Sethuraman (1994) [8]. The stick-breaking construction is based on two infinite collections of independent random variables $(v_k)_{k=1}^{\infty}$ and $(\theta_k)_{k=1}^{\infty}$:

$$v_k \mid \alpha, G_0 \sim Beta(1, \alpha), \; \theta_k \mid \alpha, G_0 \sim G_0,$$

where $Beta(a, b)$ is the Beta distribution with parameters a and b. The stick-breaking construction of G is then given by

$$G = \sum_{k=1}^{\infty} \pi_k(\mathbf{v}) \delta_{\theta_k}, \qquad (3)$$

where

$$\pi_k(\mathbf{v}) = v_k \prod_{l=1}^{k-1}(1 - v_l) \in [0.1], \; \sum_{k=1}^{\infty} \pi_k(\mathbf{v}) = 1. \quad (4)$$

In this case, we may interpret the sequence $\boldsymbol{\pi} = (\pi_k)_{k=1}^{\infty}$ as a random probability measure on the positive integers. Under the stick-breaking representation of the Dirichlet process, the atoms $\theta_k$, drawn independently from the base distribution $G_0$, can be seen as the parameters of the component distribution of a mixture model comprising an unbounded number of component densities, with mixing proportions $\pi_k(\mathbf{v})$. Sethuraman(1994) showed that G as defined in this way is a random probability measure distributed according to $DP(\alpha, G_0)$[8]. This stick breaking representation of G makes clear that the random measure G drawn from $DP(\alpha, G_0)$ is discrete. It shows explicitly that the support of G consists of a countably infinite sum of atoms located at $\theta_k$, drawn independently from $G_0$.

### B. Gaussian Dirichlet Process Mixture Model

One of the most important applications of the Dirichlet processes is as a nonparametric prior distribution of a mixture model. In particular, suppose that observation $\mathbf{y}_n$ arise as follows:

$$\varphi_n \mid G \sim G, \; \mathbf{y}_n \mid \varphi_n \sim F(\varphi_n)$$

where $F(\varphi_n)$ denotes the distribution of the observation $\mathbf{y}_n$ given $\varphi_n$. The factors $\varphi_n$ are conditionally independent given G, and the observation $y_n$ is conditionally independent of the other observations given the factor $\varphi_n$. When G is distributed according to a Dirichlet process, this model is referred to as a Dirichlet process mixture (DPM) model. Since G can be represented using a stick-breaking construction (3), the factors $\varphi_n$ take on values $\theta_k$ with probability $\pi_k(v)$. We may denote this using an indicator variable $\mathbf{z}_n$, which takes on positive integral values $\{k | k = 1, \cdots, \infty\}$ and is distributed according to $\boldsymbol{\pi} = (\pi_k)_{k=1}^{\infty}$.

Next, suppose that we have a set of d-dimensional independent multivariate observations $\mathbf{Y} = \{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_N\}$. We want to model this data by mans of nonparametric Bayesian formulation of Gaussian Dirichlet process mixture (GDPM) model. For this purpose, since the number of mixture component K is unknown, we have to consider the mixture model with countably infinite components. Therefore, we will use the Diriclet process mixture model as the prior distribution over the number of components generating the data, and we also assume the probability distribution of observations as the multivariate Gaussian distribution. Moreover, introducing a set of latent variables $\mathbf{Z} = \{z_1, z_2, \cdots, z_N\}$ indicating the component labels associated with the observation data as defined on above. Then, the GDPM model for the observed data set can be described as follows. First, we have used the multi-dimensional Gaussian distribution with parameter $\boldsymbol{\Theta}_k = (\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k)$ for the likelihoods of the observations;

$$\mathbf{y}_n \mid z_n = k; \boldsymbol{\Theta}_k \sim \mathcal{N}(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k^{-1}). \qquad (5)$$

Second for the prior distribution of total cluster memberships, we assume that

$$p(z_1, \cdots, z_N) \mid \boldsymbol{\pi}(\mathbf{v})) = \prod_{n=1}^{N} p(z_n \mid \boldsymbol{\pi}(\mathbf{v}))$$

where $p(z_n = k \mid \boldsymbol{\pi}(\mathbf{v}))$ is the prior probabilities of the cluster membership stemming from the imposed Drichlet process, that is,

$$p(z_n = k \mid \boldsymbol{\pi}(\mathbf{v})) \sim Multi(\boldsymbol{\pi}(\mathbf{v})),$$

$Multi(\boldsymbol{\pi}(\mathbf{v}))$ denotes the multinomial distribution over $\boldsymbol{\pi}(\mathbf{v})$.

Third, the probabilities of countable infinite number of components in the mixture model is given by the stick-breaking representation of the DP, that is,

$$v_k \mid \alpha, G_0 \sim \text{Beta}(1, \alpha), k = 1, \cdots, \infty$$
$$\pi_k(\mathbf{v}) = v_k \prod_{l=1}^{k-1} (1 - v_l) \in [0.1], \quad \sum_{k=1}^{\infty} \pi_k(\mathbf{v}) = 1. \quad (6)$$

Forth, Bayesian inference for the assumed GDPM model involves the assumption of a set of appropriate priors over the model parameters, and derivation of the corresponding posterior densities. We choose the conjugate-exponential prior distributions over the model parameters. Hence, we impose a joint normal-Wishart distribution over the mean vector and precision matrix for a multivariate Gaussian distribution of the model component as follows:

$$\boldsymbol{\mu}_k \mid \boldsymbol{\Lambda}_k^{-1} \sim \mathcal{N}(\boldsymbol{\mu}_k \mid \mathbf{m}_k, (\lambda_k \boldsymbol{\Lambda}_k)^{-1}) \quad (7)$$
$$\boldsymbol{\Lambda}_k \sim \mathcal{W}(\boldsymbol{\Lambda}_k \mid \omega_k, \boldsymbol{\Psi}_k).$$

Finally, taking under consideration the effect of effective mixture components of the GDPM model, we choose to also impose a hyper-prior over the innovation hyper-parameter α of the GDPM model. We use a Gamma prior with parameter $\eta_1$ and $\eta_2$:

$$\alpha \mid \eta_1, \eta_2 \sim \Gamma(\alpha \mid \eta_1, \eta_2). \quad (8)$$

Hence, the joint probability of latent variables and all parameters considered up to now can be rewritten as

$$p(\mathbf{Y}, \mathbf{Z}, \boldsymbol{\pi}(\mathbf{v}), \alpha, \boldsymbol{\mu}, \boldsymbol{\Lambda})$$
$$= p(\mathbf{Y} \mid \mathbf{Z}, \boldsymbol{\mu}, \boldsymbol{\Lambda}) \, p(\mathbf{Z} \mid \boldsymbol{\pi}(\mathbf{v})) p(\mathbf{v} \mid \alpha) p(\boldsymbol{\mu} \mid \boldsymbol{\Lambda}) \, p(\boldsymbol{\Lambda}) \quad (9)$$

where the individual factors are

$$p(\mathbf{Y} \mid \mathbf{Z}, \boldsymbol{\mu}, \boldsymbol{\Lambda}) = \prod_{n=1}^{N} \prod_{k=1}^{\infty} \mathcal{N}(\mathbf{y}_n \mid \boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k^{-1})^{z_{nk}}$$
$$p(\mathbf{Z} \mid \boldsymbol{\pi}(\mathbf{v})) = \prod_{n=1}^{N} p(z_n \mid \boldsymbol{\pi}(\mathbf{v})),$$
$$p(z_n \mid \boldsymbol{\pi}(\mathbf{v})) = \prod_{k=1}^{\infty} \pi_k(v)^{z_{nk}}, \, z_{nk} \in \{0,1\} \quad (10)$$
$$p(\mathbf{v} \mid \alpha) = \prod_{k=1}^{\infty} \alpha (1 - v_k)^{\alpha-1}, \, v_k \in [0,1]$$
$$p(\boldsymbol{\mu}_k \mid \boldsymbol{\Lambda}_k) = \mathcal{N}(\boldsymbol{\mu}_k \mid \mathbf{m}_k, (\lambda_k \boldsymbol{\Lambda}_k)^{-1}), \, p(\boldsymbol{\Lambda}_k) = \mathcal{W}(\boldsymbol{\Lambda}_k \mid \omega_k, \boldsymbol{\Psi}_k).$$
(10)

and

$$\mathcal{N}(\mathbf{y} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{d/2}} \frac{1}{|\boldsymbol{\Sigma}|^{1/2}} \times \exp\left(-\frac{1}{2}(\mathbf{y} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{y} - \boldsymbol{\mu})\right)$$

$$\mathcal{W}(\boldsymbol{\Lambda} \mid \omega, \boldsymbol{\Psi}) = B(\omega, \boldsymbol{\Psi}) |\boldsymbol{\Lambda}|^{(\omega-d-1)/2} \times \exp\left(-\frac{1}{2}\text{Tr}(\boldsymbol{\Psi}^{-1}\boldsymbol{\Lambda})\right)$$

$$B(\omega, \boldsymbol{\Psi}) = |\boldsymbol{\Psi}|^{-\omega/2} (2^{\omega d/2} \pi^{d(d-1)/4} \times \prod_{i=1}^{d} \Gamma(\frac{\omega+1-i}{2}))^{-1}.$$

## III. VARIATIONAL BAYESIAN INFERENCE

Inference for the GDPM model can be conducted based on a Bayesian setting, typically by means of variational Bayesian methods. Variational Bayesian inference implies that the actual posterior distribution $p(\boldsymbol{\Phi} \mid \boldsymbol{\Xi}, \mathbf{Y})$ over a set of all hidden variables and unknown parameters of GDPM model, $\boldsymbol{\Phi} = \{\mathbf{Z}, \mathbf{v}, \alpha, \boldsymbol{\mu}, \boldsymbol{\Lambda}\}$ given an observed date set, $\mathbf{Y}$ and the set of the hyper-parameters of the assumed priors, $\boldsymbol{\Xi} = \{\lambda, \mathbf{m}, \omega, \boldsymbol{\Psi}, \eta_1, \eta_2\}$ is approximated by a variational Bayesian posterior distribution, $q(\boldsymbol{\Phi})$. To derive the variational Bayesian distribution $q(\boldsymbol{\Phi})$, we consider a well-known equation for the log evidence $\log p(\mathbf{Y} \mid \boldsymbol{\Xi})$. This can be expressed as

$$\log p(\mathbf{Y} \mid \boldsymbol{\Xi}) = \mathcal{L}(q(\boldsymbol{\Phi})) + \mathcal{D}_{KL}(q(\boldsymbol{\Phi}) \| p(\boldsymbol{\Phi} \mid \mathbf{Y}, \boldsymbol{\Xi})) \quad (11)$$

where

$$\mathcal{L}(q(\boldsymbol{\Phi})) = \int q(\boldsymbol{\Phi}) \log \frac{p(\mathbf{Y} \mid \boldsymbol{\Phi}) p(\boldsymbol{\Phi} \mid \boldsymbol{\Xi})}{q(\boldsymbol{\Phi})} d\boldsymbol{\Phi} \quad (12)$$

and

$$\mathcal{D}_{KL}(q(\boldsymbol{\Phi}) \| p(\boldsymbol{\Phi} \mid \mathbf{Y}, \boldsymbol{\Xi}) = \int q(\boldsymbol{\Phi}) \log \frac{q(\boldsymbol{\Phi})}{p(\boldsymbol{\Phi} \mid \mathbf{Y}, \boldsymbol{\Xi})} d\boldsymbol{\Phi}. \quad (13)$$

Here, $\mathcal{D}_{KL}(q(\boldsymbol{\Phi}) \| p(\boldsymbol{\Phi} \mid \mathbf{Y}, \boldsymbol{\Xi})$ stands for the Kullback-Leibler (KL) divergence between the approximate variational posterior $q(\boldsymbol{\Phi})$ and the actual posterior $p(\boldsymbol{\Phi} \mid \mathbf{Y}, \boldsymbol{\Xi})$ and $\mathcal{L}(q(\boldsymbol{\Phi}))$ called

the variational free energy, forms a strict lower bound of the log evidence. Hence, maximizing the variational free energy $\mathcal{L}(q(\boldsymbol{\Phi}))$ is equivalent to minimizing the KL divergence. By appropriate choice of $q(\boldsymbol{\Phi})$, $\mathcal{L}(q(\boldsymbol{\Phi}))$ becomes tractable to compute and to maximize.

For computational convenience, the variational posterior $q(\boldsymbol{\Phi})$ is expressed in a factorized form, with the same form as the priors $p(\boldsymbol{\Phi} \mid \boldsymbol{\Xi})$, and each parameter represented by its own conjugate exponential prior. Furthermore, variational Bayesian inference assumes to formulate under infinite dimensional setting. But, it is actually not tractable. For this reason, we employ a common strategy in DPM literature, formulated on the basis of a truncated stick-breaking representation of the DP. That is, we fix a value K and we let the variational posterior over the stick-breaking random variables $v_k$ have the property $q(v_K = 1) = 1$. This implies that the mixture proportions $\pi_k(v)$ are equal to zero for $k > K$. Therefore, for GDPM model proposed in this paper, the variational Bayesian posterior is given as the following form:

$$q(\boldsymbol{\Phi}) = \prod_{n=1}^{N} q(z_n) q(\alpha) \prod_{k=1}^{K-1} q(v_k) \prod_{k=1}^{K} q(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k). \quad (14)$$

Then, substituting (10) and (14) into (12), we have the following variational free energy for our model;

$$\mathcal{L}(q(\boldsymbol{\Phi})) = \sum_{k=1}^{K} \int \int q(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k) \ln \frac{p(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k \mid \lambda_k, \mathbf{m}_k, \omega_k, \boldsymbol{\Psi}_k)}{q(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k)} d\boldsymbol{\mu}_k d\boldsymbol{\Lambda}_k$$
$$+ \int q(\alpha) \{ \ln \frac{p(\alpha \mid \eta_1, \eta_2)}{q(\alpha)} + \sum_{k=1}^{K-1} \int q(v_k) \ln \frac{p(v_k \mid \alpha)}{q(v_k)} dv_k \} d\alpha \quad (15)$$
$$+ \sum_{k=1}^{K} \sum_{n=1}^{N} q(z_n = k) \{ \int q(\mathbf{v}) \ln \frac{p(z_n = k \mid \boldsymbol{\pi}(v))}{q(z_n = k)} d\mathbf{v}$$
$$+ \int \int q(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k) \ln p(\mathbf{y}_n \mid \boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k) d\boldsymbol{\mu}_k d\boldsymbol{\Lambda}_k \}$$

Derivation of the optimal variational posterior distribution $q(\boldsymbol{\Phi})$ involves the maximization of the variational free energy over each one of the factors $q(\boldsymbol{\Phi}_j)$ of $q(\boldsymbol{\Phi})$ in turn, holding the others fixed. Using the calculus of variations, it can be shown that the best distribution $q(\boldsymbol{\Phi}_j)^*$ for each of the factors can be expressed as:

$$q(\boldsymbol{\Phi}_j) = \exp(E_{i \neq j}[\log p(\mathbf{Y}, \boldsymbol{\Phi} \mid \boldsymbol{\Xi})])/Z_j,$$

where $Z_j$ denotes the normalized constant of variational distribution $q(\boldsymbol{\Phi}_j)$. Hence, the update equations for the variational posteriors of each factors are given as follows.

(1) The variational posterior of mixture component indicator variable

$$q(z_n = k) =$$
$$\exp\{ E_{[\boldsymbol{\pi}(v)]}[\ln p(z_n = k \mid \boldsymbol{\pi}(v)] + E_{[\boldsymbol{\mu}, \boldsymbol{\Lambda}]}[\ln p(\mathbf{y}_n \mid z_n = k, \boldsymbol{\mu}, \boldsymbol{\Lambda}] \}$$
$$= \hat{\pi}_k(v) \times \hat{p}(\mathbf{y}_n \mid \boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k) \quad (16)$$

Where

$$\widehat{\pi_k}(v) = \exp\{ E_{[q(v)]}[\ln p(z_n = k \mid \boldsymbol{\pi}(v))] \}$$
$$= \exp\{ E_{[q(v)]}[\ln v_k] + \sum_{i=1}^{k-1} E_{[q(v)]}[\ln(1 - v_i)] \}$$
$$\hat{p}(\mathbf{y}_n \mid \boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k) = \exp\{-\frac{1}{2}\ln 2\pi + \frac{1}{2} E_{[q(\boldsymbol{\Lambda}_k)]}[\ln |\boldsymbol{\Lambda}_k|]$$
$$- \frac{1}{2} E_{[q(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k)]}[(\mathbf{y}_n - \boldsymbol{\mu}_k)^T \boldsymbol{\Lambda}_k (\mathbf{y}_n - \boldsymbol{\mu}_k)] \},$$

and

$$E_{[q(v)]}[\ln v_k] = \psi(\hat{\beta}_{k,1}) - \psi(\hat{\beta}_{k,1} + \hat{\beta}_{k,2}),$$
$$E_{[q(v)]}[\ln(1 - v_k)] = \psi(\hat{\beta}_{k,2}) - \psi(\hat{\beta}_{k,1} + \hat{\beta}_{k,2}),$$
$$E_{[q(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k)]}[(\mathbf{y}_n - \boldsymbol{\mu}_k)^T \boldsymbol{\Lambda}_k (\mathbf{y}_n - \boldsymbol{\mu}_k)]$$
$$= \frac{d}{\lambda_k} + \omega_k (\mathbf{y}_n - \mathbf{m}_k)^T \boldsymbol{\Psi}_k^{-1} (\mathbf{y}_n - \mathbf{m}_k)$$
$$E_{[q(\boldsymbol{\Lambda}_k)]}[\ln |\boldsymbol{\Lambda}_k|] = -\ln |\frac{\boldsymbol{\Psi}_k}{2}| + \sum_{k=1}^{d} \psi(\frac{\omega_k + 1 - k}{2}),$$

where $\psi(\cdot)$ is the Digamma function.

(2) The variational posterior of the DP parameters

$$q(\alpha) = \exp\{\ln p(\alpha) + E_{[q(\mathbf{v})]}[\ln p(\mathbf{v}\,|\alpha)]\}$$

where

$$E_{[q(\mathbf{v})]}[\ln p(\mathbf{v}\,|\alpha)] = \sum_{k=1}^{K-1} E_{[q(\mathbf{v})]}[\ln p(v_k\,|\alpha)] .$$

Hence,

$$q(\alpha) = \Gamma(\hat{\eta}_1, \hat{\eta}_2), \qquad (17)$$

where

$$\hat{\eta}_1 = \eta_1 + K - 1, \hat{\eta}_2 = \eta_2 - \sum_{k=1}^{K-1}[\psi(\hat{\beta}_{k,2}) - \psi(\hat{\beta}_{k,1} + \hat{\beta}_{k,2})]$$

(3) The variational posterior of stick-breaking variables

$$q(v_k) = \exp\{E_{[q(\mathbf{z})]}[\ln p(\mathbf{z}\,|\boldsymbol{\pi}(\mathbf{v}))] + E_{[q(\alpha)]}[\ln p(v_k\,|\alpha)]\}$$

where

$$E_{[q(\mathbf{z})]}[\ln p(\mathbf{z}\,|\boldsymbol{\pi}(\mathbf{v}))] = \sum_{n=1}^{N} q(z_n = k)$$

$$E_{[q(\alpha)]}[\ln p(v_k\,|\alpha)] = E_{[q(\alpha)]}[\alpha] + \sum_{l=k+1}^{K}\sum_{n=1}^{N} q(z_n = l) .$$

Hence,

$$q(v_k) = Beta(\hat{\beta}_{k,1}, \hat{\beta}_{k,2}) \quad k = 1, \cdots, K-1 \qquad (18)$$

where

$$\hat{\beta}_{k,1} = 1 + \sum_{n=1}^{N} q(z_n = k), \hat{\beta}_{k,2} = \frac{\hat{\eta}_1}{\hat{\eta}_2} + \sum_{l=k+1}^{K}\sum_{n=1}^{N} q(z_n = l) .$$

(4) The variational posterior of the likelihood function parameters

$$q(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k) = \exp\{\ln p(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k) + \sum_{n=1}^{N} q(z_n = k)(\ln p(\mathbf{y}_n\,|\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k)\} .$$

Hence,

$$q(\boldsymbol{\mu}_k, \boldsymbol{\Lambda}_k) = \mathcal{N}(\boldsymbol{\mu}_k|\mathbf{m}_k, (\lambda_k\boldsymbol{\Lambda}_k)^{-1})\mathcal{W}(\boldsymbol{\Lambda}_k\,|\boldsymbol{\Psi}_k, \omega_k , \qquad (19)$$

where

$$N_k = \sum_{n=1}^{N} q(z_n = k) , \ \lambda_k = \lambda_0 + N_k, \ \ \omega_k = \omega_0 + N_k$$

$$\bar{\mathbf{y}}_k = \frac{1}{N_k}\sum_{n=1}^{N} q(z_n = k)\,\mathbf{y}_n ,$$

$$\mathbf{S}_k = \sum_{n=1}^{N} q(z_n = k)\,(\mathbf{y}_n - \bar{\mathbf{y}}_k)(\mathbf{y}_n - \bar{\mathbf{y}}_k)^{\mathsf{T}}$$

$$\mathbf{m}_k = \frac{1}{\lambda_k}(\lambda_0\mathbf{m}_0 + N_k\bar{\mathbf{y}}_k),$$

$$\boldsymbol{\Psi}_k = \boldsymbol{\Psi}_0 + \mathbf{S}_k + \frac{\lambda_0 N_k}{\lambda_0 + N_k}(\bar{\mathbf{y}}_k - \mathbf{m}_k)(\bar{\mathbf{y}}_k - \mathbf{m}_k)^{\mathsf{T}}$$

As a last step, after updating the posterior distributions (16)-(19) using the variational Bayesian inference algorithm for the GDPM model at each iteration, we use a Bayesian rule which allocates each pixel to one of regions in accordance with their posterior probabilities to segment a given color image. That is, every pixel is assigned to the class having the highest posterior probability that the observation originated from this class.

## IV. 3D GEOMETRY ESTIMATION

In this section, we present an automatic approach for creating a 3D model based on region segmentation by statistic model from a single scene. The model is made up of several texture-mapped plannar billboards and has the complexity of a typical children's pop-up book illustration. The proposed core technology is that we are based on statistical-model geometric features defined by their orientation components in the image instead of attempting to recover precise geometry. First of all, regions are created by labeling of the segmented input image into coarse categories: "ground", "sky", and "vertical". In the second step, each label is used to "cut and fold" the image into a pop-up model using a set of simple assumptions. In general, we can show the results for creating virtual walkthroughs that is completely automatic and requires only a single photograph as input scene [13].

## V. EXPERIMENTAL RESULTS

First, to verify the application of GDPM model to image segmentation, we have used various color images data. Figure 1(a) shows the color images used at our experiment and Figure 1(b) also shows the results of segmentation for color images using proposed model. From the experimental results, we note that our algorithm manage to discriminate exactly each objects in color image.



(a) Color images (b) Segmentation results
Fig. 1 Results of segmentation for color images using the proposed approach

We can observe that the GDPM model is able to converge with the optimal likelihood function without dependent on assumed initial values for model parameters. Therefore, this model can classify or partition exactly each pixels into proper regions, and we can obtain the excellent segmented regions.

In order to test the performance of the proposed method, we use Hoiem's publicly available code to generate the 3D model from an image based MATLABE [13]. Fig. 2 shows the qualitative results of the proposed method on several images. Therefore, we can set out with the goal of automatically creating visually pleasing 3D models for a 2D scene of an outdoor image. We can create beautiful 3D scenes for various images.

Fig. 2 Input scenes and novel views taken from automatically
generated 3D models

## VI. CONCLUSIONS

In this paper, we present automatically creating visually pleasing 3D models from a single 2D image of an outdoor scene. The proposed approach can observe single-view modeling paves the way for a new class of applications. First, in order to segment of ROI from natural scene, we apply new segmentation method based GDPM model, which can automatically determinate the number of mixture components at a unsupervised segmentation. The method uses the variational Bayesian inference method recently very often used, and we have conducted to segment various color images by using the trained GDPM model. Therefore, the experimental results indicate that the proposed method can be effective in 3D modeling with natural single scene.

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